IV. The Expanding Universe and The Microwave Background Radiation:

1. Models of the Universe: If one assumed how mass-energy was distributed throughout the Universe, then General Relativity (GR) told one how the space-time geometry of the entire Universe was structured. Of course nobody knew how mass-energy was distributed throughout the Universe – but by trying some simple models one could find out which models came closest to predicting a Universe that had parts that looked like the part we could observe.

Einstein's first model assumed that on the largest distance scales matter was uniformly and staticly distributed throughout the Universe. In this he was simply following the hoariest of tradition. He also assumed that the volume of space was finite although *without any boundary*. The meaning of this assumption we will discuss shortly, but the reason for the assumption was to avoid having to deal with an *infinite* amount of matter uniformly distributed throughout infinite space. Such an arrangement was known to be unstable. Small upward fluctuations in local mass density could precipitate runaway gravitational condensations around the local disturbance. With the matter primarily in the form of stars one also encounters **Olber's paradox**, which we mentioned in chapter **I**, according to which the night sky, in an infinite space with uniformly distributed stars, should not be dark, but blindingly bright. In a space with finite volume and finite matter density there will only be a finite number of stars.

While we can't picture a finite volume with no boundary in our minds eye (some mathematicians claim they can) we can understand the idea by analogy with the *surface* of a sphere. The surface (not the interior of the sphere) is the analogue of space and the area of the surface is the analogue of the volume of space. The area is finite but the surface has no boundary. The surface *is*, itself, a boundary for the interior of the sphere, but the surface, itself, *has* no boundary. In the previous chapter we used this kind of analogy to help us understand curved geometry since the spherical surface is clearly curved and the straightest possible paths on it are great circle arcs which beings living in the surface (not 'on' the surface but 'in' the surface) would interpret as straight line paths. Here we are using, not portions of the spherical surface to analogize curved space, but the entire spherical surface to analogize **closed unbounded space** with finite volume.

Anyhow, Einstein found that in his theory of GR this model was disturbingly dynamical, i.e., it could be expanding or contracting (In the analogy the spherical surface is literally expanding or contracting) but it would not stay fixed. If it was expanding the attractive aspect of gravity would gradually slow the expansion down and possibly reverse it eventually and if it were contracting gravity would accelerate the contraction towards ultimate catastrophe. But in this matter Einstein was a thorough conservative. He believed, along with almost everyone else, that the Universe as a whole was static. It did not expand and it did not contract. So he changed his theory by adding a term in the basic equations which introduced a repulsive aspect to gravity on the largest distance scales which would just balance and maintain a static Universe. This was called the **cosmological constant** term.

A few other workers quickly offered alternative models of the Universe that also followed from Einstein's equations and were not static as Einstein's model was. The situation was ripe for significant input of new data concerning the large scale structure of the Universe.

In 1929 Hubble published his findings on the recessional velocities of distant galaxies (or **nebulae** as they were then called since their character as large collections of stars was not yet established) indicating that the Universe was expanding. Einstein immediately recognized his error in introducing the cosmological constant term instead of accepting the dynamical predictions of his original theory and dubbed it " – the greatest blunder of my life."

2. The Big Bang: Among those who accepted the expansion of the Universe, the question immediately arose of how to understand the past history of the Universe. In particular, how old is the Universe and what is the Universe expanding from?

If, in the past, the distant galaxies had the same velocities relative to one another that they have now, then the reciprocal of the Hubble constant, $1/H_0$, is how long the galaxies have been rushing apart and how long space has been expanding from a common point. This follows from writing Hubble's law as,

$$d = v\,(1/H_0)\ .$$

Since d is the distance of a receding galaxy from us and v is its speed, it would take ($1/H_0$) time for the galaxy to get that distance, d, away with a constant speed, v. Everything would have started from a point at a time, $1/H_0$ ago. The Universe would be

$$1/H_0 = 1/((71 \text{ km/sec})/\text{Mpsc}) \sim 1/((43\text{mi/sec})/1.9\text{x}10^{19}\text{mi})$$

$$\sim 4.4 \text{ x } 10^{17}\text{sec} \sim 14 \text{ billion years} = 14\text{Gyr}$$

old! Note that the *point* of the origin is not a point *in* space. The point *is* the whole of space since space, itself, was, in this model, just a point some 14 billion years ago.

But the assumption of constant recessional velocity between any two given galaxies is too simplistic. Once the expansion began we would expect gravity to slow it down and the more densely packed the mass-energy in the past, the more deceleration gravity would produce. Thus for any two presently receding galaxies, the closer they were to each other in the past, the faster they would have been receding from each other. This noticeably reduces the estimate of how long ago the Universe was a point and the expansion began. Estimates as low as 10 billion years were made (**Fig. IV.1**).

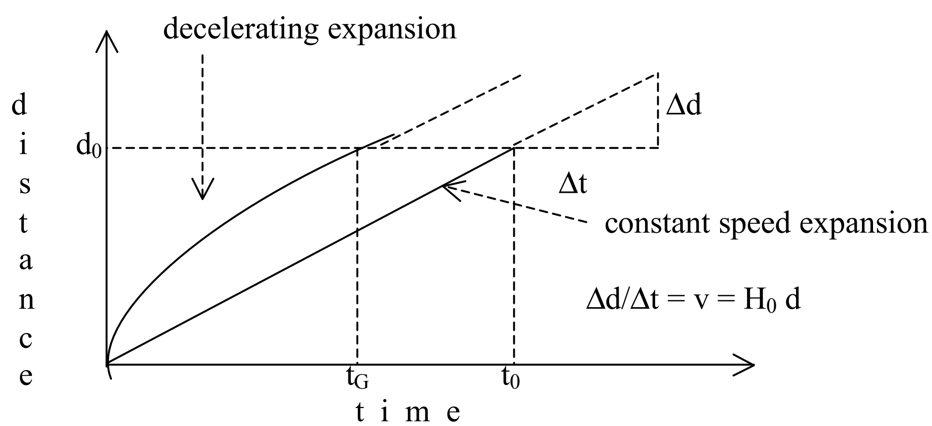

Fig. IV.1: Age of model universe with constant speed expansion, t_0, and with gravitationally decelerated expansion, t_G. If the separation of distant galaxies goes as $D(t)/D(t_0) = (t/t_0)^n$, with $0 < n < 1$ for decelerating expansion, then Hubble's parameter is, $H = n / t$ (**Appendix A**).

Furthermore, the closer one goes back to the beginning, the faster everything had to be moving apart. This is one feature that gives the beginning the character of an explosion – **the Big Bang!**(BB).

With a current value of about (71 km /sec)/Mpc for the Hubble constant, the Hubble Law tells us that galaxies presently one billion lgt.yrs away (1Glgt yr ~ 310 Mpc) have a recession velocity of about 22,000 km/sec, or about $1/14^{th}$ the speed of light. It would then also seem to tell us that galaxies presently further away than 14Glgt yr would have recession velocities exceeding the speed of light!! That doesn't seem consistent with the Relativity Theory claim for the supremacy of the speed of light.

Admittedly, if the Universe is younger than 14 billion years, which presently seems just likely, then light emitted from any galaxy *when* it was more than 14Glgt yr away could not have had time to reach us so we would not have seen such galaxies yet. On the other hand, back when the universe was only 5Gyr old, say, galaxies that were then more than only 5Glgt yr away from us could have been (depending on some details), according to Hubble's law, already receding at superluminal speeds and we might have seen the light from some of them by now! - - or would we?! But whether we can see them or not, the mere notion that there may *be* such superluminal galaxies out there is a bit unsettling. What's going on?!

In the first place it's still the case that no piece of matter or energy can overtake or even catch a photon in vacuum. The vacuum speed of light relative to any distribution of matter and/or energy *in the cosmological vicinity of the light* is still the universal constant,

$$c \simeq 186{,}000 \text{ mi/sec} \simeq 300{,}000 \text{ km/sec},$$

and upper limit for *motion through space*!

But, in general relativity (GR), on astronomical cum cosmological distance scales, the distance between objects can change (increase or decrease), not because the objects are moving through space, but because *the space between them is expanding or shrinking*! That is what GR tells us is happening in the expansion of the universe. The galaxies and galactic clusters are not receeding from one another by moving through space, they are being carried away from one another by the expansion of the space between them! And *there is no theoretical limit on how fast that expansion* *can occur*! Indeed, if the expansion is uniform everywhere in our universe, as it so far appears to be, then if A and B are separating at speed, v, and B and C are separating at speed, v, and A, B, and C are consecutively ordered in a line, then A and C must be separating at speed, 2v, whatever v might be. In particular, v might be 2/3 c, whereupon, 2v = 4/3 c.

The reader may well balk at this notion and have good reason to do so. In fact, a reader familiar with Einstein's first relativity theory, the so-called Special Relativity theory (SR), would have more reason to balk than a not familiar reader. First, how does one distinguish between *moving through space* and *being carried by space*? What markers does pure space, the vacuum of space, provide for telling whether one is moving through it or not? Second, in SR we learn that when compounding relative velocities, as in those between A and B and B and C to get that between A and C, one doesn't just add them! Instead the rule, for parallel velocities, is,

$$v_{AC} = (v_{AB} + v_{BC}) / [1 + (v_{AB}v_{BC}/c^2)] ,$$

and this guarantees that if v_{AB} and v_{BC} are both subluminal, so will v_{AC} be! This rule is what guarantees the supremacy of vacuum light speed in SR for ordinary matter-energy. So why are we now using pre-SR velocity composition in cosmology which is governed by GR?!

These are very good questions and a full answer would take us too far into the technical details of GR. But reasonable partial answers are accessible. In the SR case, each of the "observers", A, B, and C, are employing their own coordinate systems or reference frames. The velocities are, thus, relative velocities between distinct reference frames. In the cosmological, expanding universe case, all the galaxies, the "observers", are using one and the same coordinate system. The so-called co-moving coordinate system, which covers the whole universe and expands, itself, along with the separation between the galaxies. The recession velocities are all referred to that same coordinate system and this, ultimately, justifies composing them by simple addition. As for distinguishing between moving through space versus being carried along by expanding space, the cosmic microwave background (CMB) does, in fact, provide a marker. We will be talking about that background in more detail later, but for now, let's just say that if you're being carried by space, the CMB looks spherically symmetric because it, also, is carried by space. If you're moving through space, the background displays a dipole moment distribution, looking slightly blue shifted in the direction you're moving and slightly red shifted in the opposite direction, the more so the faster you're moving. Our Sun and solar system moves through space as our galaxy rotates and we see a dipole moment on the CMB. There is, then, a sense in which, at the cosmological level, GR reintroduces a concept of absolute motion, which SR was at pains to abolish. Indeed, more than one cosmologist has claimed the CMB to define the *rest frame of the universe*! For more details on *our* motion through space see the **Appendix B**.

So we've made some progress in understanding how sufficiently distant galaxies could be receding from us at superluminal speeds. But how can we ever receive light from such galaxies. If we do just subtract (because the light is directed towards us while the galaxy is moving away) the speed of light from the superluminal speed of recession, won't the light still be moving away from us rather than towards us? Yes it will - - *initially*, but that will change with time.

But before we spell that out, it will be helpful to consider some general issues concerning how gravity from matter and energy determines the overall structure of the universe.

According to GR the overall curvature of the Universe depends on the average density of mass-energy in the Universe. If this average density is above a certain critical value, then space will be positively curved, analogous to the surface of a sphere, and it would have finite volume analogous to the finite area of the surface of a sphere. Below the critical value space would be negatively curved, analogous to an infinitely extended saddle shaped surface (hyperboloidal), and with infinite volume. If the average mass-energy density was exactly at the critical value (which at first glance seems unlikely) space would be basically flat with no overall curvature and with infinite volume (there would still be regions of local curvature in the vicinity of stars and galaxies and black holes). Before Einstein almost no one gave a second thought to any conception of space except that it was flat and infinite. But now that the Universe was expanding from some prior more condensed state it was easier to get ones mind around the idea that the volume of space was finite, analogous to the surface of a sphere. So early on, in the absence of much data, that was the preferred view (**Fig. IV.2**).

On the other hand, if we eventually conclude in favor of an expanding universe with an overall flat space (which seems to be the direction in which accumulating data is now leading us), the corresponding two-dimensional surface-space analogy diagram would look something like **Fig. IV.3**.

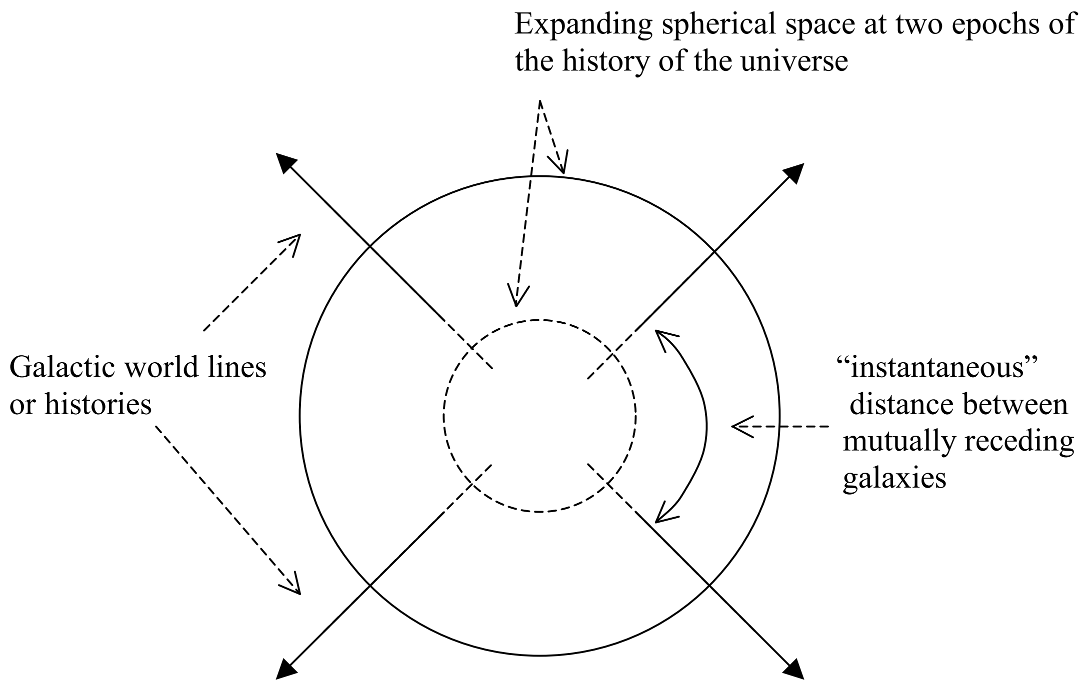

Fig. IV.2: The two dimensional surface-space analogy for the positively curved, closed, expanding spherical space in Einstein's General Relativity.

In this **Fig. IV.2** the temporal evolution of the universe is represented by the succession of ever larger concentric spherical surfaces carrying all the points on the surfaces (galaxies) radially away from the point center which represents the BB. Whether the evolution is gravitationally decelerated or not corresponds to whether the radius of the sphere (the radius of curvature of the universe) slows down as it increases or not.

On the other hand, in **Fig. IV.3** the vertical line represents both the world line of the arbitrarily chosen reference galaxy, treated as being at rest, and the time axis. Consequently, there is no question as to how rapidly the horizontal flat spaces are evolving up the diagram (each space is just all of space at a particular time) and the question of whether the expansion of space is decelerating or not is answered by whether the worldlines of the receding galaxies are straight lines (no deceleration) or curve towards the vertical as they rise (deceleration). The figure portrays no deceleration.

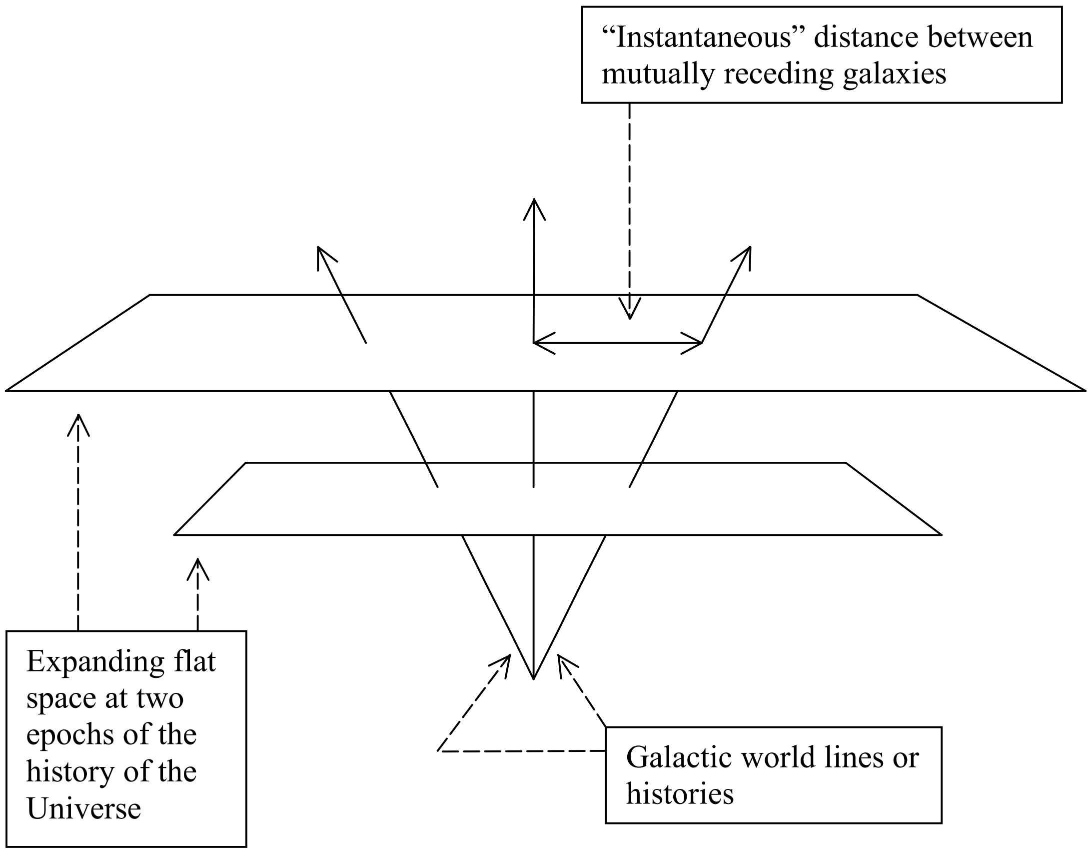

Fig. IV.3: The two dimensional, surface-space analogy for a zero curvature, infinite, expanding flat space of Einstein's General Relativity (neglecting deceleration).

Consider, in this flat space case, the cosmological hypothesis which asserts uniformity of matter-energy distribution on the largest scales and invariance of perspective regardless of the choice of reference galaxy. That hypothesis implies that the recession speeds of distant galaxies must exceed all bounds at sufficiently large distances, just as the Hubble Law indicates. Otherwise the matter-energy distribution, even if initially uniform, would become non-uniform with time. For the closed universe, on the other hand, even though the Hubble law would hold there also, the recession velocities are limited by the maximum distance between any two galaxies at a given time and the rate of expansion of the universe. That upper limit may or may not exceed the vacuum speed of light.

We are now in a position to understand how light emitted in our direction from a superluminally receding galaxy can, eventually, reach us. The situation is simplest in an infinite, flat space universe expanding with no deceleration. So we will focus on that situation (**Fig.IV.4**).

Imagine a photon emitted from or just crossing a distant galaxy that is receding from us at twice the speed of light. The photon's speed relative to that galaxy is just light speed, c, and if it's moving in our direction, it's velocity relative to our galaxy is + c, i.e., a speed, c, *away from us*! For the present, the photon is, itself, receding from us. But as it moves away from the source, or crossing galaxy, in our direction, it encounters galaxies that are not receding from us quite as fast as the original galaxy. Being light, the photon maintains, at c, its speed relative to whatever galaxy it is in or is crossing. Consequently, the photons speed away from us is slowing and its speed away from the original galaxy is increasing. Eventually, the photon reaches a galaxy receding from us just at light speed, c. At that galaxy *the photons velocity relative to us is zero*! It is momentarily standing still relative to us. But it's still moving at c relative to its local or ambient galaxy and "soon" it reaches a galaxy moving away from us at only c/2. The photon is now making progress towards us at a velocity of – c/2, where the minus sign indicates the photon is now decreasing its distance from us. By the time the photon reaches our galaxy (the reference galaxy in the figure) its velocity relative to us will be – c.

Clearly, the time required for the photon to reach us is substantially longer than just its original distance away divided by c! One can either say its velocity relative to us is time dependent and part of the time in the wrong direction, or that the photon had to move through ever increasing amounts of space to reach us due to the ongoing expansion while in transit. Nevertheless, it turns out that no matter how far away or how fast away from us the original galaxy was, if the expansion rate is constant or decelerating, then the photon will reach us eventually. On the other hand, if the expansion is accelerating, then there will be some critical distance beyond which no photon aimed in our direction would ever be able to reach us. The increasing rate of expansion would never allow such a photon to reach a region of

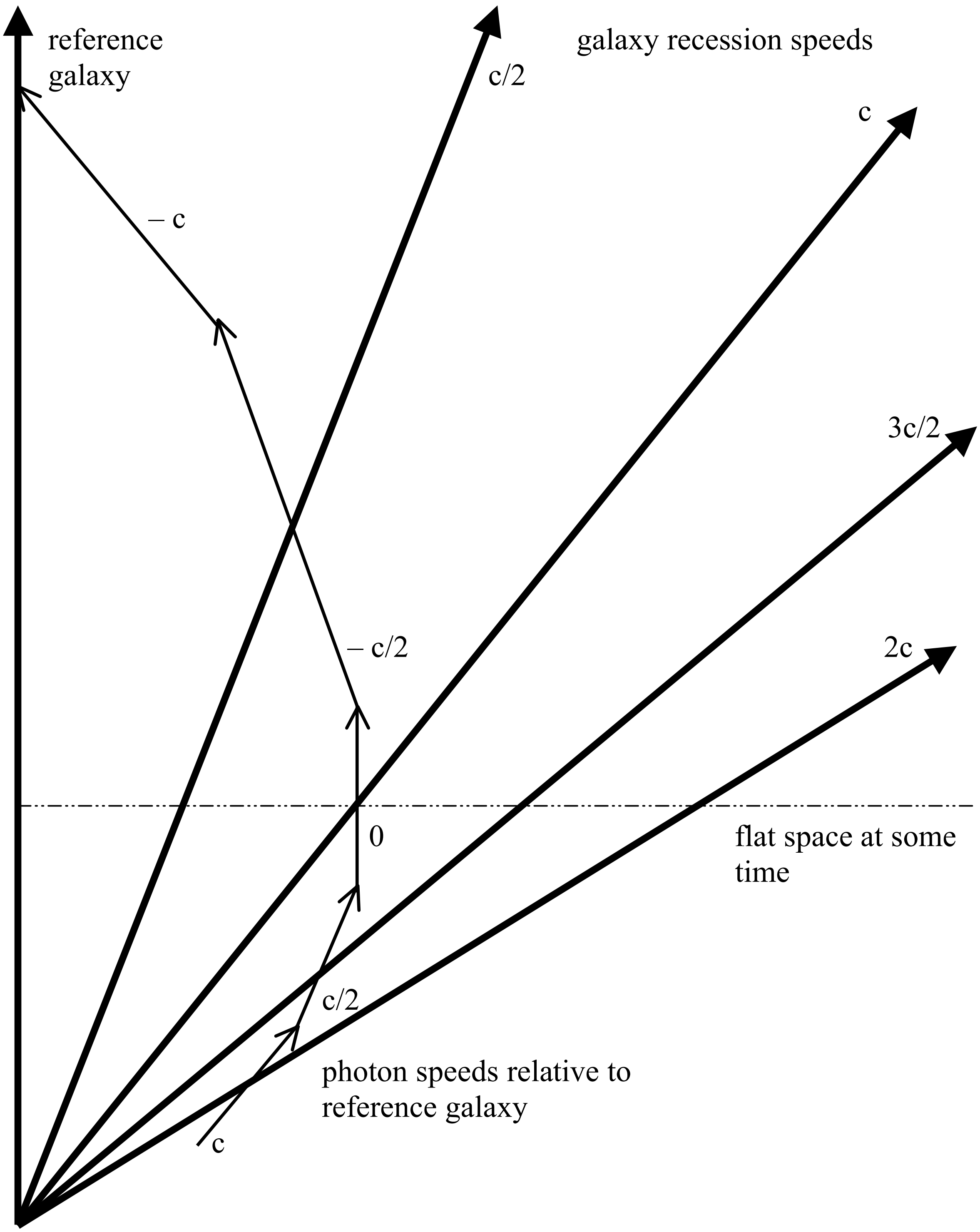

Fig. IV.4: Mutually receding, non-decelerating, galaxy world lines (bold) and (stepwise) photon world line always moving at light speed relative to the local galaxy being crossed. Initially the photon recedes from the reference galaxy. Eventually it reaches it.

space in which its velocity relative to us could become negative. This critical distance defines a horizon that acts like the event horizon of a black hole in that we can never receive matter-energy or information from beyond it. It is called the particle horizon.

Similar conclusions hold in an expanding closed universe if the rate of expansion is fast enough to generate superluminal recession speeds.

3. Cosmological Red Shift: He didn't know it at the time, but when Hubble was measuring the red shifts of receding nebulae, the greater the red shift and the further away the nebulae were, the less the shift was due to a Doppler effect! Just as the receding galaxies are not moving through space away from us but are being carried away by the expanding space between us, so the light we receive from distant galaxies is not shifted to lower frequencies by the velocity of their recession, as Doppler would have it, but by the *stretching* of the wavelength of the light as it travels through the expanding space. This stretching is called the **cosmological red shift**. Doppler shifting of wavelengths and frequencies is something that happens during the emission and/or reception of light, and, in between, the traveling light has a fixed, albeit different, wavelength relative to its source and its receiver. Cosmological shifting occurs during the trip and is determined by how much spatial expansion has occurred between departure from the source and arrival at the receiver. Of course, if we have sources and receivers that are in motion through space as well as being immersed in expanding space, both Doppler shifting and cosmological shifting will contribute. But for receding galaxies at sufficient cosmological distances the cosmological shift is the overwhelmingly dominant contribution!

The quantitative expression of this cosmological red shift is extremely simple. Suppose light of frequency and wavelength, f_0 and λ_0, is emitted from a source galaxy when the source is a distance d_0 away from the receiver galaxy. Suppose next, that the distance between the two galaxies when the light is received is, d, and that the change is due to expansion. Then the received frequency and wavelength, f and λ, satisfy,

$$f / f_0 \;=\; d_0 / d \;=\; \lambda_0 / \lambda \,.$$

So if, by examining the frequency of the spectral absorption lines in the received light, we determine the amount of red shift the light has been subject to, we can infer the expansion that has occurred since its emission. Using the best model one has of the dynamical evolution of the expanding universe, one can infer how long it has been since that amount of most recent expansion began. This tells us both how long ago the light was emitted and how far away the source galaxy was at the time of emission. But how does one determine a good model of the dynamical evolution of the expanding universe? For that we must consider the CMB. Where did it come from and what is its structure?

4. Temperature and the Cosmic Microwave Background: Besides the high velocity with which matter and energy emerge from the BB, another feature contributing to the explosive character of the BB is that as we trace back in time we expect the ambient temperature of the Universe to be higher and higher.

At present the obvious, main constituents of the universe would seem to be the galaxies and their stars and the dust and molecules of vast intergalactic clouds. If we imagine running the Universe expansion backwards, we recognize that in the distant past the constituents are much closer to one another and will be subject to more frequent encounters and collisions. This means a greater percentage of the kinetic energy of the universe is randomized and the temperature is higher. Indeed, in the presence of decelerating expansion the *total* amount of kinetic energy would be greater in the past and the randomized percentage would contribute to even higher temperatures.

According to our best theories of the formation of stars and galaxies, if we go back to ($t_{Universe}$ < 300Myr) after the BB, galaxies and stars have not yet had time to form, the ambient temperature is ~ 1,000 ^{0}K, and the constituents of the universe are only atoms and molecules and photons. Closer still to the BB ($t_{Universe}$ < 380kyr) and it is too hot (T > 3,000 ^{0}K) for neutral atoms and molecules to exist. There is little more than electrons, photons, protons and Helium nuclei. In this situation the photons can't move very far between interactions with the electrically charged electrons, protons and nuclei. So taken en masse, the radiation is constrained to move with the matter. This is truly a "blooming, buzzing confusion"!

At less than three minutes after the BB it's even too hot ($T > 1G^0K$) for the Helium nuclei to hold together. Further back yet and the exotic denizens of the elementary particle zoo, the quarks and gluons and vector mesons are the only 'heavy' particles, but photons and neutrinos dominate the scene with electrons, muons, tauons and almost equal amounts of antimatter plentiful.

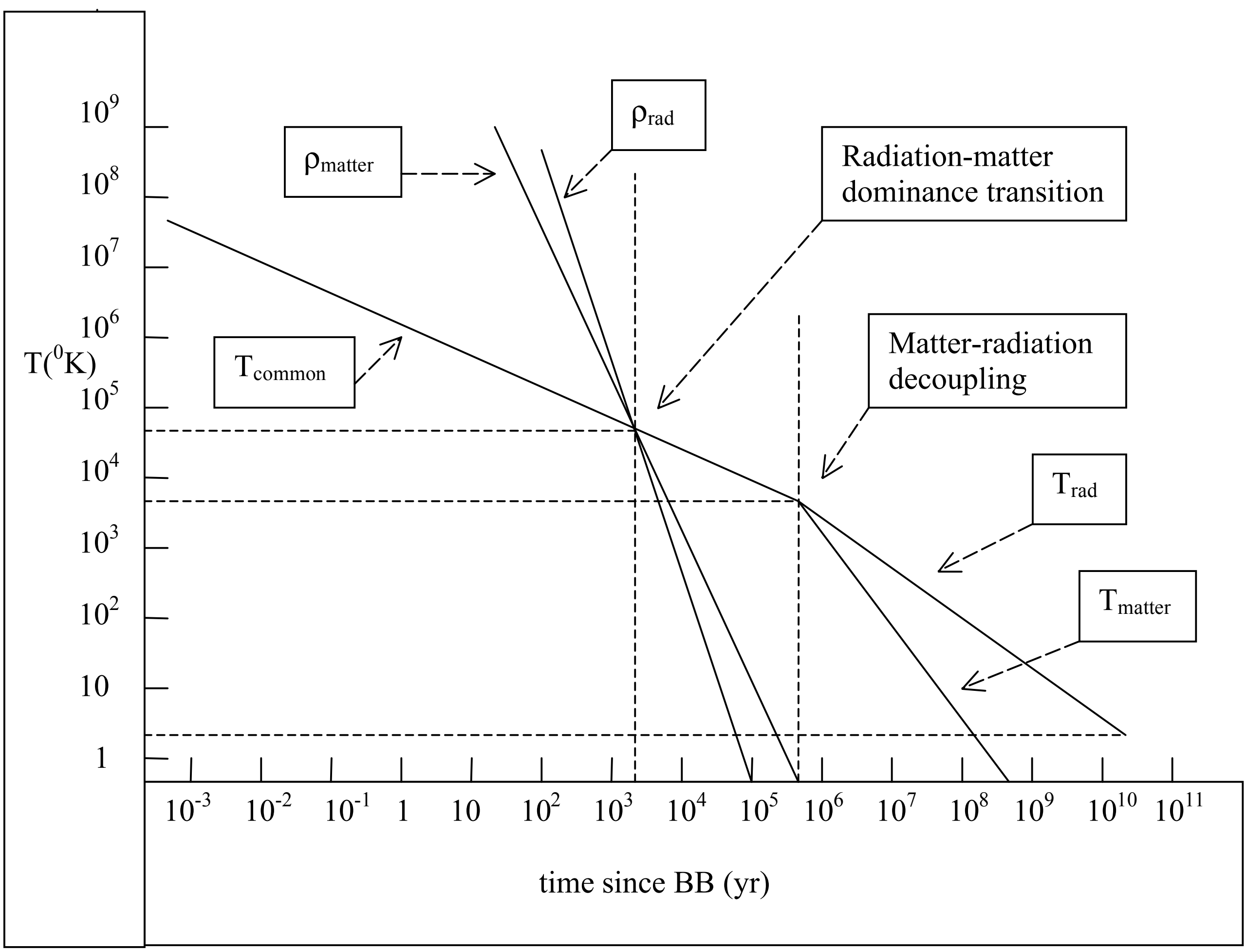

Fig. IV.5: Time dependence of temperatures and energy densities in the expanding Universe.

In fact, for the first few thousand years after the BB, there is so much radiation that the era is called the **radiation dominated Universe**. There's so much 'light' that if you could survive being there, you couldn't "see" anything but blinding 'light'!

Reversing the clock again back to normal time flow and returning to about 380,000 yr after BB, the temperature has cooled to about $3{,}000^0$K and electrons and nuclei (about 74% ^{1}H and 25% ^{4}He with traces of heavier nuclei) could combine to form electrically neutral atoms. The photons can now travel considerable distances without interacting with matter and the radiation effectively decouples from matter and these two ingredients of the Universe now cool at different rates (**Fig. IV.5**).

In fact the decoupled radiation proceeds to cool with the expansion to what we now recognize as the ~ 3^0K **,** CMB radiation discovered in 1965 by **Arno Penzias** and **Robert Wilson**. They were preparing a microwave horn antenna for satellite communications and were frustrated at not being able to eliminate a residual static noise from their signals. They even vigorously cleaned accumulated bird droppings from the horn to try to eliminate the noise as such material can interact with microwave signals. Eventually they realized that the noise was coming from the Universe and equally in all directions. Subsequent measurements showed the radiation to be thermal, i.e., having a definite temperature (see **Fig.III.1**), and the temperature was about 2.7 ^{0}K, corresponding, via Wien's Law, to a peak wavelength of about 1 mm.

From this result and one other measurement we can try to understand some features of **Fig. IV.5**.

As mentioned before, temperature is a measure of the average randomized kinetic energy of the constituents of a system

$$k\,T \propto \langle K_{ran} \rangle,$$

where k is **Boltzmann's constant**, k = 86 μeV/^{0}K (1μeV being the kinetic energy an electron or a proton would acquire upon being accelerated through an electric potential difference of one millionth of a volt).

If the speed of the randomized motion is a sizeable fraction of the speed of light (called relativistic motion), then K_{ran} is proportional to the momentum, p_{ran}, of the motion. If the speed is very small compared to the speed of light (called non-relativistic motion), then K_{ran} is proportional to the square of the momentum, p_{ran}^2. After the decoupling at T ~ 3,000 ^{0}K and $t_{Universe}$ ~ 380 kyr, the random motion of matter is overwhelmingly non-relativistic (see below) and so,

$$k\,T_{matter} \propto < p_{ran}^2 >,$$

while radiation, composed of photons and, by definition, always relativistic, satisfies,

$$k\,T_{rad} \propto < p_{ran} >.$$

The different dependencies on averaged respective momenta is important because, in the expanding Universe, momenta, like the frequencies of light, tend to decrease with the reciprocal of the length scale, $L(t_{Universe})$, of the Universe, i.e.,

$$p \sim [1/L(t_{Universe})].$$

This can be understood, qualitatively, as a consequence of the moving particles having ever decreasing momentum relative to the receding matter they are moving through. A bullet pass in football which has less momentum relative to the downfield running back who catches it than to the quarterback who threw it provides an analogy. A second explanation that comes from the quantum world is that *all* particles, material and photons alike, have wavelike aspects about them and can be assigned a wavelength that is inversely proportional to their momenta. In particular,

$$\lambda = h/p,$$

where *h* is Planck's constant from quantum physics. All of these wavelengths are equally stretched by the expansion of space proportional to the length scale. Hence all the momenta are inversely proportional to the length scale.

Now suppose the expansion was without deceleration, as in the simple model we examined in section **2**. Then the Universe expands at a constant rate, $L(t_{Universe}) \propto t_{Universe}$, and we have,

$$k\,T_{rad} \propto \langle p_{ran} \rangle \sim [1/L(t_{Universe})] \propto (1/t_{Universe})$$

and

$$k\,T_{matter} \propto \langle p_{ran}^{\;2} \rangle \sim [1/L(t_{Universe})]^2 \propto (1/t_{Universe})^2$$

and $$t_{Universe\ now} = 1/H_0 = 14\text{Gyr}.$$

This allows us to claim,

$$(1/1{,}000) \sim (2.7\ {}^0\text{K}/3{,}000\ {}^0\text{K}) = (T_{CMB}/T_{decoupling}) \simeq (t_{decoupling}\ /14\ \text{Gyr}),$$

which requires $t_{decoupling} \sim 14$ Myr, *much too late* according to **Fig. IV.5** !

If, instead of no deceleration, we assume the deceleration corresponding to a simple flat space model expanding under gravity, we find,

$$L(t_{Universe}) \propto t_{Universe}^{\;2/3}.$$

But in this model, $t_{Universe\ now} = 2/3H_0 = (2/3)14\ \text{Gyr} = 9.3\text{Gyr}.$

Hence we would now have,

$$(1/1{,}000) \sim (2.7\ {}^0\text{K}/3{,}000\ {}^0\text{K}) \sim (T_{CMB}/T_{decoupling}) \sim (t_{decoupling}\ /9.3\text{Gyr})^{2/3}.$$

This yields,

$$t_{decoupling} \sim (9.3\text{Gyr}/32{,}000) \sim 290{,}000\ \text{yr},$$

a much better agreement with current estimates of the decoupling time and **Fig. IV.5**!

For the present ambient temperature of non-relativistic matter using this model with deceleration, we have,

$$(T_{matter}/3{,}000\ {}^0\text{K}) \sim (t_{decoupling}/9.3\text{Gyr})^{4/3} \sim (1/32{,}000)^{4/3} \sim 10^{-6},$$

or $$T_{matter} \sim (1/330)\ {}^0\text{K}.$$

If your puzzled by this extremely low value for the present temperature of matter, remember that we're talking about the *ambient* temperature in the Universe as a whole. Where gravitational attraction has given rise to stars and galaxies, temperatures are much higher. But these are local hot spots where, among other things, occasional living organisms can warm their tootsies. Far from these warm hearths, in the vast voids between galaxies it's a very cold Universe.

Or so it was thought until very recently. It now appears that star formation, which began about 200 Myr after the BB, and galaxy formation, that started about 800 Myr later still, reionized and heated not only the nearby and interstellar gas within galaxies, but the extremely tenuous intergalactic gas at vast distances as well. The mechanisms for this process are only dimly understood at present and research on the problem is intense.

For non-relativistic matter most of its energy comes from the Einstein equivalent of its rest mass, m, via, $E = m c^2$. The kinetic energy we've been discussing above is comparatively insignificant for

$$k T_{matter} << m c^2 \quad .$$

For electrons that means $T_{matter} << 5.9\ G^0K$! For a proton it's another factor of a thousand higher still. As we assumed above, the matter in the Universe since very early on is non-relativistic. Assuming that, roughly speaking, the amount of matter remains constant, it follows that the energy density of matter varies as the reciprocal of the shrinking volume scale, i.e.,

$$\rho_{matter} \sim N\ m / V \sim (1 / L(t_{Universe}))^3.$$

For photons, however, the energy is $E = p\ c$, where p is the momentum which we have seen satisfies $p \sim (1 / L)$. Assuming that, roughly speaking, the number of photons remains constant (a much subtler matter, but justifiable), the energy density of radiation will vary as

$$\rho_{rad} \sim N\ p / V \sim (1 / L(t_{Universe}))^4.$$

At present the matter energy density greatly exceeds the radiation energy density and we say we are in the **matter dominated era** of the Universe. The current estimates (from observations) yield (very roughly),

$$(\rho_{rad} / \rho_{matter})_{now} \sim \ (1/10{,}000)$$

But, as we consider earlier epochs of the Universe, where $L(t_{Universe})$ is much smaller than now, ρ_{rad} will increase faster than ρ_{matter}. Using the flat space deceleration model we expect,

$$(\rho_{rad} / \rho_{matter}) \sim (1/L(t_{Universe})) \sim (1/t_{Universe})^{2/3}.$$

Consequently,

$$10{,}000 = [1/(1/10{,}000)] \sim [(\rho_{rad} / \rho_{matter})_{transition}/(\rho_{rad} / \rho_{matter})_{now}]$$

$$= [t_{U\text{-}now} / t_{U\text{-}transition}]^{2/3} \sim [9.3\text{Gyr} / t_{U\text{-}transition}]^{2/3},$$

or, $$t_{U\text{-}transition} \sim 9.3\text{Gyr} /(10{,}000)^{3/2} \sim 9{,}300 \text{ yr},$$

once again in the right ballpark (**Fig. IV.5**)!! Our *best* estimates place the transition from radiation domination to matter domination at about 2,500 yr after the BB.

We're finally ready to turn to the unsolved problems of the 21st century, **The New Cosmology** proper.

Appendix A: Hubble parameter for power law expansion

We denote the time dependent distance between galaxies A and B by, $D_{AB}(t)$. We say that a model of the expanding Universe follows a power law expansion if, for any two cosmological times, t and t_0, we have,

$$D_{AB}(t) / D_{AB}(t_0) = (t / t_0)^n,$$

where n, the exponent, is positive and constant (If n were negative, we would have power law contraction). If $V_{AB}(t)$ is the recession speed between the A and B galaxies, then the power law expansion yields (this step comes from elementary calculus)

$$V_{AB}(t) / D_{AB}(t_0) = n\, t^{n-1}/ t_0^{\,n}.$$

If n – 1 is negative, the recession speed is slowing down with time and we have decelerating expansion. If n – 1 is positive the recession speed is increasing with time and we have accelerating expansion. For $n = 1$ we have constant recession speed between any two galaxies.

For all of these cases the Hubble parameter is given by,

$$H(t) = V_{AB}(t) / D_{AB}(t) = (V_{AB}(t) / D_{AB}(t_0)) / (D_{AB}(t) / D_{AB}(t_0))$$

$$= (n\, t^{n-1} / t_0^{\,n}) / (t^n / t_0^{\,n}) = n / t.$$

So for any power law expansion, the Hubble parameter slowly decreases as the Universe gets older and its reciprocal is proportional to the age of the Universe.

Appendix B: Our proper motion *through* space

Speaking roughly (between one and two significant figures) and keeping in mind the varying directions of the motions involved, our Earth orbits the Sun at about 30 km/sec. The Sun orbits the center of the galaxy at about 220 km/sec. The galaxy moves around the Local Group of galaxies at about 50 km/sec and the Local Group moves in the Virgo supercluster of galaxies at about 200 km/sec. The net effect of these motions is to give the Earth a proper motion *through* space with an average speed of about 400 km/sec.

With the current value of the Hubble parameter, (71 km/sec)/Mpsc, a distant galaxy would have to be at least 15 Mpsc away before the cosmological expansion of space dominates over our motion through space.