

Cofunction Identities:

$$\begin{aligned} \sin\left(\frac{\pi}{2} - x\right) &= \cos x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x & \sec\left(\frac{\pi}{2} - x\right) &= \csc x \\ \cos\left(\frac{\pi}{2} - x\right) &= \sin x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x & \csc\left(\frac{\pi}{2} - x\right) &= \sec x \end{aligned}$$

Addition Formulas:

$$\begin{aligned} \sin(x + y) &= \sin x \cos y + \cos x \sin y \\ \cos(x + y) &= \cos x \cos y - \sin x \sin y \\ \tan(x + y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y} \end{aligned}$$

Subtraction Formulas:

$$\begin{aligned} \sin(x - y) &= \sin x \cos y - \cos x \sin y \\ \cos(x - y) &= \cos x \cos y + \sin x \sin y \\ \tan(x - y) &= \frac{\tan x - \tan y}{1 + \tan x \tan y} \end{aligned}$$

Sums of Sines and Cosines:

$A \sin x + B \cos x = k \sin(x + \phi)$ where $k = \sqrt{A^2 + B^2}$ and ϕ satisfies

$$\cos \phi = \frac{A}{\sqrt{A^2 + B^2}} \text{ and } \sin \phi = \frac{B}{\sqrt{A^2 + B^2}}$$

Double-Angle Formulas:

$$\begin{aligned} \sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x} \end{aligned}$$

Half-Angle Formulas:

$$\begin{aligned} \sin \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{2}} \\ \cos \frac{x}{2} &= \pm \sqrt{\frac{1 + \cos x}{2}} \\ \tan \frac{x}{2} &= \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x} \end{aligned}$$

Formulas for lowering powers:

$$\begin{aligned} \sin^2 x &= \frac{1 - \cos 2x}{2} \\ \cos^2 x &= \frac{1 + \cos 2x}{2} \\ \tan^2 x &= \frac{1 - \cos 2x}{1 + \cos 2x} \end{aligned}$$

Product-to-Sum Formulas:

$$\begin{aligned} \sin x \cos y &= \frac{1}{2}(\sin(x + y) + \sin(x - y)) \\ \cos x \sin y &= \frac{1}{2}(\sin(x + y) - \sin(x - y)) \\ \cos x \cos y &= \frac{1}{2}(\cos(x + y) + \cos(x - y)) \\ \sin x \sin y &= \frac{1}{2}(\cos(x - y) - \cos(x + y)) \end{aligned}$$

Sum-to-Product Formulas:

$$\begin{aligned} \sin x + \sin y &= 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} \\ \sin x - \sin y &= 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2} \\ \cos x + \cos y &= 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \\ \cos x - \cos y &= -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2} \end{aligned}$$