The Effectiveness of Mathematics in the Sciences

For many decades (if not longer) it has been widely ('though not universally) accepted that the effectiveness of mathematics in the sciences, particularly the physical sciences, is a puzzle. A puzzle that is hard to explain and nigh onto a wonderful miracle! Some famous scientists have written famous essays to this effect.[1] The argument, essentially, goes something like this: How can this 'game' of consistently manipulating symbols representing invented abstract concepts have anything to do with the structure of reality as discovered by science?

I strongly disagree with this widely held view and I regard the effectiveness of mathematics in the sciences as to be expected. For me the only puzzle is why the physical sciences had, for so long, been the exclusive domain in which the effectiveness in question was so obvious? Happily, this is no longer the case and the life sciences and social sciences are becoming more and more infused with mathematical formulations and techniques at a rapid rate! Even introductory college textbooks on the mathematical aspects and methods of the life and social sciences have appeared.

But why do I think this development was and is to be expected? It has to do with the nature of mathematics, or, more exactly, with an essential aspect of the nature of mathematics as I see that subject. For, while many different views exist concerning the ultimate nature or exhaustive classification of mathematics, it is, I think, almost universally agreed that, whatever else is involved, mathematics involves the study of systems of relationships abstracted, to varying degrees, from the details of the things related.

Whether the branch of mathematics is general algebra in which the relationships attendent upon the operations of composition via generalized versions of addition and subtraction, multiplication and division, exponentiation and taking roots are studied; or geometry in which the relations among the parts and constituents of generalized spaces and manifolds of arbitrary dimensions are studied; or analysis in which the relations among the constructs emerging from infinite limit processes or

1. Wigner, Eugene P. (1960) "The Unreasonable Effectiveness of Mathematics in the Natural Sciences", *Communications in Pure and Applied Mathematics,* v. 13 No. 1; reprinted in *Symmetries and Reflections Scientific Essays* MIT Press 1967.

non-standard infinitary structures are studied, etc., systems of relationships and their study are pervasive throughout mathematics.

Is it then any surprise that a scientist, struggling to understand relationships observed among the parts and aspects of nature and natural phenomena may discover, in the panorama of mathematical studies, a branch or subdivision of that broad field which is well suited to express and analyze the relationships observed in nature. To be sure, at a critical juncture in the early days of the scientific revolution, after Galileo and Kepler had found the ancient Greek's geometry of parabolas and ellipses just the ticket for expressing and analyzing projectile motion and planetary orbits respectively, the requisite mathematics for the grounding of a general mechanics had to be discovered by the leading scientists, themselves. But they, Descartes, Newton and Leibniz, were also the leading mathematicians of their day and were up to the task of producing Analytic Geometry and the Calculus. We may well regard their genius as a miracle, but, given their genius, the existence of a mathematics of the relationships they needed for their scientific studies is not a source of bafflement. If a system of relationships can be manifested by nature, an abstracted version of such a system of relationships, which we would recognize as a subdivision of mathematics, is bound to exist.

Ever since the days of Descartes, Newton and Leibniz the breadth and depth of mathematics found to be extremely useful and important for both the investigation and expression of understanding of the structure of nature has grown vastly and without cessation. All indications are that the process will continue so long as the sciences themselves continue to grow. And the process is not a puzzle but fully to be expected from the very nature of the mathematical enterprise!