1. Solve the following equations completely

   (a) $\sqrt{3}\tan x + 1 = 0$
   Solution:
   $\sqrt{3}\tan x = -1$
   $\tan x = -\frac{1}{\sqrt{3}}$
   Hence $x = \frac{5\pi}{6} + k\pi$

   (b) $2\cos x \sin x - \sqrt{2}\cos x = 0$
   Solution:
   $\cos x(2\sin x - \sqrt{2}) = 0$
   Hence $\cos x = 0$ or $2\sin x - \sqrt{2} = 0$
   Thus $x = \frac{\pi}{2} + k\pi$ or $\sin x = \frac{\sqrt{2}}{2}$
   Thus $x = \frac{\pi}{2} + k\pi$ or $x = \frac{\pi}{4} + 2k\pi$ or $x = \frac{3\pi}{4} + 2k\pi$

   (c) $\sin^2 x = 2\sin x + 3$
   Solution:
   $\sin^2 x - 2\sin x - 3 = 0$
   Substitute $u = \sin x$:
   $u^2 - 2u - 3 = 0$
   $(u-3)(u+1) = 0$
   Hence $u = 3$ or $u = -1$
   Thus $\sin x = 3$ or $\sin x = -1$
   The first one does not have a solution, and the second one gives $x = \frac{3\pi}{2} + 2k\pi$

2. Find all solutions in the interval $[0, 2\pi)$

   (a) $2\sin 3x + 1 = 0$
   Solution:
   $2\sin(3x) = -1$
   $\sin(3x) = -\frac{1}{2}$
   Hence $3x = \frac{7\pi}{6} + 2k\pi$ or $3x = \frac{11\pi}{6} + 2k\pi$
   Thus $x = \frac{7\pi}{18} + \frac{2k\pi}{3}$ or $x = \frac{11\pi}{18} + \frac{2k\pi}{3}$
   Now the solutions between 0 and 2π are:
   $\frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}$

   (b) $\cos 3x = \sin 3x$
   Solution:
   Divide both sides by $\cos(3x)$ to get:
   $\tan(3x) = 1$
   Hence $3x = \frac{\pi}{4} + k\pi$
   Thus $x = \frac{\pi}{12} + \frac{k\pi}{3}$
   Now the solutions between 0 and 2π are:
   $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$

   (c) $2\cos^2 x + \sin x = 1$
   Solution:
   First replace $\cos^2 x$ by $1 - \sin^2 x$:
   $2(1 - \sin^2 x) + \sin x = 1$
   $2 - 2\sin^2 x + \sin x = 1$
   $2\sin^2 x - \sin x - 1 = 0$
   Now substitute $u = \sin x$:
   $2u^2 - u - 1 = 0$
   $(2u + 1)(u - 1) = 0$
   Hence $u = -\frac{1}{2}$ or $u = 1$
   Thus $\sin x = -\frac{1}{2}$ or $\sin x = 1$
   Then the solutions between 0 and 2π are:
   $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$