

1.1 Basic Equations

- Fractional Equations:
 - Find numbers that would make the denominator equal to 0
 - Multiply by LCD
- Using Radicals: Solution of $x^{\frac{n}{m}} = a$ is
 - $x = \pm \sqrt[n]{a^m}$ if n is even
 - $x = \sqrt[n]{a^m}$ if n is odd

1.2 Modeling with Equations

- Read the problem several times
- Figure out what you have to find
- *Define variable(s)*
- Write down all the information, sketch a diagram if necessary

1.3 Quadratic Equations

- Factoring
- Completing the square
- Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Discriminant: $D = b^2 - 4ac$
 - if $D > 0$ then the equation has two real solutions
 - if $D = 0$ then the equation has one real solution (of multiplicity 2)
 - if $D < 0$ then the equation has two complex solutions

1.4 Complex Numbers

- Addition/Subtraction: $(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$
- Multiplication: $(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$
- Conjugate of $a + bi$: $a - bi$
- Division: Multiply by conjugate: $\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \cdot \frac{c-di}{c-di} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$
- Square roots of negative numbers: $\sqrt{-a} = \sqrt{a} \cdot i$

1.5 Other Types of Equations

- Polynomial Equations
- Factoring by grouping
- Equations with Radicals: Isolate the root on one side, then square/cube/etc.
- Substitution

1.6 Inequalities

- Make the right side equal to 0
- Find the critical points: where the left side is either 0 or undefined
- Put the critical points on the number line
- Use test points
- If multiplying/dividing by a negative number reverse the direction of the inequality

1.7 Absolute Value Equations and Inequalities

- Equations:
 - Isolate the absolute value on one side
 - $|x| = C$ implies $x = \pm C$
- Inequalities:
 - Isolate the absolute value on one side
 - $|x| < C$ implies $-C < x < C$
 - $|x| > C$ implies $x > C$ or $x < -C$

2.1 The Coordinate Plane

- Distance formula: $d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

2.2 Graphs of Equations in two Variables

- Finding intercepts:
 - x -intercepts: set y equal to 0
 - y -intercepts: set x equal to 0
- Equation of a circle: $(x - x_C)^2 + (y - y_C)^2 = r^2$
- Symmetry:
 - x -axis: equation unchanged if y is replaced by $-y$
 - y -axis: equation unchanged if x is replaced by $-x$
 - origin: equation unchanged if x is replaced by $-x$ and y by $-y$

2.4 Lines

- Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Slope of a vertical line is undefined, slope of a horizontal line is 0
- Point-Slope Form: $y - y_P = m(x - x_P)$
- Slope-Intercept Form: $y = mx + b$
- Vertical line: $x = a$, Horizontal line: $y = b$
- Parallel lines: same slope
- Perpendicular lines: $m_2 = -\frac{1}{m_1}$

3.1 What is a function

- Domain: all possible x -values. Finding the domain: Find potential problems that could make the function undefined such as
 - Roots (even power): radicand cannot be negative
 - Fractions: denominator cannot be 0
- Range: all possible y -values: Finding the range: Graph the function

3.2 Graphs of Functions

- Piece-wise defined functions
- Vertical line test: if a vertical line crosses two points of the graph, then the graph is not a function
- Test equations if they define a function:
 - Solve for y
 - Check if y is unique
- Memorize the graphs of the functions on page 226

3.3 Increasing and Decreasing Functions, Average Rate of Change

- Increasing and Decreasing Functions
- Average rate of change: $= \frac{f(b)-f(a)}{b-a}$

3.4 Transformations of Functions

- Vertical shift: $f(x) + c$
- Horizontal shift: $f(x - c)$
- Reflection in x -axis: $-f(x)$
- Reflection in y -axis: $f(-x)$
- Vertical stretch($c > 1$)/shrink($c < 1$) by factor c : $cf(x)$
- Horizontal stretch($c < 1$)/shrink($c > 1$) by factor $1/c$: $f(cx)$
- Even function: symmetric with respect to y -axis: $f(x) = f(-x)$
- Odd function: symmetric with respect to origin: $-f(x) = f(-x)$

3.5 Quadratic Functions, Maxima and Minima

- $f(x) = ax^2 + bx + c$
- Maximum if $a < 0$, Minimum if $a > 0$
- x -value of Max/Min is $-\frac{b}{2a}$

3.6 Combining Functions

- Addition/Subtraction/Multiplication/Division of functions: Add/Subtract/etc. the y -values
- Composition of functions: $(f \circ g)(x) = f(g(x))$
- Domains of combined functions

3.7 One-to-one Functions and their Inverses

- One-to-one functions: no two different x -values have the same y -value
- Horizontal line test: if a horizontal line crosses two points of the graph, the function is not one-to-one
- Inverse function: $f^{-1}(y) = x \leftrightarrow f(x) = y$
- Finding inverses:
 - Reverse x and y
 - Solve for y
- Properties:
 - $f(f^{-1}(x)) = x$
 - $f^{-1}(f(x)) = x$

4.1 Polynomial Functions and their Graphs

- End behavior:
 - Look at leading coefficient/exponent and check sign
 - If polynomial is factored, check sign of each factor and multiply
- Graphing a polynomial:
 - Factor
 - Find x - and y -intercepts
 - Find end behavior
 - Either use test points between the intercepts or memorize the shape around zeros depending on the multiplicity:
 - If multiplicity is 1, then it crosses the x -axis in a straight line
 - If multiplicity is even, then it turns back around
 - If multiplicity is odd > 1 , then it "squiggles" through the x -axis

4.2 Dividing Polynomials

- Long Division: Make sure to fill in missing powers
- Synthetic Division: Only works for division by $(x - c)$. Again make sure to fill in 0's for missing powers
- Remainder Theorem: to find $P(c)$ carry out a synthetic division for c , the remainder is $P(c)$
- Factor Theorem: c is a zero of $P \leftrightarrow (x - c)$ is a factor of $P(x)$

4.3 Real Zeros of Polynomials

- Rational Zeros Theorem: The possible rational zeros of a polynomial are of the form $\frac{p}{q}$ where p is a factor of the constant coefficient a_0 and q is a factor of the leading coefficient a_n
- How to find all zeros of a polynomial:
 - Try previous factoring methods like substitution or grouping, if this does not work then:
 - List all possible rational zeros using the Rational Zeros Theorem
 - Test the possible zeros
 - If you find a zero, factor it out
 - Repeat from the top until your polynomial is quadratic, then factor/complete the square/quadratic formula

4.4 Complex Zeros and the Fundamental Theorem of Algebra

- Fundamental Theorem of Algebra: every polynomial of degree n has precisely n zeros (zeros of multiplicity k are counted k times)
- Conjugate Zeros Theorem: If a complex number is a zero of polynomial with real coefficient, then its conjugate is also a zero

4.5 Rational Functions

- Horizontal asymptotes: n is the degree of the numerator, m is the degree of the denominator
 - $n > m$: no horizontal asymptote
 - $n = m$: horizontal asymptote is $y = \frac{a_n}{b_m}$
 - $n < m$: horizontal asymptote is $y = 0$
- Vertical asymptotes: zeros of the denominator (that do not cancel with the numerator)
- Graphing rational functions:
 - Factor numerator and denominator
 - Find x - and y -intercepts
 - Find horizontal and vertical asymptotes
 - Either use test points between intercepts/vertical asymptotes or use the shape around vertical asymptotes/intercepts to determine the shape of the graph
- Slant asymptote: only exists if the degree of the numerator is one greater than the degree of the denominator: use long/synthetic division

5.1 Exponential Functions

- $f(x) = a^x$, memorize the graph:
 - Horizontal asymptote $y = 0$
 - no vertical asymptote
 - Domain = $(-\infty, \infty)$
 - Range = $(0, \infty)$
- Compound interest formula: $A(t) = P(1 + \frac{r}{n})^{nt}$
- Continuously compounded interest: $A(t) = e^{rt}$

5.2 Logarithmic Functions

- Definition of logarithm: $\log_b a = x \leftrightarrow b^x = a$
- Properties:
 - $\log_b 1 = 0$
 - $\log_b b = 1$
 - $\log_b b^x = x$
 - $b^{\log_b x} = x$
- $f(x) = \log_b x$, memorize the graph:
 - Vertical asymptote: $x = 0$
 - no horizontal asymptote
 - Domain = $(0, \infty)$
 - Range = $(-\infty, \infty)$
- Finding the domain of logarithmic function: logarithms only defined for positive numbers
- Common log: $\log x = \log_{10} x$
- Natural log: $\ln x = \log_e x$

5.3 Laws of Logarithms

- $\log_b(xy) = \log_b x + \log_b y$
- $\log_b \frac{x}{y} = \log_b x - \log_b y$
- $\log_b x^y = y \log_b x$
- no laws for $\log_b(x + y)$ or $\log_b x \cdot \log_b y$
- Change of base: $\log_b x = \frac{\log_c x}{\log_c b}$ where c can be any positive number

5.4 Exponential and Logarithmic Equations

- Solving exponential equations:
 - Isolate the exponential term on one side
 - Take logarithm of both sides:
 - If there is only one exponential term, use that base for the log
 - If there is an exponential term on both sides, use either the common or natural log
 - Pull the exponent to the front and solve the equation
- Solving logarithmic equations:
 - If there are multiple logarithmic terms, combine them into one using logarithmic laws
 - Isolate the logarithmic term on one side
 - Raise the base of the logarithm to the left and the right side of the equation
 - Use the property $b^{\log_b x} = x$ to get rid of the log
 - Solve the equation
- Two special cases of exponential equations:
 - Combination of exponential and polynomial terms: try to factor
 - Sum of multiple exponential terms: try to use substitution

6.1 Angle Measure

- Relationship between Degrees and Radians:
 - convert from degrees to radians by multiplying by $\frac{\pi}{180}$
 - convert from radians to degrees by multiplying by $\frac{180}{\pi}$
- Coterminal angles: Angle between 0° and 360° degrees (or 0 and 2π)
- Length of a circular arc: $s = r\theta$ (θ in rad)
- Area of a circular sector: $A = \frac{1}{2}r^2\theta$ (θ in rad)
- Linear Speed and Angular Speed: $\omega = \frac{\theta}{t}$ and $v = \frac{s}{t}$
- Relationship between linear and angular speed: $v = r\omega$

6.2 Trigonometry of Right Triangles

- Trigonometric Ratios:
 - $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 - $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
 - $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
 - $\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$
 - $\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$
 - $\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$
- Values of the trig ratios for angles 30° , 45° and 60°
- Solving right triangles

6.3 Trigonometric Functions of Angles

- Memorize in which quadrants each trig function is positive
- Reference angles: Acute angle formed by x -axis and terminal side
- Using reference angles to evaluate trig functions
- Reciprocal Identities:
 - $\csc \theta = \frac{1}{\sin \theta}$
 - $\sec \theta = \frac{1}{\cos \theta}$
 - $\cot \theta = \frac{1}{\tan \theta}$
 - $\tan \theta = \frac{\sin \theta}{\cos \theta}$
 - $\cot \theta = \frac{\cos \theta}{\sin \theta}$
- Pythagorean Identities:
 - $\sin^2 \theta + \cos^2 \theta = 1$
 - $\tan^2 \theta + 1 = \sec^2 \theta$
 - $\cot^2 \theta + 1 = \csc^2 \theta$
- Expressing trig functions in terms of other trig functions
- Evaluating trig functions using identities
- Area of a Triangle: $\frac{1}{2}ab \sin \theta$ (where θ is the angle between a and b)

6.4 Law of Sines

- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Solving triangles:
 - SAA
 - SSA (either no solution, one solution or two solutions)

6.5 Law of Cosines

- Law of Cosines:
 - $a^2 = b^2 + c^2 - 2bc \cos A$
 - $b^2 = a^2 + c^2 - 2ac \cos B$
 - $c^2 = a^2 + b^2 - 2ab \cos C$

Solving triangles:

- SSS
- SAS
- Navigation: Bearing
- Heron's Formula: Area of a triangle is $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

7.1 The Unit Circle

- Terminal Points
- Reference Numbers

7.2 Trigonometric Functions of Real Numbers

- Definitions of trig functions using unit circle
- Domains of trig functions
- Reciprocal Identities (see 6.3)
- Pythagorean Identities (see 6.3)
- Even-Odd Properties:
 - $\sin(-x) = -\sin x$
 - $\cos(-x) = \cos x$
 - $\tan(-x) = -\tan x$
 - $\csc(-x) = -\csc x$
 - $\sec(-x) = \sec x$
 - $\cot(-x) = -\cot x$

7.3 Trigonometric Graphs

- Periodic Properties:
 - $\sin(x + 2\pi) = \sin x$
 - $\cos(x + 2\pi) = \cos x$
- Graphs of sin and cos
- Transformations
- $y = a \sin k(x - b)$ has amplitude $|a|$, period $\frac{2\pi}{k}$, and phase shift b
- $y = a \cos k(x - b)$ has amplitude $|a|$, period $\frac{2\pi}{k}$, and phase shift b

7.4 More Trigonometric Graphs

- Periodic Properties:
 - $\tan(x + \pi) = \tan x$
 - $\cot(x + \pi) = \cot x$
 - $\csc(x + 2\pi) = \csc x$
 - $\sec(x + 2\pi) = \sec x$
- Graphs of tan, cot, csc, sec
- Transformations

8.1 Trigonometric Identities

- Cofunction Identities:
 - $\sin\left(\frac{\pi}{2} - x\right) = \cos x$
 - $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
 - $\tan\left(\frac{\pi}{2} - x\right) = \cot x$
 - $\csc\left(\frac{\pi}{2} - x\right) = \sec x$
 - $\sec\left(\frac{\pi}{2} - x\right) = \csc x$
 - $\cot\left(\frac{\pi}{2} - x\right) = \tan x$
- Simplifying trig expressions
- Proving trig identities

8.2 Addition and Subtraction Formulas

- Addition Formulas:
 - $\sin(x + y) = \sin x \cos y + \cos x \sin y$
 - $\cos(x + y) = \cos x \cos y - \sin x \sin y$
 - $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- Subtraction Formulas:
 - $\sin(x - y) = \sin x \cos y - \cos x \sin y$
 - $\cos(x - y) = \cos x \cos y + \sin x \sin y$
 - $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
- Sums of Sines and Cosines: change $A \sin x + B \cos x$ to $k \sin(x + \phi)$
 - First calculate $k = \sqrt{A^2 + B^2}$
 - ϕ satisfies $\cos \phi = \frac{A}{k}$ and $\sin \phi = \frac{B}{k}$

8.3 Double-Angle, Half-Angle, and Product-Sum Formulas

- Double-Angle Formulas:

- $\sin 2x = 2 \sin x \cos x$

- $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$

- $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

- Half-Angle Formulas:

- $\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$

- $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$

- $\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$

- Formulas for lowering powers:

- $\sin^2 x = \frac{1 - \cos 2x}{2}$

- $\cos^2 x = \frac{1 + \cos 2x}{2}$

- $\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$

- Product-to-Sum Formulas:

- $\sin x \cos y = \frac{1}{2}(\sin(x + y) + \sin(x - y))$

- $\cos x \sin y = \frac{1}{2}(\sin(x + y) - \sin(x - y))$

- $\cos x \cos y = \frac{1}{2}(\cos(x + y) + \cos(x - y))$

- $\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$

- Sum-to-Product Formulas:

- $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

- $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

- $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$

- $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

8.4 Inverse Trigonometric Functions

- Graphs of \sin^{-1} , \cos^{-1} , \tan^{-1}

- Domains and Ranges of inverse trig functions

- Evaluating expressions involving inverse trig functions

8.5 Trigonometric Equations

- Methods to solve trig equations:
 - Factoring
 - Substitution
 - Using trig identities
 - Squaring (check answers!)