V: The New Cosmology

1. Dark Matter: Of all the new features that characterize what I'm calling "The New Cosmology", the one for which evidence first started to accumulate is the existence of what we now call **dark matter**. As long ago as the 1930's evidence was obtained that the stars within galaxies and the galaxies within galactic clusters were moving too fast to be kept within the larger conglomerate by the gravity that could be generated by the mass within that conglomerate. At first the evidence was difficult enough to gather reliably that it was easier to be skeptical of the evidence than to believe the challenge to mass estimates or our understanding of gravity. But as the years passed and the technology and extent and consistency of the measurements improved it became clear that stars within galaxies and galaxies within clusters moved as if the galaxies were more massive than we could account for - - **by about a factor of six !**

In **III: Stars** we talked about the spectra and temperatures and nuclear reactions and evolution of stars but said very little about the masses of stars. How can anyone asses the masses of stars and, thereby, the masses of galaxies? To answer that question we have to dwell a little on some quantitative aspects of gravity. For this purpose Newton's comparatively simple theory will suffice. According to that theory any two masses, M_1 and M_2, orbiting each other at an 'average' distance, d, from each other due to mutual gravitational attraction and completing their orbit in the time, T, will satisfy the relationship,

$$d^3 / T^2 = (G / 4\pi^2)(M_1 + M_2),$$

where G is Newton's **universal constant of gravitation** which, in Einstein's theory as well, continues to characterize the strength of gravity.

In **I: The Scale of Things** we commented on Kepler's discovery that d^3 / T^2 had the same value for all the planets orbiting the Sun. That result follows from the previous equation when one remembers that all the planetary masses are much smaller than the Sun's mass so that $(M_{Sun} + M_{planet}) \sim M_{Sun}$. If G is known and d and T are measured, this result can be used to calculate the mass of the Sun. Similarly, in the Earth – Moon system, the approximation that $(M_{Earth} + M_{Moon}) \sim M_{Earth}$ enables one to use the equation to estimate the mass of the Earth. In fact, G was measured in an extremely delicate measurement in 1798 by **Henry Cavendish** and we can therefore estimate the masses,

$$M_{Earth} \sim 0.4 \text{ GGMslugs} \sim 6 \times 10^{24} \text{ kgm} ,$$

$$M_{Sun} \sim M_{Sun} \sim 300{,}000\ M_{Earth} \sim 2 \times 10^{30} \text{ kgm.}$$

So far, so good, but what about other stars? What do we know about objects, the acceleration of which is dominated by other stars? Well, in **binary star systems** we have a pair of stars accelerating around one another. The major difference between binary star systems and the systems we've just discussed is that the stars in the binary system are of comparable mass. In that case additional equations can be brought to bear, which always hold but are not important in cases where one of the masses is much larger than the other. The equations refer to a point along the line joining the orbiting masses, called the **center of mass** of the system, which does not accelerate due to the gravity of the masses (**Fig. V.1**). If it accelerates at all, it's due to forces from outside the system. That point lies on the line a distance d_1 from M_1 and a distance d_2 from M_2 where,

$$M_1\, d_1 = M_2\, d_2 ,$$

and

$$d = d_1 + d_2 .$$

If we get to observe a distant mutually orbiting binary star system from the right angle and with sufficient precision, we can identify the location of the center of mass, measure d_1 and d_2 and then use the equations to calculate M_1 and M_2.

Fortunately, astronomers have been lucky enough, over many years, to observe very many binary star systems and they have contained a sufficient variety of star types so that close correlations between stellar mass and luminosity for main sequence stars on the H-R diagram, (**Fig. V.2**), have been established.

By counting and statistical estimates of the number of different star types in galaxies, one can estimate the total stellar mass in a galaxy. Other methods also enable the estimation of the substantial mass of dust clouds in galaxies.

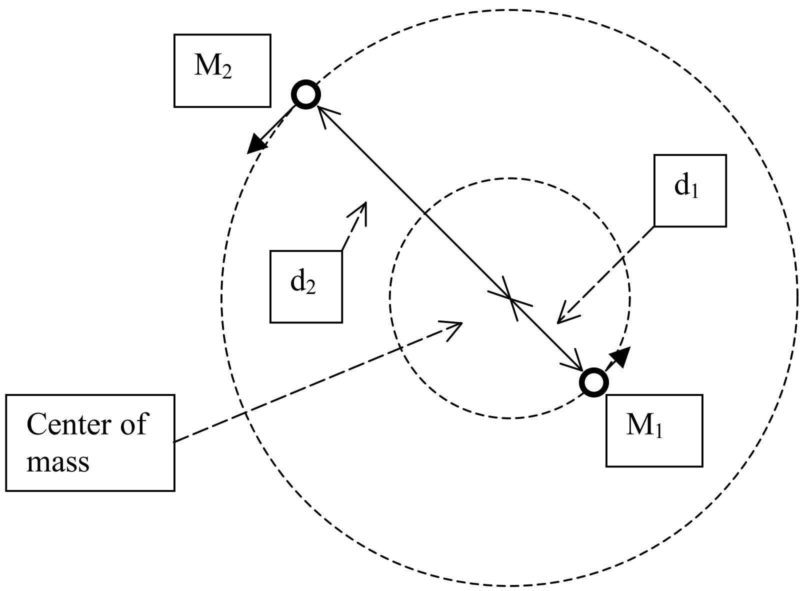


Fig. V.1: Binary star system orbiting about common center of mass. In this example the orbits are circles (usually they're ellipses) and $M_1 > M_2$. As stated in the text, $M_1 d_1 = M_2 d_2$ and $(d_1 + d_2)^3/T^2 = (G/4\pi^2)(M_1 + M_2)$.

Again, in **I. The Scale of Things** we saw (**Fig. I.7**) that our galaxy has a sizeable spherical center full of stars surrounded by a large and comparably thin disk. This is the standard configuration of **Spiral galaxies** and most of the stellar mass, by far, is concentrated in the central sphere. This leads us to expect that the orbital motion of stars in the disks of spiral galaxies would be dominated by the gravitation of the central sphere. If so, then the Kepler-Newton relation we've been discussing for binary star systems should hold in the form,

$$R_{\text{disk star}}{}^3 / T_{\text{disk star}}{}^2 = (G / 4\,\pi^2)\, M_{\text{central sphere}} \;,$$

where $R_{\text{disk star}}$ and $T_{\text{disk star}}$ are the orbital radius and period of a disk star. This requires the orbital period to be proportional to the cube of the square root of the radius, just as is the case for the planets in our solar system (**Fig. V.3a**)(See also p.6 of **I. The Scale of Things**).

In fact, however, the disk stars of spiral galaxies appear to have orbital periods that are simply proportional to the radius itself. In other words, the

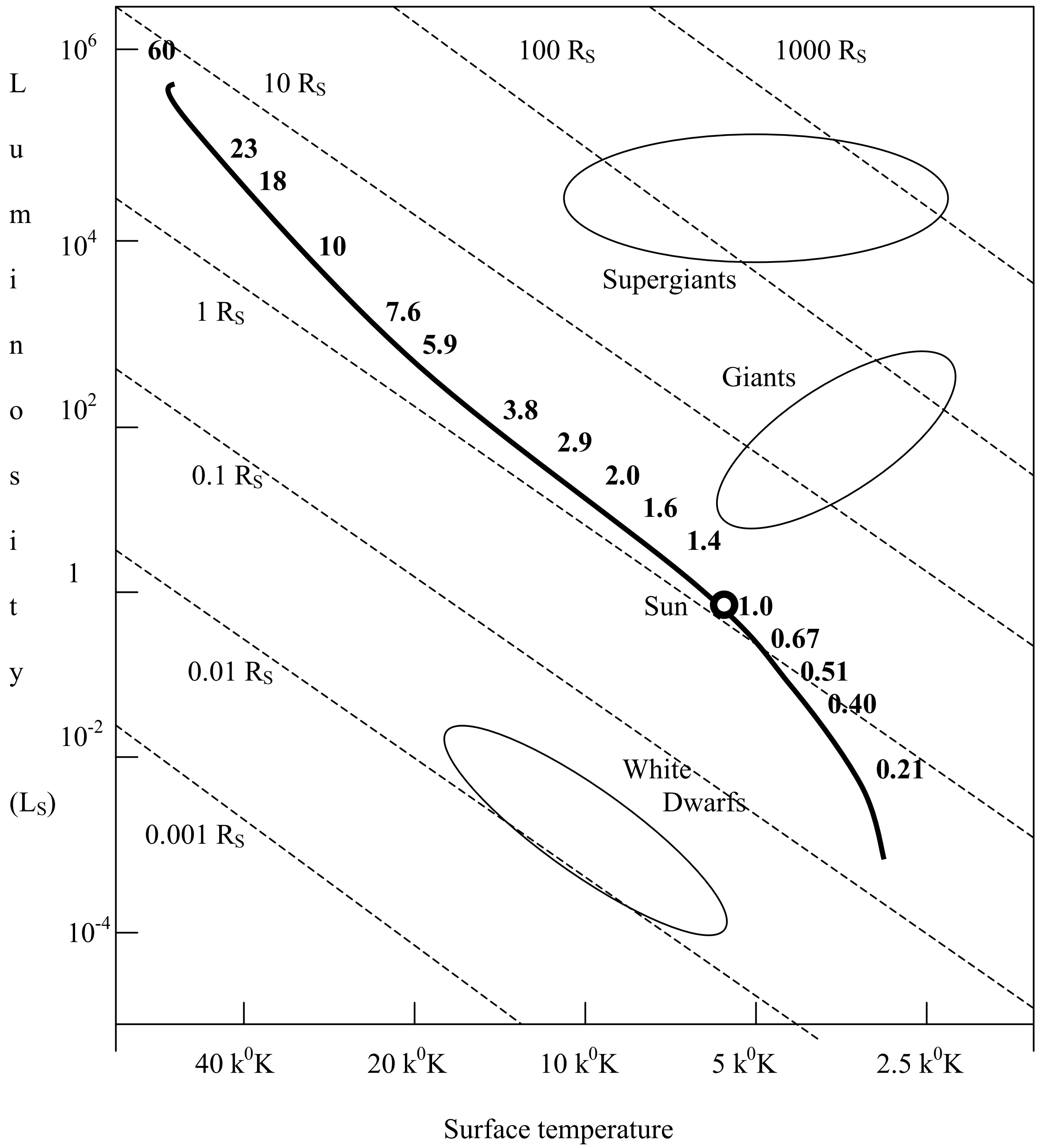
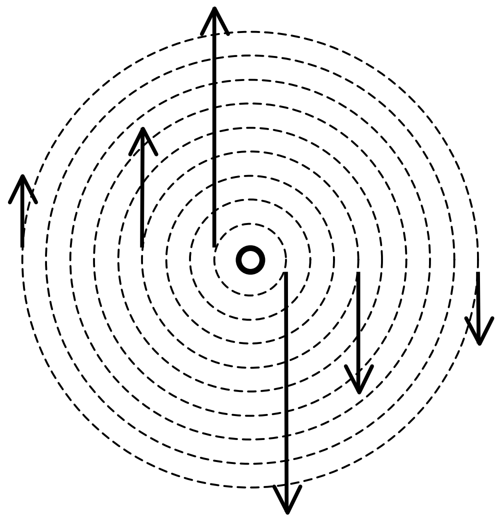


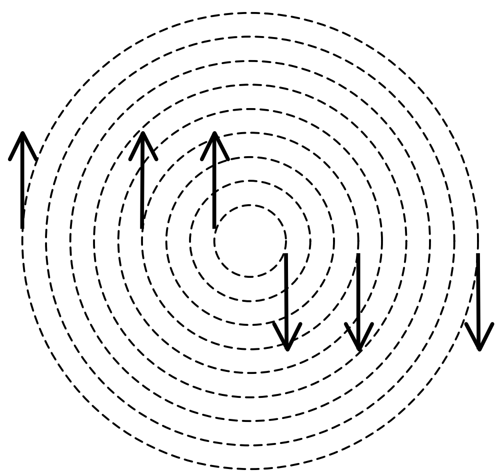
Fig. V.2: The Herzsprung – Russell diagram with typical stellar masses indicated in bold face as multiples of solar mass for main sequence stars.

orbital speeds of the disk stars are *independent* of radius until one gets quite close to the central sphere (**Fig. V.3b**). Notice that this is quite different again from the speeds of points on a rotating solid disk which *increase* proportional to the distance from the center (**Fig. V.3c**).



$$v^2 / r = G\,M / r^2 \qquad \text{or} \qquad v = [G\,M / r]^{1/2}$$

Fig. V.3a: Circular orbital velocities due to centrally concentrated gravitating mass, M.



$$M(r) = C_0\, r :$$

$$v^2 / r = G\,M(r) / r^2 = G\,C_0 / r\,, \quad \text{or} \quad v = [G\,C_0]^{1/2}$$

Fig. V.3b: Circular orbital velocities of disk stars in spiral galaxies indicative of gravitating mass proportional to the radius.

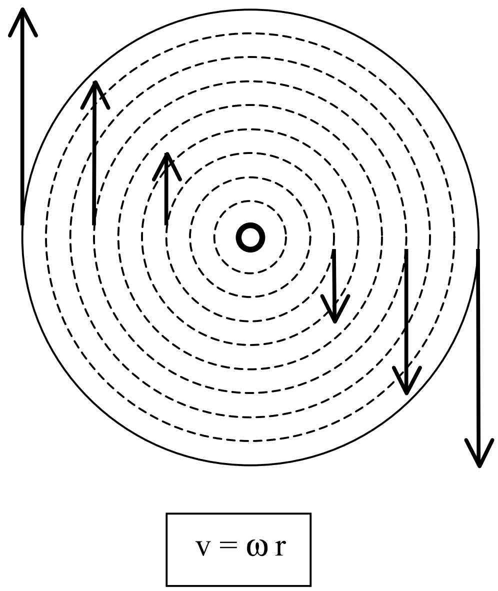


Fig. V.3c: Circular velocities due to rotation of a rigid disk.

In the case of elliptical galaxies the analysis is a bit more complex because the visible mass is not dominated by a central concentration. But the conclusion is the same. The stars within the galaxies move too fast to be held within the galaxies by gravitation of the visible mass alone. Finally, entire galaxies bound gravitationally within galactic clusters are moving sufficiently fast in, presumably, closed orbits that their masses must be much larger than their visible mass to account for their mutual gravity. The data indicates that the requisite additional mass appears to be distributed throughout large *halos* around the individual galaxies with a density that decreases with the inverse square of the distance from the center of the galaxy except for leveling off near the center.

When the numbers are plugged into the equations we find that the additional invisible mass required is about *six times* greater than the ordinary visible mass which includes the stars and the dust clouds. This is a big effect, not a small correction! We call the additional mass **dark matter**. What might it be?!

Many hypotheses have been offered and most are being vigorously investigated. One proposal is that dark matter consists of small very dim stars and/or black holes with masses between $0.01M_{Sun}$ and $1M_{Sun}$. These objects are collectively called Massive Compact Halo Objects, or MACHOS. Another proposal is that dark matter consists of exotic quantum particles of elementary particle theory called Weakly Interacting Massive Particles, or WIMPS. A third proposal suggests that neutrinos, which are very plentiful and very hard to detect and some types of which have recently been found to have very small rest masses, are responsible. So far the evidence is vague and allows one only to make rough estimates of the fractions that the proposed ingredients might contribute to dark matter. We just don't know yet.

A proposal that especially interests me, partly because of its heretical nature, has been advanced by **Mordehai Milgrom**, as long ago as 1983. Milgrom argues that *there is no dark matter* and, instead, it is the laws of mechanics that need to be changed for the circumstances being dealt with here. This proposal, called MOND for Modified Newtonian Dynamics, argues that **Newton's second law of motion,**

$$\mathbf{F} = m\,\mathbf{a}\,,$$

relating resultant, or total, force, **F**, acting on an object, to the mass, m, and acceleration, **a**, of the object (use of boldface means that the *direction* of the force and acceleration is the same), has never been tested at the very small accelerations of stars in galaxies and galaxies in clusters. Milgrom then points out that if, at those very small accelerations, we had,

$$F \sim a^{2},$$

the behaviour of stars and galaxies could be largely accounted for in terms of the visible mass. A lot of the data certainly fits this idea, but not quite all of the data. But *none* of the current hypotheses fit well with *all* the data and I think Milgrom's idea deserves more attention than it's getting. But physicists and astronomers, like everyone else, are reluctant to give up cherished views. So most will pursue MACHOS and WIMPS and neutrinos, etc., unless the data eventually forces them to consider the likes of MOND.

To get a feel for the accelerations involved, let's calculate the Sun's acceleration in its orbit around our galaxy. Assuming a circular orbit with constant speed, v_{Sun}, and radius, r_{Sun}, the acceleration, a_{Sun}, is

$$a_{Sun} = v_{Sun}{}^{2} / r_{Sun} .$$

The speed, on the other hand is just

$$v_{Sun} = 2 \pi r_{Sun} / T_{Sun} .$$

Consequently,

$$a_{Sun} = (2 \pi r_{Sun} / T_{Sun})^{2} / r_{Sun} = 4 \pi^2 r_{Sun} / T_{Sun}{}^{2} .$$

We know that $r_{Sun} \sim 27{,}000$ lgt yr and $T_{Sun} \sim 240$ Myr. So we get,

$$a_{Sun} \sim (40 \text{ x } 270 / 24 \text{ x } 24) \text{ lgt yr} / (\text{Myr})^2$$

$$\sim (0.2 \text{ nm / sec}) / \text{sec} \;\; !!$$

This is about 20 millionths of a millionth of the acceleration of a falling rock near the surface of the Earth. Such small accelerations have never been investigated in a laboratory.

2. Inflation: Sometime after the discovery of the CMB, which we discussed in **Ch. IV**, it was recognized that it possessed properties that were puzzling. It was too uniform directionally! Except for a tiny asymmetry due to a blue Doppler shifting in the direction our Sun was moving towards and a red Doppler shifting in the direction our Sun was moving from, the CMB had the same wavelength spectrum in all directions. Of course, to a first approximation, it was expected that the spectrum would, in all directions, be that of thermal radiation that had cooled to 2.7 ^{0}K as a consequence of the expansion of space since the decoupling time when neutral atoms were formed about 380,000 yr after the BB. And the dominant correction to that approximation was expected to be the Doppler effect due to the Sun's motion. But then, after these features were taken into account, one expected to see small directional deviations due to differing circumstances in the different parts of the fledgling Universe just before the decoupling. No such directional deviations of the expected order of magnitude are seen. For reasons explained below, this is called the **horizon problem**.

It is regarded as a serious problem rather than just a curiosity because it seems impossible that any ordinary process could damp out the differences one would normally expect in the distant early Universe in different directions. The argument goes like this. Consider CMB radiation coming to us from opposite directions. In each direction that radiation decoupled from matter and started cooling at its own rate about 9.3Gyr ago and has just reached us, at the midpoint from the sources, now (**Fig. V.4**). Since light travels through space as fast or faster than any other influence and since

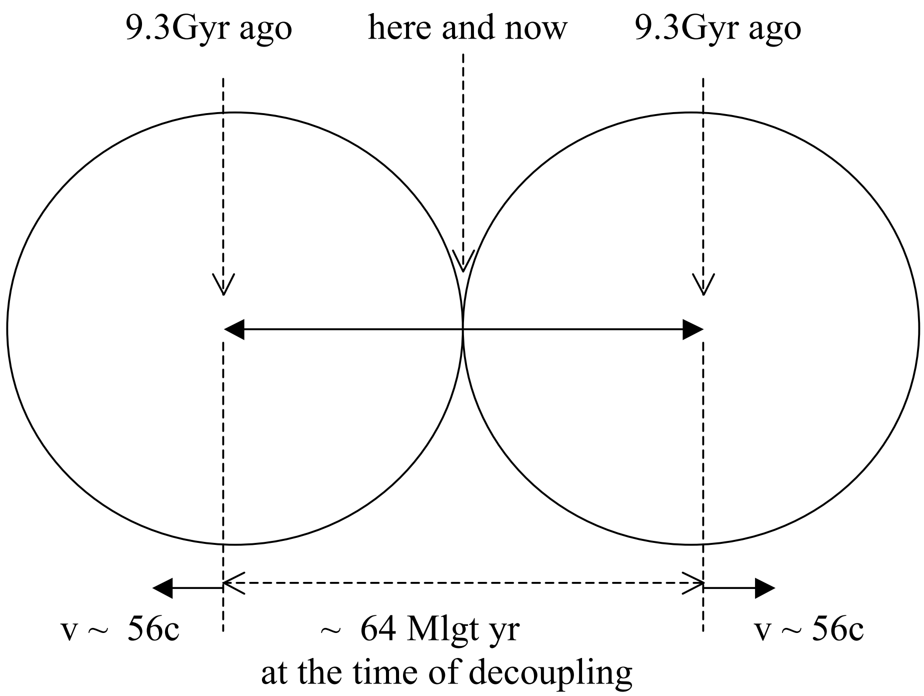
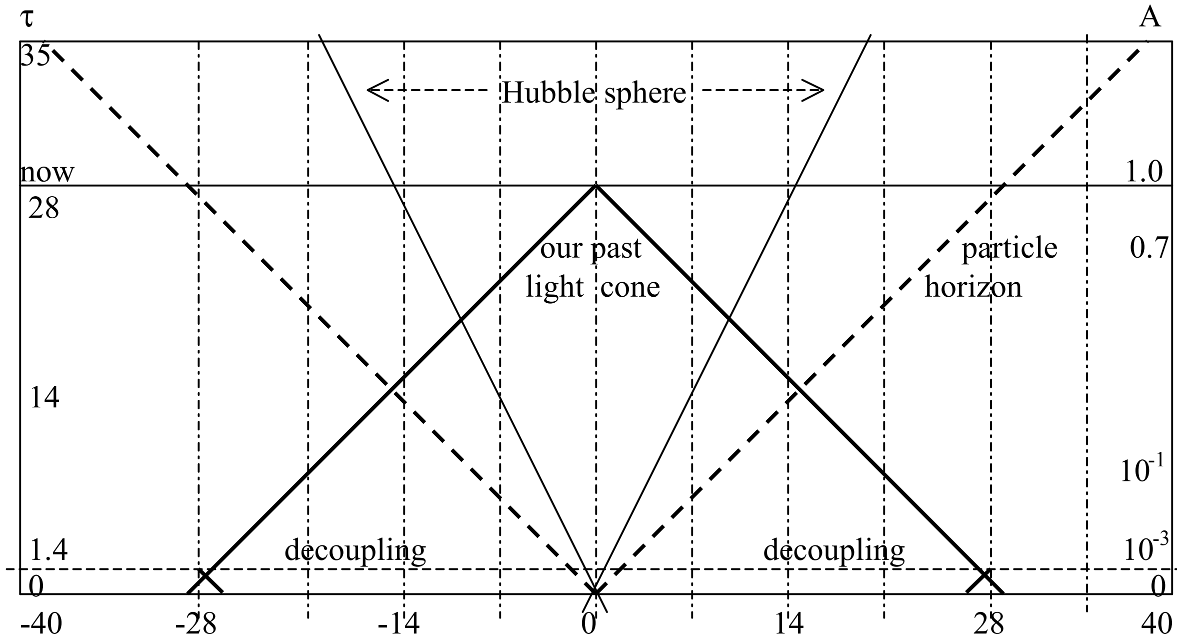


Fig. V.4: Light spheres expanding for ~ 9.3 Gyr from decoupling time sources in opposite directions just reaching us here and now. Since the BB was less than 400 Kyr = 4 x 10^{-4}Gyr earlier than decoupling these directionally opposed sources can not have influenced each other prior to decoupling or suffered common influence from the BB. Notice the recession speeds!

the BB was less than 400 Kyr before the decoupling, there is no way that those source regions could have influenced each other since the BB or have been influenced by common regions of the BB. In other words, their backward light cones (within which all past influences must come) have no overlap (**Fig. V.5**). Admittedly, because of the expansion of space during the time since decoupling, the sources were closer together then than now but, due to the expected deceleration of the expansion, they were moving apart much faster (very superluminally) between the BB and decoupling than they have been since. Consequently, the light from them has only just reached us at the midpoint now – that's why we can see it now –



Proper distance = scale factor x co-moving distance = A r = $(t/9.3\text{Gyr})^{2/3}r$

$$\text{Conformal time} = \tau(t) = \int_0^t dt'/A(t') = 3(9.3\text{Gyr})^{2/3}\, t^{1/3}$$

$H_0 = 1/14\text{Gyr}$ $\qquad\qquad$ $t_U = 9.3\text{Gyr}$

Fig. V.5: Conformal diagram of the (standard, pre inflation) evolution of the expanding Universe. The virtue of the conformal diagram is that the world lines of light rays are straight (conformal speed of light = c). The vertical dashed lines represent receding galaxies at their fixed commoving distances. The past light cone lines represent light rays that we see now. The Hubble sphere lines represent distances beyond which galaxies are receding superluminally (in proper distance / proper time). The dashed particle horizon represents the maximum commoving distance any particle can have covered since the BB. The dashed horizontal line just above the bottom represents the time of decoupling of radiation from matter, at a scale factor of about 10^{-3}, and we notice how distant the two small backward light cones from this time are from each other. For more detail see **Appendix B**.

– and therefore the directionally opposite sources have never been in touch since the BB. In fact, it turns out that the same argument can be made for CMB radiation coming from directions as little different as just over 2^0. So why does the decoupled and cooled CMB coming from these sources look so similar?

Some comments are in order to assist in reading **Fig. V.5**. This is analogous to a Mercator projection map of the Southern hemisphere of the Earth. The BB is the entire horizontal axis at the bottom just as such a Mercator map would represent the South pole as a horizontal line rather than a point. The vertical dotted dashed lines represent mutually receding galaxy world lines at their fixed co-moving coordinates just as Mercator maps represent lines of longitude which actually come together at the poles. The vertical axis on the left here is *conformal time* which goes from zero at the bottom to a bit above 35Gyr at the top where cosmological time is about 20Gyr. On the right side the vertical axis is the scale factor comparing expansion at various conformal times to the present state of expansion. The proper distance between receding galaxies at any time is the product of the commoving distance and the scale factor.

Another puzzle generated by the standard BB – Expanding Universe theory is the so-called **flatness problem**. According to the theory there is a critical value for the average density of matter in the Universe that would result in space being geometrically flat on the largest distance scales. This is a result of a dynamical balance between gravitational attraction from the matter and spatial expansion. This critical density for flat space decreases as space expands since any fixed amount of mass-energy occupies larger and larger volume as space expands. If, now, the average matter density is less than critical, then space should be negatively curved. In both situations the volume of space is infinite. If, on the other hand, the average matter density is greater than the critical value, then space should be positively curved and its volume, finite.

When the best estimates were made for the average matter density, including the dark matter we were discussing in the previous section, the value came out to be about 0.3 times the critical value. So, apparently, space should be negatively curved and infinite. However, Einstein's dynamics for an expanding Universe are such that in order for the ratio of mass density to critical density to be 0.3 now, that ratio would have had to be just *minutely* below 1.0 near the BB. It works both ways. If the ratio were larger than 1.0 near the BB, it would be much larger than 1.0 now. Whichever way the ratio differs from 1.0 near the BB, that difference would be greatly exaggerated in the subsequent evolution of the Universe.

The question, then, is why was the ratio so very close to 1.0 near the BB without being exactly 1.0? You might well shrug your shoulders and say – it just *was*! It's just an accident that the ratio happened to have the value near the BB that results in the value of 0.3 now. But if they can, most scientists tend to prefer to see situations in ways that demand an explanation. For most physicists and cosmologists the required extreme closeness of the ratio to 1.0 near the BB is too severe to be just a coincidence.

Accordingly, in 1984, **Allen Guth** argued that a possible phenomena, occurring *very early* after the BB and called by Guth "**inflation",** could account for both the horizon problem of the CMB and the flatness problem. Inflation may be likened to a period of enormously rapid expansion of a tiny 'bubble' of space (including much more than would become the presently observable Universe) that occurred about 10^{-36}sec after the BB. Furthermore, it lasted for only about 1000 x 10^{-36}sec. during which the linear dimensions of the expanding 'bubble' of space may have doubled every 5 x 10^{-36}sec, leading to two hundred doublings in radius for a total expansion by a factor of more than 1 followed by fifty zeros, 10^{50}, **Fig. V.6**!

Setting aside for the moment the question of *why* anything so outrageous as this should occur, let us look at some of the consequences. The depression of the scale factor by 10^{-50} as we pass backwards in time from 10^{-33}sec to 10^{-36} sec results in an enormous growth in the conformal time variable from the BB to 10^{-33}sec. This completely changes the character of the conformal diagram, **Fig. V.5**, so that the backward lightcones of the CMB emission points have ample overlap, **Fig. V.7**. Thus the horizon problem disappears because now the regions of the Universe in opposite directions emitting CMB radiation from the decoupling epoch that reaches us now *were* in thermal contact prior to inflation since those regions were soooo much closer together originally than could be the case with ordinary (and by comparison, snail's pace) expansion. The flatness problem disappears because the brief inflationary period drives almost any initial curvature of the expanding space extremely close to zero in analogy to the way in which any fixed small area of a balloon surface gets flatter and flatter as the balloon expands.

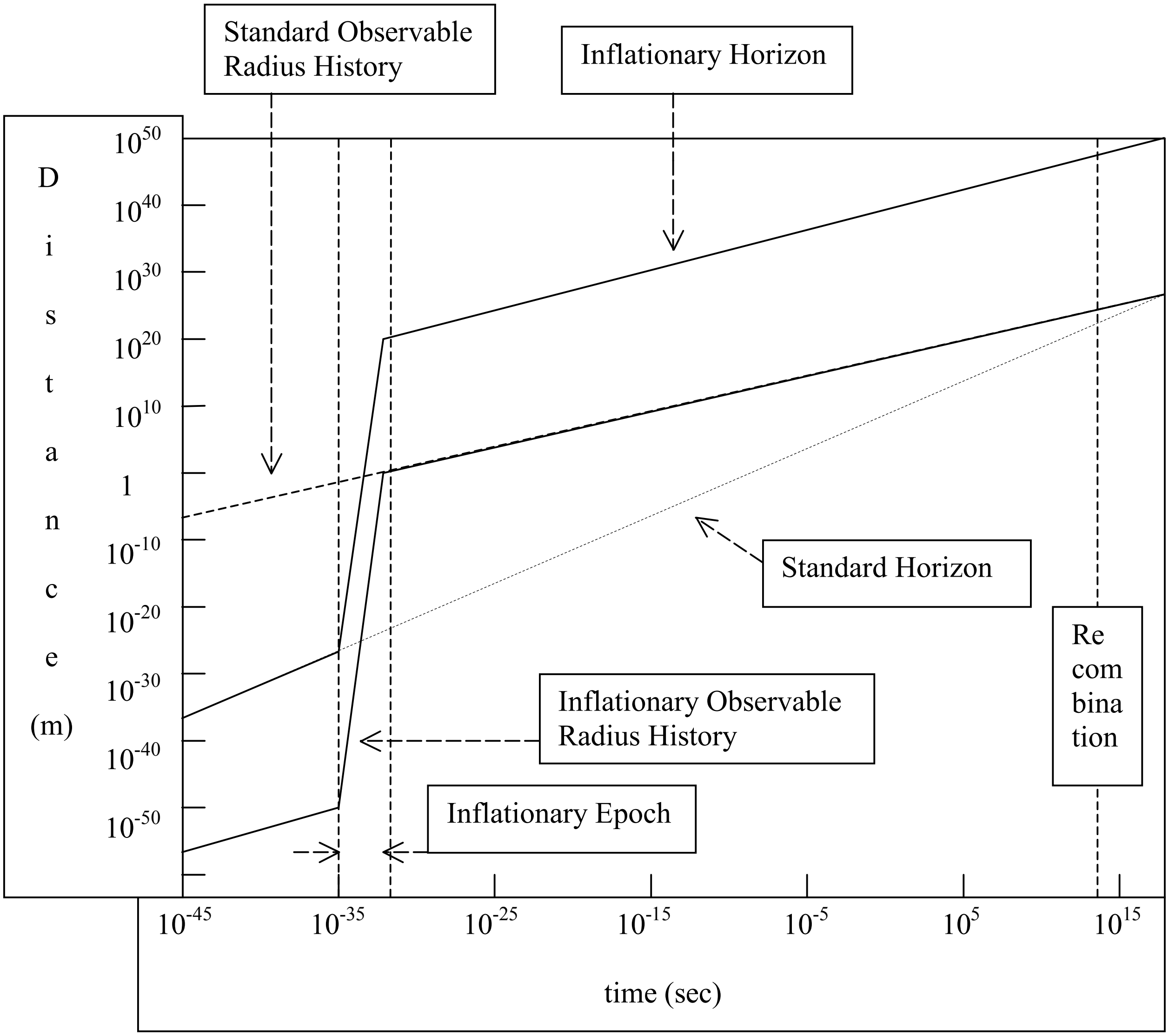


Fig. V.6: Inflationary Expansion compared to Standard Expansion. The curves (solid for Inflation, dashed for Standard) represent the horizon beyond which we can not observe and the past history of the distance of that portion of space *presently* just at the standard horizon.

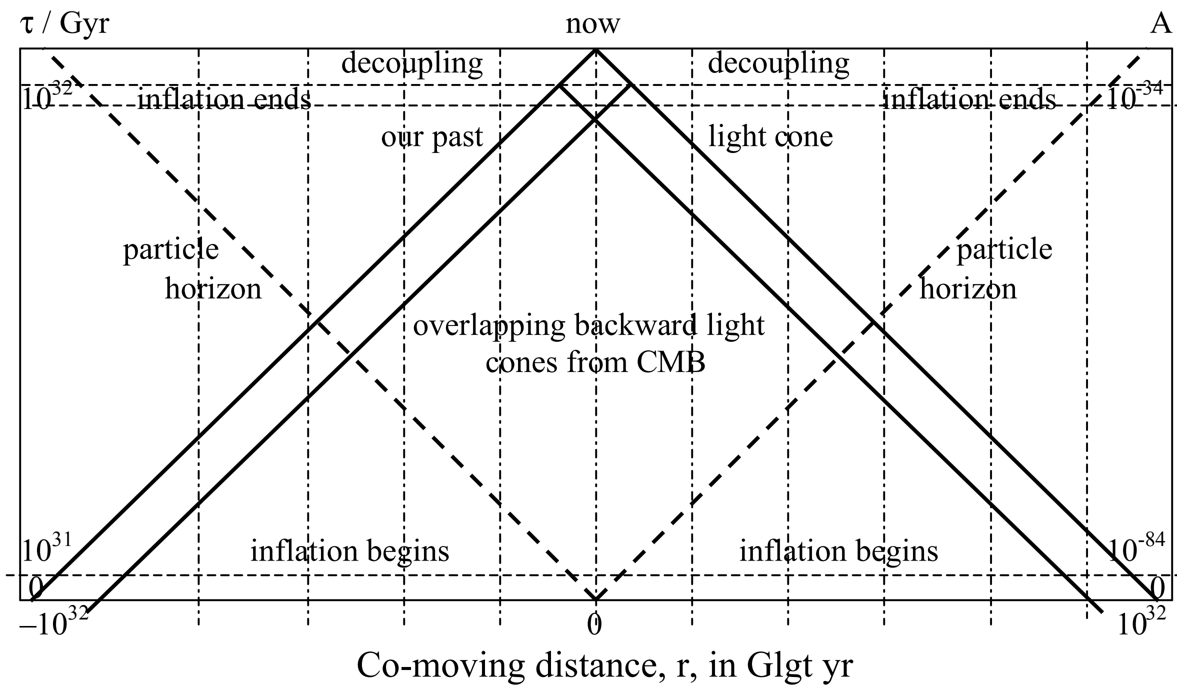


Fig. V.7: Conformal diagram of the inflationary evolution of the Universe. All of the previous conformal diagram is crammed into the top region above the second dashed line (greatly exaggerated). The backward light cones from the CMB emissions from opposite directions now have enormous overlap.

There were other problems, as well, with the ordinary expanding Universe, which we have not discussed and which inflation seemed to eliminate, especially after the theory was revamped into the ‘New Inflationary Scenario’. Because of these successes, physicists were prepared to take seriously the reasoning Guth offered for the occurrence of early inflationary expansion. A sketch of his arguments follows.

Prior to the beginning of inflation the Universe was *extremely* hot!! Guth was able to argue from the recognized laws of **quantum field theory**, (an amalgam of Quantum Theory and Special Relativity which, in the present context, is being somewhat artificially squeezed into the environment of General Relativity) that as the temperature dropped below about 10^{27} ^{0}K a natural ‘phase change’ for space itself could occur but might well be delayed until a significantly cooler temperature was reached. This phase change was called a change from the “false vacuum” to the “true vacuum”. This would be analogous to cooling liquid water below its freezing temperature without the phase change to ice taking place. With delicacy this can be done and the water is then said to be **supercooled**. By analogy we might call supercooled water, “false ice”. Eventually, the change to “true” ice starts at some tiny nucleation site and then expands through the water with extreme rapidity! Similarly, the inflating ‘bubble’ of space is the result of a delayed, supercooled phase change spreading explosively. Much more explosively than the original BB! And just as freezing water liberates so-called **latent heat**, the phase transition of space liberates great quantities of energy that becomes the matter and radiation of the Universe within the expanding ‘bubble’.

The effect this process has on Einstein’s equations of General Relativity is to briefly reinstate the cosmological constant, Einstein’s “greatest blunder”. The core of GR is encapsulated in an equation of the form,

$$E = (8\pi\, G / c^4)\, T\ .$$

E stands for a collection of 10 quantities characterizing the geometrical curvature features of space-time and is called the **Einstein tensor**. T stands for a corresponding collection of 10 quantities that characterize the distribution and flow of energy, momentum and stresses within space-time and is called the **stress-energy-momentum tensor**. G is Newton’s universal gravitation constant and c is the vacuum speed of light. This equation has solutions describing expanding Universes that are decelerating and it has solutions describing collapsing Universes that are accelerating. But it has no solutions describing static Universes which Einstein believed to be the reality. To allow for the static case Einstein changed the equation to read,

$$E = (8\pi\, G / c^4)\, T + \Lambda\, g,$$

where Λ is the cosmological constant and g is a collection of 10 basic geometrical quantities called the **metric tensor** and out of which the members of the collection E are built. If Λ is positive it results in space being filled with an energy that repels itself instead of gravitationally attracting itself. With the right value for Λ this repulsive energy can just balance the attractive gravity of ordinary energy-matter to yield a precarious static Universe.

Inflation is initiated by the delayed phase change effectively boosting Λ to a high enough value to overwhelm gravity and produce extreme expansion.

The inflating 'bubble' cools rapidly and with cooling Λ settles back to zero and inflation gives way to ordinary decelerating expansion.

Or does it?

3. Accelerating Expansion: The discovery of expansion of the Universe by Hubble employed two relationships. The Doppler shift relationship that relates the lengthening of received wavelengths, λ, to their source or emitted values, λ_0, when emitted from a source receding with speed, v,

$$z \equiv (\lambda - \lambda_0) / \lambda_0 \simeq v / c \ ,$$

and the astronomically observed proportionality between the spectral shift, z, and the distance of stellar and galactic sources (when they're sufficiently far away),

$$z \simeq H_0 (d / c) \ .$$

Combined these yield Hubble's Law,

$$v \simeq H_0 d \ .$$

Now it turns out that, according to Special Relativity (SR), the first relationship for Doppler shift must be corrected if z gets close to 1 or exceeds 1 as it does for very distant sources because $v > c$ is forbidden in SR. On the other hand, once expansion of the Universe was recognized, General Relativity (GR) required Hubbles law, itself, to be exact,

$$v = H_0 d \ ,$$

even though it leads to the conclusion that sufficiently distant galaxies recede from us at superluminal speeds, i.e., $v > c$. As we saw earlier, in **IV: The Expanding Universe and The Microwave Background Radiation**, this is because *they are receding due to the expansion of space rather than due to moving through space*. Remember that we've just seen an instance of pervasive (albeit very brief) hyperluminal expansion of space in the Inflationary Scenario. In any case the SR version of the Doppler shift relation already requires a corresponding correction to the proportionality between the spectral shift, z, and the distance, d.

Again as we saw in **IV**, GR requires us to revise our understanding of z as being due primarily to a stretching effect of spatial expansion during the transit of the light rather than a Doppler effect due to recession of the source. This is a gradual shift in understanding that sets in at ‘intermediate’ distances. For relatively short distances the Doppler shift effect is dominant and is replicated by expansional stretching. But for the greatest distances we have been exploring in recent decades the spatial expansion effect is completely dominant.

The improved approximation for the relation between z and d is (**Fig. V.8**),

$$z \simeq (H_0 d / c) + (1/2)(1 + q_0)(H_0 d / c)^2 + O((H_0 d / c)^3) ,$$

where q_0 is called the **deceleration parameter** and has the values 0 and ½, respectively, for the simple un-accelerating and decelerating models of flat expanding Universes we considered in **IV.**

This looks a bit complicated but it just tells us that unless $(H_0 d / c)$ is rather small we need to include contributions to z from at least the square of $(H_0 d / c)$ if not higher powers. We will be ignoring the higher powers and so we drop the last term on the right.

How small is small? Well if $d \simeq 1$ Mpc then $(H_0\, d / c) \simeq (1/4000) = 0.00025$. That’s small! If $d \simeq 1$ Gpc = 1000 Mpc, then $(H_0 d / c) \simeq 1/4$. This yields

$$z \simeq (1/4) + (2/64)$$

for the un-accelerated model Universe and

$$z \simeq (1/4) + (3/64)$$

for the simple decelerating model. Rough measurements of z and d would not require the correction terms but fine measurements would, especially if one wanted to measure the deceleration parameter, q_0 (**Fig. V.8**).

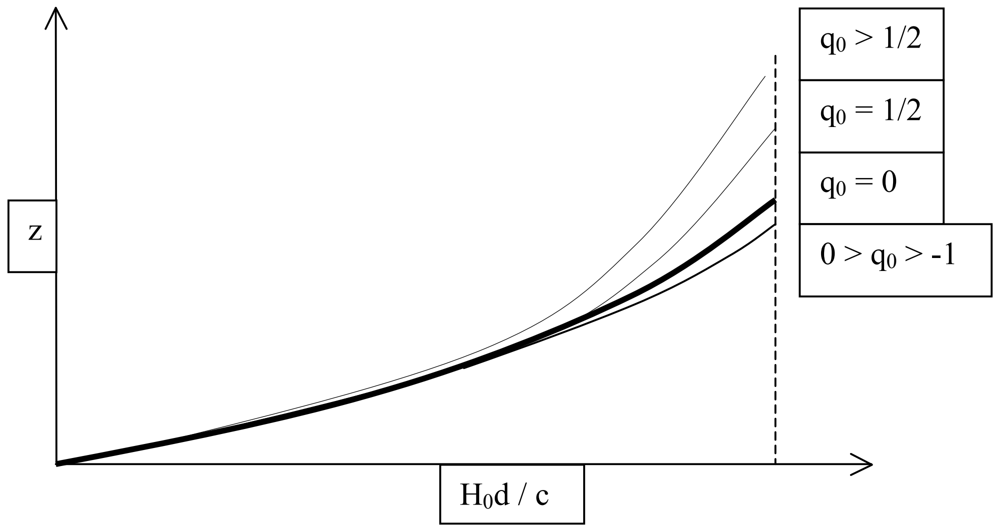


Fig. V.8: Graph of the spectral shift, z, versus the distance parameter, H_0d / c, for 'small' positive, zero and negative values of the deceleration parameter, q_0 , according to the equation,

$$z = (H_0d / c) + (1/2)(1 + q_0)(H_0d / c)^2 .$$

In 1997 that's exactly what several groups of astrophysicists set out to do, measure the deceleration parameter, q_0, of the Universe. It was 'understood' that the Universe was decelerating, i.e., the expansion was slowing down and, therefore, $q_0 > 0$. The pervasiveness of gravity guaranteed that. The question was, by how much? Determining q_0 would also help in the assessment of the matter and energy density in the Universe which determines the large scale structure and ultimate fate of the Universe. If the density is below a critical value, the Universe is negatively curved spatially, like a saddle surface, and will expand forever, asymptotically *approaching* but never dropping below some minimum, non-zero, rate of expansion. If the density is at the critical value, space is flat and the expansion continues forever, but at ever slowing rates asymptotically approaching zero. This corresponds to $q_0 = 1/2$. If the density is above the critical value, space is positively curved, like the surface of a sphere, and the expansion eventually stops and turns into a contraction, ultimately ending in **the Big Crunch!**

As mentioned in the previous section matter and energy assessments suggested the Universe was below critical density but detailed analysis of the Cosmic Microwave Background suggested that space was flat. A measurement of q_0 might help to resolve this discrepancy.

To determine q_0 the observers examined type 1A Supernovae at distances between 7 Glgt.yr ~ 2 Gpc and 3 Glgt.yr ~ 1 Gpc. Type 1A Supernovae occur when a dead, white dwarf star, in orbit with a nearby active star, has pulled enough matter out of the nearby star and onto itself that the resulting pressure and temperature reignites the white dwarf into an enormous thermonuclear explosion that literally blows the white dwarf apart. Within about three weeks of first ignition the luminosity exceeds a billion times that of our Sun and then slowly declines over months (**Fig. II.12**). This may seem like a sluggish explosion to you, but on stellar and galactic time scales this is a blink of an eye.

Estimates are that such supernovae occur about once a second somewhere in the observable Universe. And yes, sometimes they occur in our own galaxy (see the **Appendix A**). Catching them is another matter. Most are very dim because so far away or obscured by dust or whatever. We observe about one a month. When we do catch them we can assess their distance by their dimness, i.e., their apparent luminosity, AL. Knowing their intrinsic luminosity, IL, (from **Fig. II.12**) we have,

$$AL \simeq IL / d^2 ,$$

or

$$d \simeq [IL / AL]^{1/2}$$

(Strictly speaking this simple relationship only holds in flat space. With negative curvature AL is less and with positive curvature AL is greater for a given d. Unfortunately, we can't delve into this subtlety here).

The data contained a great surprise! For any given z the Supernovae were dimmer than expected. They were farther away than expected. Put another way, for any given distance, d, the spectral shift, z, was less than expected. The light had not been stretched as much as expected. The Universe had not expanded during the trip as much as expected.

Now you may wonder, since they were measuring the distances and the spectral shift to determine q_0 , why did they *expect* anything in particular.

The point was that while they didn't *know* q_0 well, they *expected* it to be close to the critical value of 1/2 for a flat space. They *expected* it to be a little bit lower (negatively curved open space), but not by much. It might have been a very little bit higher (positively curved closed space), but certainly not by much. What they found was that q_0 was much lower than 1/2! What they found was that q_0 was *negative*!

A negative q_0 means the Universe expansion isn't decelerating at all. It's *accelerating*! The Universe had not expanded as much as expected during the trip for the light from the Supernovae because it was expanding more slowly during most of the trip than it is now. Instead of gravity retarding the expansion, something has overwhelmed the large scale gravity in the Universe and is speeding the expansion up! What might it be?

Remembering our discussion of Inflation, we know that a positive cosmological constant in Einstein's equations can, under the right conditions, produce accelerated expansion. The conditions are that the value of the constant be sufficient to overwhelm (by the right amount) the gravity due to the average mass-energy-pressure densities in the Universe. During the Inflationary epoch the constant was comparatively enormous. The present value for the constant, to match the measurements, has to be small, and if it has 'always' had that same small value since Inflation stopped, then long ago, when the matter-energy-pressure densities were much higher, ordinary gravity would have been dominant and the expansion would have been decelerating. In other words the accelerating expansion is, in the cosmological scale of things, a comparatively recent development. Indeed, the examination of more and more distant Supernovae, as well as other data, suggest just such a decelerating expansion followed, about 9 Gyr ago, by a transition to acceleration. In this way we presently assess the time since the BB to be about 13.7 Gyr. (**Fig. V.9, 10**).

But what gives the cosmological constant its non-zero, positive value? There are many theories. As yet there are no convincing answers. Not knowing the cause, we give it a name. We call it **Dark Energy**. Unlike ordinary energy, it does not attract, it repells. Preliminary estimates place it at about 71% of of the energy in the Universe. Along with the 23% contributed by Dark Matter that makes 94% of all the stuff in the Universe – and we don't know

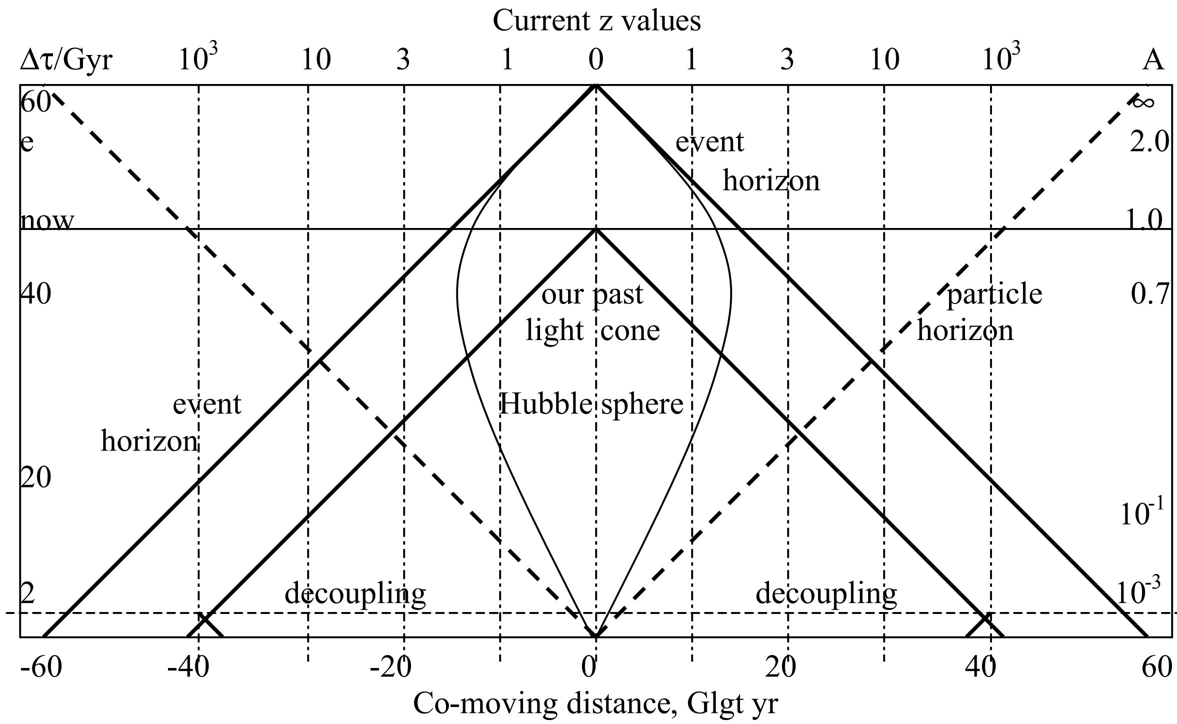


Fig. V.9: Conformal diagram of the post inflationary evolution of the first decelerating and then accelerating expanding Universe. Here, however, while the vertical lines would, in a proper space-time graph, come together at the bottom line for the BB, as we approach the top line they would, as a consequence of accelerating expansion, spread apart at increasing rates as proper time goes to infinity. The conformal time difference axis on the left now adds to the inflationary values of **Fig. V.7** from zero at the bottom to a bit above 60 Gyr at the top where proper time goes to infinity. Notice that the Hubble sphere does not expand indefinitely as in **Fig. V.5** , but instead, at a scale factor of about 0.7, slows its expansion and begins to contract. This is the sign of accelerated expansion and gives rise to the event horizon, from beyond which we will never receive photons or anything else as the rate of expansion forever pulls that region beyond our purview.

what either of them are! Like ordinary energy, Dark Energy influences space-time geometry and the present estimates place the total at just enough to make space flat on the largest scales. **IF** the cosmological constant remains *constant*, **AND** our current theories continue to hold, then, unlike ordinary energy, the density of Dark Energy will not decrease as the

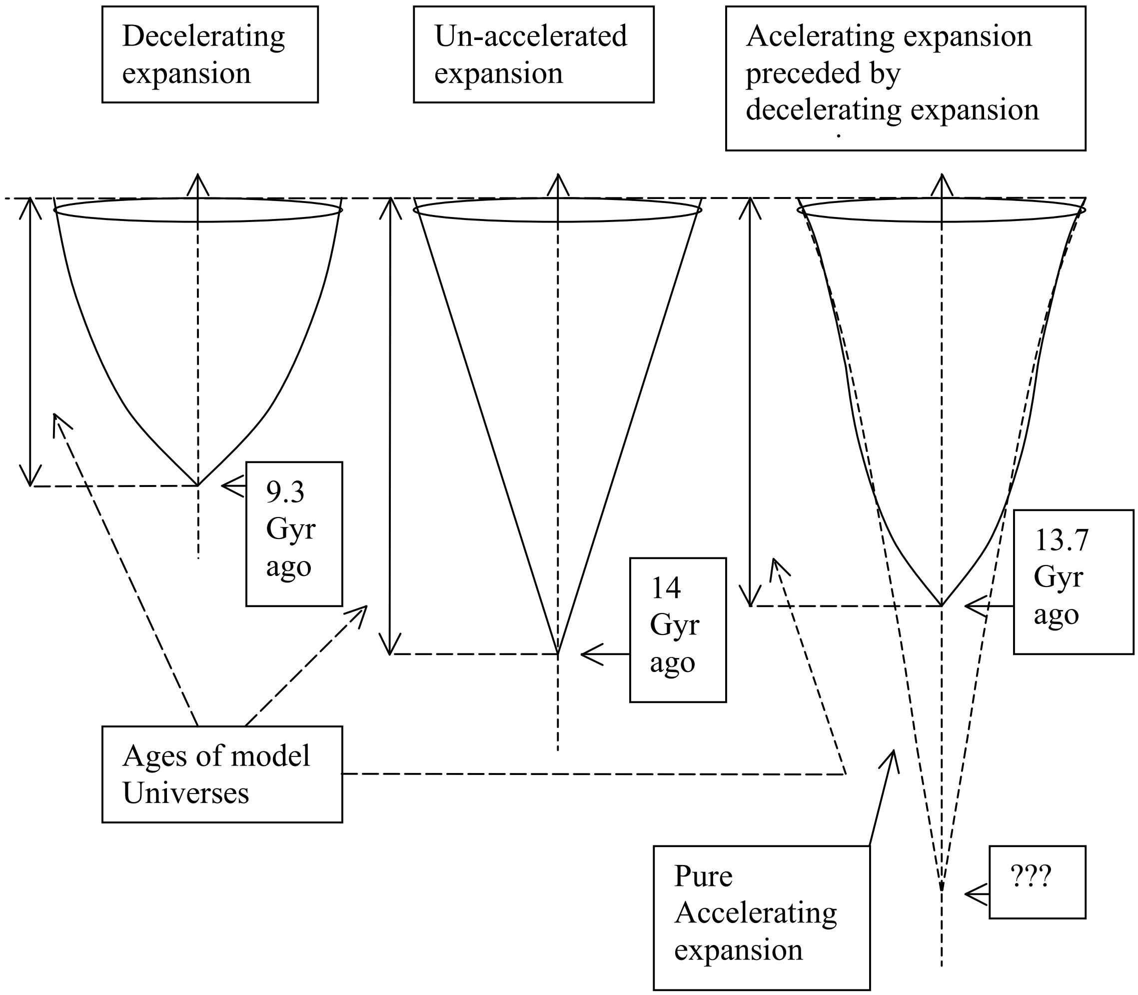


Fig. V.10: Models of expanding Universes. The figures and numbers for the decelerating and un-accelerated model are taken from our discussion in **IV**. The decelerating-then-accelerating model reflects best current assessments. The purely accelerating model is just for comparison. Ideally each figure should display the same rate of widening at the top to reflect the current value for H_0.

Universe expands and the acceleration will grow and grow until the rate of expansion pulls galactic clusters and isolated galaxies so far apart from each other that the night sky will, *in about 100 billion years*, be barren by comparison with the present. **IF** the cosmological constant increases with time **AND** our current theories continue to hold, then expansion will eventually tear everything apart, galaxies, stars, planets, rocks, molecules, atoms, protons, electrons, photons, superstrings, *everything*!! **The Big Rip!!**

But long before any of these scenarios play out, and assuming that our species or some successor sentient species, somewhere, survives, it will probably have been found that our current theories have not continued to hold. Better approximations to the truth will have replaced them.

Appendix A: Supernovae in the Milky Way

We will use, for comparison, one billion, or 10^9, as a representative peak intrinsic luminosity ratio of a supernova to the Sun and $2^{14} \sim 16{,}000$ as the ratio of the apparent brightness of the Sun to that of the full Moon. If such a supernova occurred just one light year away, it would appear between 1/4 and 1/5 as bright as the Sun. It would be deadly, probably stripping away our whole atmosphere with the intense stream of high energy particles and radiation it would send our way. If it occurred 64 lgt y away, it would appear as bright as the full Moon and would almost certainly destroy our Ozone layer. Some astrophysicists regard 100 lgt y away as the smallest safe distance while others put real safety at 3,000 lgt y away. Some astronomer/paleontologists believe a Milky Way supernova may have been a factor in the Ordovician-Silurian mass extinction event of some 450 My ago.

Current estimates are that supernovae occur in the Milky Way about once every 50 years. We miss most of them because of obstruction by dust clouds and not looking in the right direction at the right time. Remember, in some directions they could be 84,000 lgt y away and still be within the Milky Way. In May of 2008 we stumbled upon the remnants of a supernova in the constellation of Sagitarius, 26,000 lgt y away, that was already 140 y old. From our perspective the remnants were still increasing in brightness because the glowing embers were emerging from a dust cloud. Historically important for the development of astronomy was the very unusual, almost back to back occurrence of vivid supernovae in 1572 and 1604 viewed by Tycho Brahe and Johannes Kepler, respectively.

A number of astronomers monitor the skies for hints of prospective Milky Way supernovae. Presently, several candidates are being watched. The closest is IK Pegasi, only 150 lgt y away. It’s a binary system with a white dwarf, so it will be a Type 1A supernova. It's expected to blow any time within the next several million years!

Appendix B: Conformal time and photon speeds

We will use the notation, dt, dr, dτ, d(anything) to denote very, very tiny (ideally infinitesimal) changes in proper time, t, co-moving radial distance, r, conformal time, τ, or anything, respectively.

According to GR, if an object has a tiny change, dr, in its co-moving radius, r, during the tiny proper time interval, dt, then, for a flat space, the combination,

$$c^2dt^2 – A^2(t)dr^2 ,$$

where A(t) is the scale factor, is never negative. For a photon moving through space towards or away from us, i.e., towards or away from the center, r = 0, of the coordinate system, we always have the *limiting value*,

$$c^2dt^2 – A^2(t)dr^2 = 0 .$$

The *proper* radial distance at time, t, to any object, D(t), is given by,

$$D(t) = A(t)\ r(t) ,$$

and an infinitesimal change in D(t) is given by,

$$dD(t) = dA(t)\ r(t) + A(t)\ dr(t) ,$$

(we drop the term dA(t) dr(t) which is a product of two infinitesimals).

If D(t) is the proper distance to an 'approaching' photon, that photon's velocity is,

$$V(t) = dD(t)/dt = (dA(t)/dt)\ r(t) + A(t)\ (dr(t)/dt) ,$$

but the limiting value equation tells us that,

$$dr(t)/dt = \pm\ c/A(t).$$

Consequently,

$$V(t) = (dA(t)/dt)\, r(t) - c\ ,$$

for 'approaching' photons. If $dA(t)/dt$ is large enough, the photon will recede from us, at least for awhile. But r(t) is decreasing and eventually the photon may reach us. In any case we notice that,

$$A(t)\, dr(t)/dt = \pm\, c = dr(t)/(dt/A(t)).$$

If we now define conformal time, $\tau(t)$, such that $\tau(0) = 0$ and,

$$d\tau(t) = dt/A(t)\ ,^*$$

then,

$$dr(t)/d\tau(t) = dr(\tau)/d\tau = \pm\, c\ ,$$

i.e., the co-moving velocity of light with respect to conformal time is always $\pm$ c, as our conformal diagrams indicated.

*For those familiar with calculus, the two defining conditions for conformal time result in,

$$\tau(t) = \int_0^t dt'/A(t')\ .$$