

1. Find the midpoint and distance of the points  $(2, 3)$  and  $(-3, 5)$

Solution:

$$\text{Midpoint: } \left( \frac{2+(-3)}{2}, \frac{3+5}{2} \right) = \left( -\frac{1}{2}, 4 \right)$$

$$\text{Distance: } \sqrt{(-3-2)^2 + (5-3)^2} = \sqrt{5^2 + 2^2} = \sqrt{29}$$

2. Find a point on the  $y$ -axis that is equidistant from the points  $(2, 0)$  and  $(3, -5)$

Solution:

Since the point is on the  $y$ -axis it has the form  $(0, y)$ .

$$\text{The distance from the first point is } \sqrt{(2-0)^2 + (0-y)^2} = \sqrt{4 + y^2}$$

$$\text{The distance from the second point is } \sqrt{(3-0)^2 + (-5-y)^2} = \sqrt{9 + 25 + 10y + y^2} = \sqrt{34 + 10y + y^2}$$

Since it is equidistant from both points, we get  $4 + y^2 = 34 + 10y + y^2$

We solve this equation:

$$4 = 34 + 10y$$

$$10y = -30$$

$$y = -3$$

Hence the point is  $(0, -3)$ .

3. Find the  $x$ - and  $y$ -intercepts and test for symmetry:  $y = 16 - x^4$

Solution:

$x$ -intercepts: set  $y = 0$ :  $0 = 16 - x^4 \rightarrow x^4 = 16 \rightarrow x = \pm 2$

$y$ -intercepts: set  $x = 0$ :  $y = 16 - 0^4 = 16$

Symmetry:

$x$ -axis:  $-y = 16 - x^4$  not the same, so no symmetry

$y$ -axis:  $y = 16 - (-x)^4 = 16 - x^4$  same, so there is symmetry

origin:  $-y = 16 - (-x)^4 = 16 - x^4$  not the same, so no symmetry

4. Find the center and radius of the circle  $x^2 + y^2 - 2x + y + 1 = 0$

Solution:

Complete the square:

$$x^2 - 2x + 1 + y^2 + y + \frac{1}{4} = -1 + 1 + \frac{1}{4}$$

$$(x - 1)^2 + (y + \frac{1}{2})^2 = \frac{1}{4}$$

Hence the center is  $(1, -\frac{1}{2})$  and the radius is  $\sqrt{\frac{1}{4}} = \frac{1}{2}$

5. Find the slope-intercept form of the line through the point  $(3, -2)$  perpendicular to the line through  $(1, 4)$  and  $(-2, 3)$

Solution:

First find the slope of the line:  $m = \frac{3-4}{-2-1} = \frac{-1}{-3} = \frac{1}{3}$

Since we are looking for the perpendicular line, the slope is

$$m = -\frac{1}{\frac{1}{3}} = -3$$

Now use the point-slope form to set up an equation of the line:

$$y - (-2) = -3(x - 3)$$

Bring it into slope-intercept form:  $y = -3x + 9 - 2 = -3x + 7$

6. Find the domains of the following functions:

$$(a) f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x - 2 & \text{if } 0 < x \leq 2 \end{cases}$$

Solution:

The function is only defined for  $x < 0$  and  $0 < x \leq 2$ , hence the domain is  $(-\infty, 0) \cup (0, 2]$

$$(b) g(x) = \sqrt{x^2 - 4x}$$

Solution:

The term under the root cannot be negative, so  $x^2 - 4x \geq 0$

Solve this inequality:

$$x(x - 4) \geq 0$$

Hence the critical points are 0 and 4.

Check the intervals:

between  $-\infty$  and 0: test  $-1$ : positive  $\rightarrow$  part of the domain

between 0 and 4: test 2: negative  $\rightarrow$  not part of the domain

between 4 and  $\infty$ : test 5: positive  $\rightarrow$  part of the domain

Hence the domain is  $(-\infty, 0] \cup [4, \infty)$