

1. Solve the following equations:

(a)  $x^{4/3} - 3x^{4/3} - 4 = 0$

Solution:

Use substitution:  $u = x^{2/3}$

Then  $u^2 - 3u - 4 = 0$

Factor:  $(u - 4)(u + 1) = 0$

Hence  $u = 4$  or  $u = -1$

Now find  $x$ :  $x^{2/3} = 4$  or  $x^{2/3} = -1$

From the first equation:  $x = \pm 4^{3/2} = \pm(\sqrt{4})^3 = \pm 8$

The second equation does not have any real solutions, but it would have the complex solutions  $\pm i$

(b)  $\sqrt{2x + 1} + 1 = x$

Solution:

First isolate the root:  $\sqrt{2x + 1} = x - 1$

Now square both sides:  $2x + 1 = (x - 1)^2 = x^2 - 2x + 1$

Now move everything to one side:  $x^2 - 4x = 0$

Factor:  $x(x - 4) = 0$

Hence  $x = 0$  or  $x = 4$

Check solutions:

0:  $\sqrt{2 \cdot 0 + 1} + 1 = 1 + 1 = 2 \neq 0 \rightarrow$  no solution

4:  $\sqrt{2 \cdot 4 + 1} + 1 = 3 + 1 = 4 \rightarrow$  solution

Hence the only solution is 4.

2. Solve the following inequalities:

(a)  $-2 < 8 - 2x < -1$

Solution:

Subtract 8:  $-10 < -2x < -9$

Divide by  $-2$  (this will reverse the direction of the inequality):

$$5 > x > \frac{9}{2}$$

Hence the solution set is  $(\frac{9}{2}, 5)$

(b)  $\frac{3}{x-1} - \frac{4}{x} \geq 1$

Solution:

Move everything to one side:  $\frac{3}{x-1} - \frac{4}{x} - 1 \geq 0$

Combine terms:  $\frac{3x-4(x-1)-x(x-1)}{x(x-1)} \geq 0$

$$\frac{3x-4x+4-x^2+x}{x(x-1)} \geq 0$$

$$\frac{-x^2+4}{x(x-1)} \geq 0$$

$$\frac{-(x+2)(x-2)}{x(x-1)} \geq 0$$

Hence the critical numbers are  $-2, 0, 1, 2$

Now put these values on the number line and test points inbetween:

$-3$ :  $\frac{-(-3+2)(-3-2)}{-3(-3-1)} < 0 \rightarrow$  not part of the solution

$-1$ :  $\frac{-(-1+2)(-1-2)}{-1(-1-1)} > 0 \rightarrow$  part of the solution

$\frac{1}{2}$ :  $\frac{-(\frac{1}{2}+2)(\frac{1}{2}-2)}{\frac{1}{2}(\frac{1}{2}-1)} < 0 \rightarrow$  not part of the solution

$\frac{3}{2}$ :  $\frac{-(\frac{3}{2}+2)(\frac{3}{2}-2)}{\frac{3}{2}(\frac{3}{2}-1)} > 0 \rightarrow$  part of the solution

$3$ :  $\frac{-(3+2)(3-2)}{3(3-1)} < 0 \rightarrow$  not part of the solution

Now check the endpoints:

$0$  and  $1$  are not part of the solution, since it is undefined

$-2$  and  $2$  are part of the solution, since it is  $\geq$

Hence the solution set is  $[-2, 0) \cup (1, 2]$

3. Solve  $3|x + 5| + 6 = 15$  for  $x$

Solution:

Isolate the absolute value:

$$3|x + 5| = 9$$

$$|x + 5| = 3$$

Thus  $x + 5 = 3$  or  $x + 5 = -3$

Hence  $x = -2$  or  $x = -8$

4. Solve  $4|x + 2| - 3 > 13$

Solution:

Isolate the absolute value:

$$4|x + 2| > 16$$

$$|x + 2| > 4$$

Now  $x + 2 > 4$  or  $x + 2 < -4$

Thus  $x > 2$  or  $x < -6$

Hence the solution set is  $(-\infty, -6) \cup (2, \infty)$