

II. Wave Particle Duality

In late nineteenth century physics a particle was a particle and a wave was a wave and never the twain shall meet. This changes dramatically in the quantum world! So we need to consider this deep divide.

1. Classical particles and waves: In classical physics a **particle** was a *tiny piece of matter* (the extreme limit of a ‘point’ particle was supposed to occupy, at any moment, just a single point of space). A particle moved along a *continuous* path through space in such a way that, at any moment of time, its **acceleration** was equal to the total **force** acting on it divided by its **mass**, i.e., $\mathbf{a} = \mathbf{f} / m$, or

$$\mathbf{f} = m \mathbf{a} \quad \leftarrow \text{Newton's 2nd Law; THE basic equation of classical physics.}$$

In particular, if two or more particles arrived at the same place at the same time then, at that place and time, one just had a collection of several particles. They might collide with one another and move apart or they might attract one another (due to forces between them) and stay close together. The mass of such a system of particles is just the sum of the individual particle masses and the other physical properties associated with the individual particles (eg. **kinetic energy**) are just ‘added’ together to get the corresponding property of the collection. The particles are *distinguishable individuals* and they don’t lose their identity by close association.

A **wave** was regarded as a very different kettle of fish! A wave was not a piece of matter and a wave was rarely ‘tiny’. Waves might be *generated* by pieces of matter, i.e. particles, as in the case of water waves or sound waves or ‘amber waves of grain’. But such waves were not *just* the collection of the pieces of matter that generated them. They were, rather, organized patterns of motion that the particles, taken as a whole, engaged in (**Fig. 2.1a, b, c**).

And in the case of other kinds of waves, the material basis of composition seemed even more indirectly related. Consider a heat wave (not the meteorological kind but a directed, propagating wave of oscillating temperature). Temperature had been recognized as a measure of average *random* molecular kinetic energy. So such a heat wave might be said to be generated by moving molecules. But the most useful way to understand

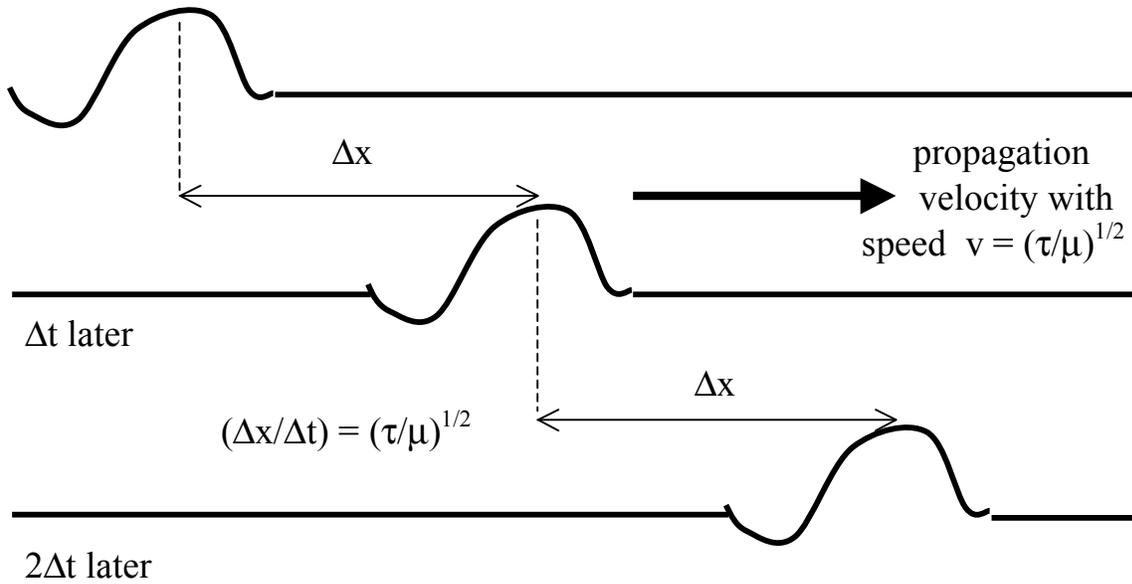
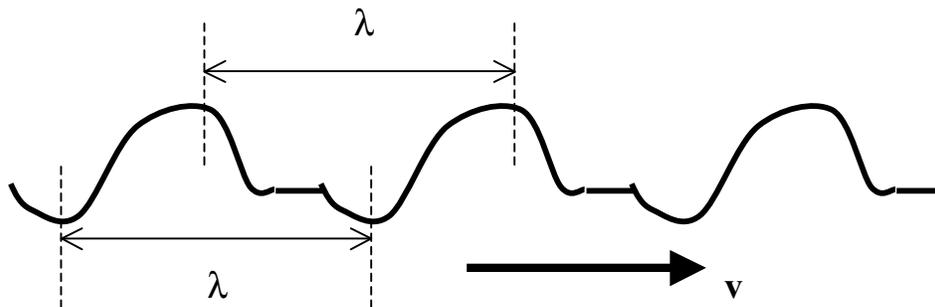


Fig. 2.1a: Transverse wave (particle motion perpendicular to wave motion) pulse traveling along taut string with tension, τ , and mass density, μ .



$$v = f\lambda \quad (1/f) = \text{time required to travel the wavelength, } \lambda.$$

Fig. 2.1b: Periodic transverse wave traveling down a taut string with wavelength, λ , and frequency, f .

such a wave was as an *organized* pattern of propagating oscillations of *random* kinetic energy! Somewhat more abstract than a collection of particles.

And electromagnetic waves, such as light waves and the then recently discovered radio waves for example, were even less directly tied to matter.

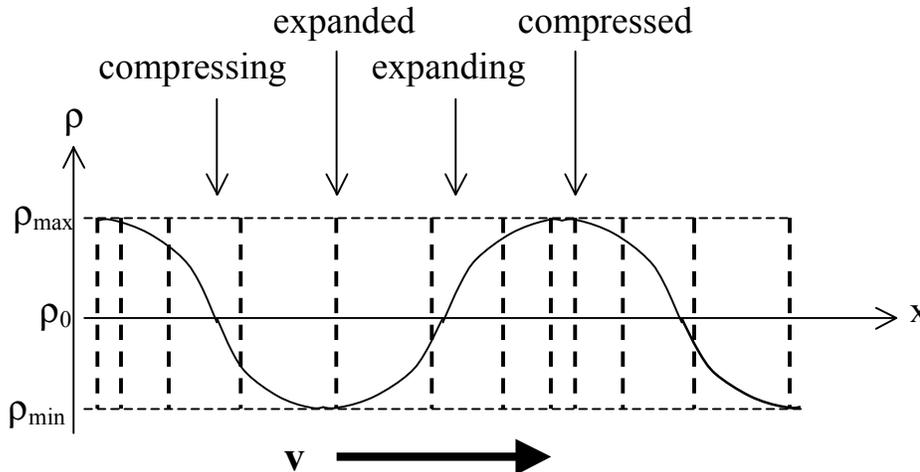


Fig. 2.1c: Longitudinal (particle motion parallel to wave motion) sound wave propagating to the right through a continuous medium. The consecutive vertical dashed lines contain equal amounts of matter between them. The solid curve indicates the progressive variation in mass density.

To be sure, almost everyone believed that there was a material basis to electromagnetic waves (**Fig. 2.2**) and, in the best academeze, it was dubbed the **Luminiferous Aether**. But no one had ever been able to display any direct evidence of its existence and, as Einstein (who administered the coup

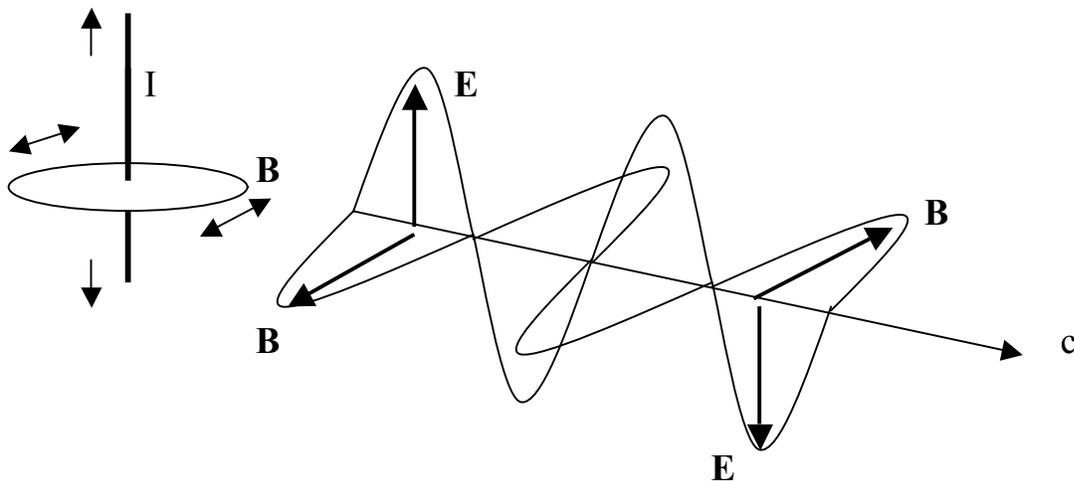


Fig. 2.2: Oscillating electromagnetic field propagating away from source consisting of oscillating current in vertical antenna wire

de gras to the Aether in his Special Relativity Theory) later reminisced, “One got used to operating with these [waves] as independent substances, without finding it necessary to give oneself an account of their mechanical nature; thus mechanics as the basis of physics was being abandoned, almost unnoticeably, because its adaptability to the facts presented itself finally as hopeless.” In particular, waves, whether lending themselves to a mechanical (particle based) interpretation or not, all displayed common properties and modes of behavior that were starkly different from those of particles and which encouraged people to think of them as ‘independent substances’.

2. Superposition and interference: FIRST: when portions of different waves of the *same kind* come together in the same region of space at the same time one does *not* have a collection of distinct waves. Instead the original waves merge to form a single total or **resultant wave** and the original waves cease to have any individual physical existence.

If the original waves are ‘small’ or ‘weak’ enough then the resultant wave is described as a **superposition** of the original waves. This means that at any point in space and time where the waves have merged, a set of quantities called **amplitudes** are evaluated for the resultant wave by *adding*, either algebraically or vectorially, the corresponding amplitudes that the original waves would each have had if they were the only wave present. These amplitudes essentially measure how ‘big’ or ‘strong’ the waves are (**Fig. 2.3a,b**).

Water waves and sound waves have just one, algebraic, amplitude each, namely relative height and relative pressure respectively. Electromagnetic waves have two, vector, amplitudes, namely the electric field and the magnetic field (**Fig. 2.2**). Notice that these amplitudes can all take negative as well as positive values. This is always true of wave amplitudes. To measure the strength or intensity of a wave with an intrinsically positive quantity we use the squares of the amplitudes. But this means that when adding amplitudes to superimpose waves the resultant squared amplitude may be smaller as well as larger than the original squared amplitudes. When the amplitudes add up to give larger squared amplitudes we call it **constructive interference (Fig. 2.3a)**. When they add up to give smaller squared amplitudes we call it **destructive interference (Fig. 2.3b)**.

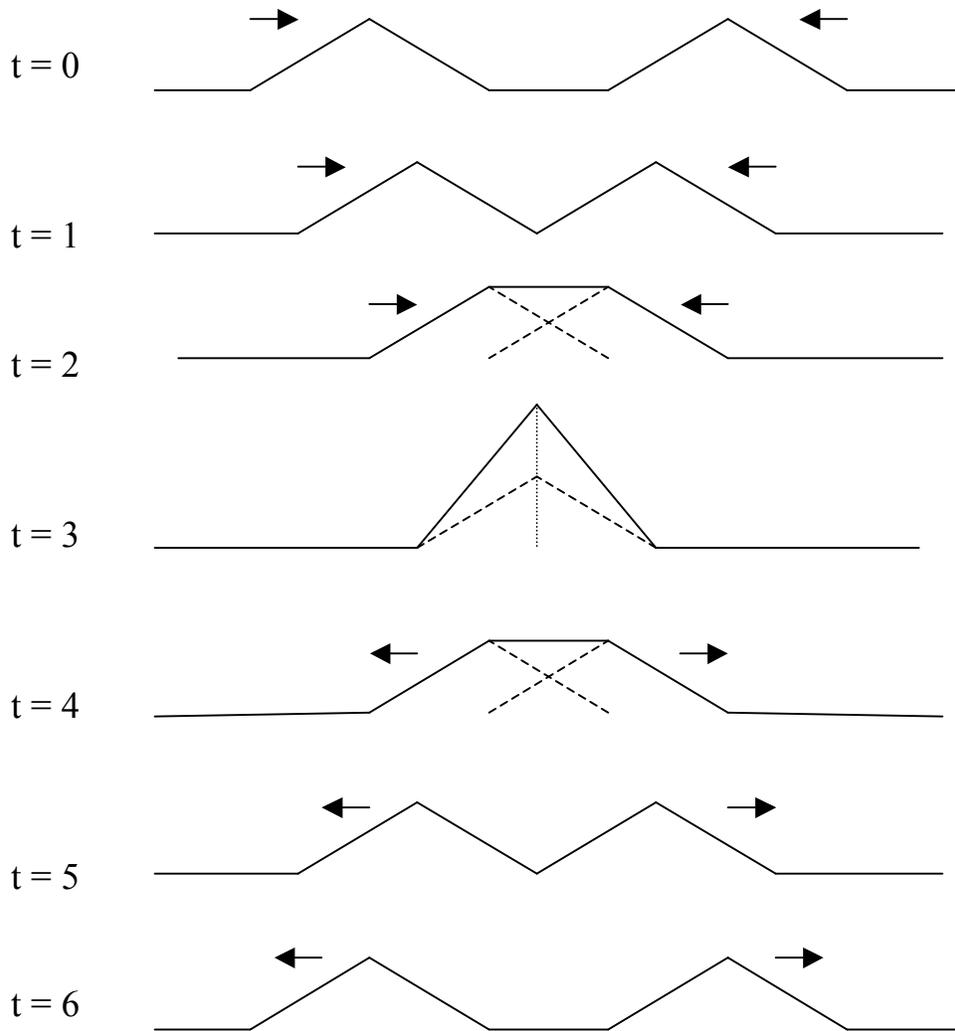


Fig. 2.3a: Superposition and *constructive* interference of two approaching and then receding wave pulses.

We will confine our discussions to cases where the contributing wave amplitudes form the resultant wave amplitudes by simple addition. This is called the superposition regime. For electromagnetic waves superposition is the correct way to form the resultant wave regardless of how strong the contributing waves are, so far as we know. But for sound waves and water waves, for example, superposition gives the wrong answer if the

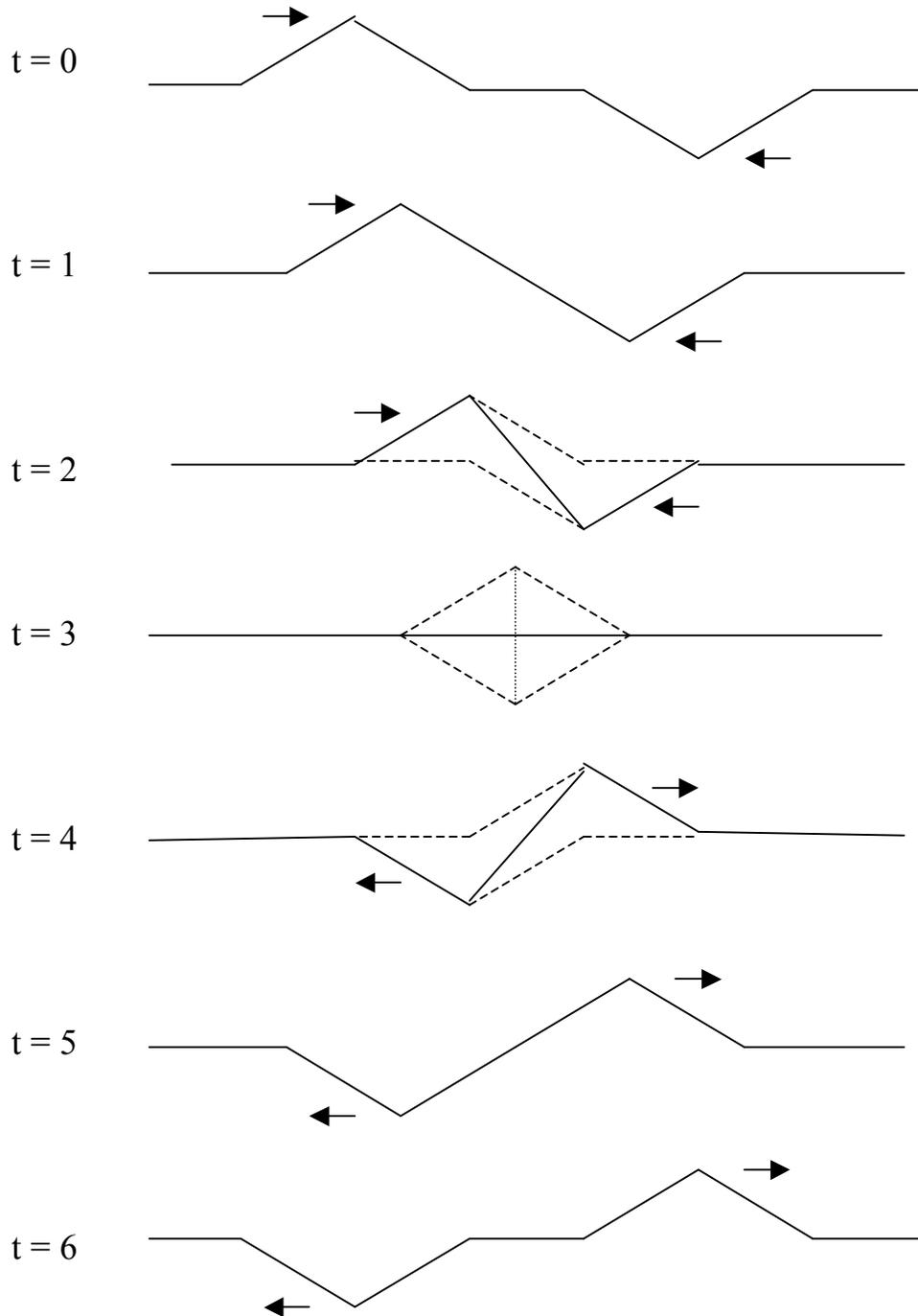


Fig. 2.3b: Superposition and *destructive* interference of two approaching and then receding wave pulses.

contributing waves are too strong. And the right answer in those cases turns out to be terribly complicated – and not very well understood – even today!

SECOND: if the amplitudes of a wave vary from point to point in space and moment to moment in time in a very special smooth way, called sinusoidally, (**Fig. 2.4**) the wave is called a **harmonic** wave and has a definite **frequency** and **wavelength** and **propagation speed**. Such a wave repeats itself again and again. The wavelength is the distance between consecutive repetitions in space at any moment in time. The **frequency** is the number of consecutive repetitions per second at any point in space. The *universal and fundamental relationship* between frequency, f , wavelength, λ , and propagation speed, v , for harmonic waves, is,

$$f \lambda = v.$$

THIRD: using the method of superposition one can analyze any resultant wave into *hypothetical* contributing waves in an infinite variety of ways. This is one sense in which the *actual* contributing waves lose their physical existence when they merge. However, if one considers analysis into harmonic waves only, then there is always one and only one way to analyze *any* wave into harmonic waves. This procedure is called **Fourier analysis**,

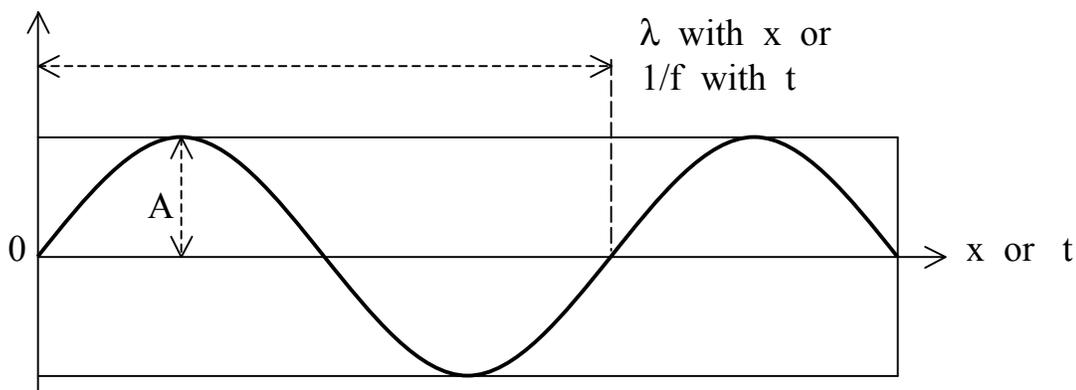
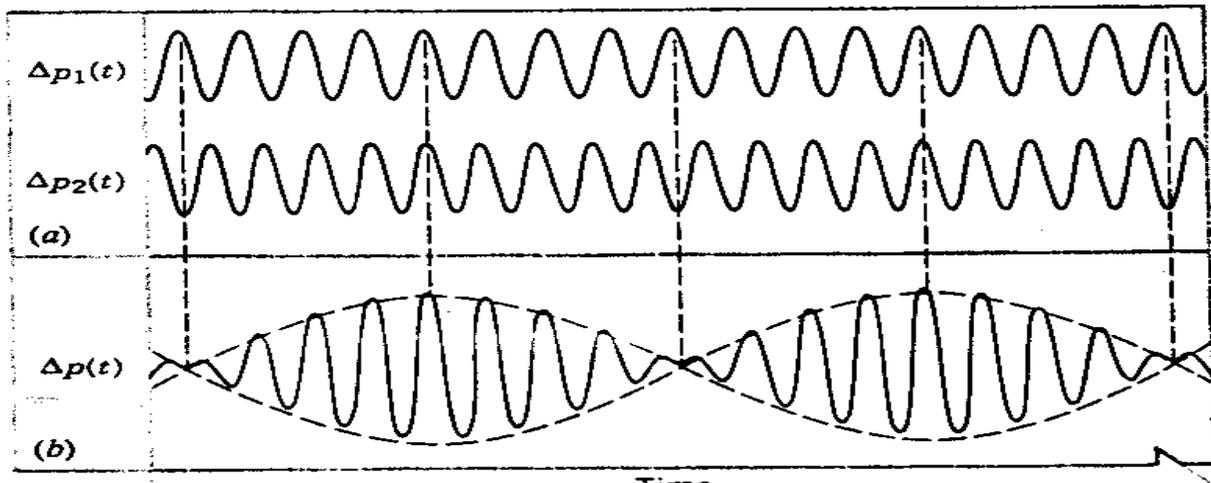


Fig. 2.4: Periodic, sinusoidal (harmonic) wave with amplitude, A , wavelength, λ , and frequency, f , identified.



The top harmonic wave has wavelength, λ , and frequency, f , and the middle harmonic wave has wavelength, λ' , and frequency, f' . If the horizontal axis represents distance covered, then the interval between the consecutive vertical dashed lines is $4\lambda = 4.5\lambda'$. If the horizontal axis represents time elapsed, then the interval is $4/f = 4.5/f'$. In either case the resultant wave, in space or time, is the bottom curve displaying **beat phenomena**.

Fig. 2.5: Superposition of harmonic waves with similar wavelengths or frequencies resulting in the “beat phenomena”.

after Joseph Fourier¹ who discovered it in the early 1800's while researching the propagation of heat waves.

FOURTH: if a resultant amplitude associated with a particular kind of wave is called A , and the propagation speed of the wave is v , then, in the superposition regime, the rate at which the wave carries energy through space, the so called **energy flux density**, is proportional to

$$v A^2.$$

3. Photons: Imagine very low intensity light of precise frequency striking a metal surface. The sum of the squares of the amplitudes of the light is $E^2 + B^2$ (the square of the magnitude of the electric field, \mathbf{E} , plus the square of the magnitude of the magnetic field, \mathbf{B}) and the speed of propagation is c , the speed of light. So the **energy flux density** of the light is proportional to

$$c (E^2 + B^2),$$

and would be small for low intensity light, i.e. small E and B.

Now the metal contains more electrons than atoms and many of those electrons, so called **conduction electrons**, are easily moved around from atom to atom by electric and magnetic fields. That's what makes a metal a metal. So these electrons can absorb energy from the incident light and once they have absorbed enough energy some of them will fly off the surface of the metal.

One would expect that both the *time* one would have to wait before any electrons absorbed enough energy to leave the surface and the *maximum* energy with which they leave would depend strongly on the intensity of the light and to some degree on the frequency. Once the process got going the number of electrons leaving per second would increase with the intensity of the light.

But in fact, that is not what happens. What does happen is called the **photo-electric effect**.

What happens is *first*, that if the frequency is below a certain *critical value*, f_0 , dependent on the type of metal, then *no electrons are liberated* from the surface at all, regardless of the intensity and regardless of how long one waits. *Second*, when the frequency of the light is above this critical value there is *no discernable time delay* before electrons start leaving the surface regardless of how weak the intensity of the light. *Third*, the *maximum* energy of the emitted electrons is proportional to the difference of frequencies, $f - f_0$, and the proportionality constant is Planck's constant, h , i.e.

$$E_{\max} = h (f - f_0) \quad \text{for} \quad f > f_0 ,$$

and E_{\max} is *completely independent of the intensity of the light!* *Fourth*, the only thing that depends on the intensity of the light is the number of electrons emitted per second.

In 1905, the same year Einstein published his Special Theory of Relativity and his analysis of Brownian motion as a means of measuring properties of atoms, and his $E = M c^2$ connection between energy and mass, he also proposed that for understanding this photo-electric effect we should think of

the energy in a light wave as distributed discontinuously through the wave in tiny packets. The energy of these individual packets would be given by,

$$E = h f,$$

and $h f_0$ is the minimum energy required to liberate a surface electron.

The intensity of the wave was proportional to the density of these packets in the wave. If $f > f_0$, collisions between these packets and electrons in the metal can give rise to the emitted electrons and, regardless of how low the density of packets was, *some* collisions might occur immediately. These packets came to be called **photons**. As a bonus this idea made it easier to understand the success of Planck's analysis of thermal radiation. Thus light was to be regarded as *possessing both wave-like properties and particle-like properties*. This was the beginning of **wave-particle duality**.

The biggest problem with this picture was how to understand interference phenomena.

As I mentioned above, constructive and destructive interference is a natural consequence of the superposition of amplitudes when waves merge. But if photons are going to handle the task of carrying the energy of electromagnetic waves then they have to be roughly plentiful where the squared amplitudes are large and they have to be few and far between where the squared amplitudes are small. Now imagine two merging waves, each with large squared amplitudes. There will be places in the merging region where they destructively interfere, at least momentarily. At these places the plentiful photons from each wave have got to, somehow, become few and far between. It's almost as though they have to annihilate one another! But only at these places! However, the electromagnetic theory of the day required that they have no direct interaction with one another at all.

The matter was brought to a head in 1908 when G. I. Taylor² produced interference phenomena with light beams so weak that, energetically speaking, there could never be more than a few photons in the whole system at any time! How in the world would one lone photon know where to go so that it and its cohorts could, in the fullness of time, produce an interference pattern?!(**Fig. 2.6**)

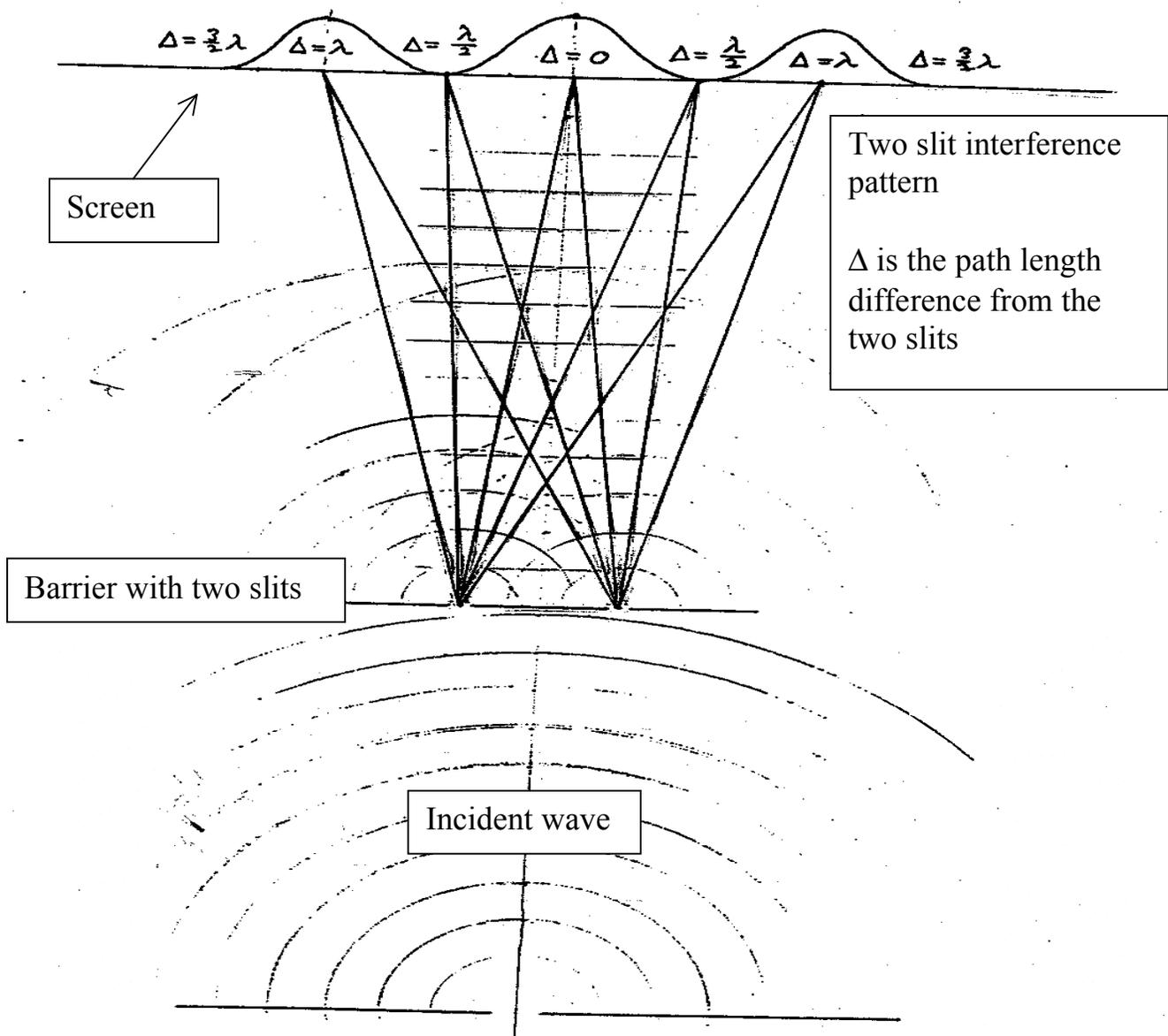


Fig. 2.6: Two slit interference phenomena

In 1922 A. H. Compton³ scattered high frequency electromagnetic waves (X-rays) off of atomic electrons and measured the drop in frequency of the deflected waves as the associated photons lost some of their energy to the electrons. Classical deflected waves would not have changed their frequency, only their amplitude!

Today, with available technology to detect individual photon interactions, the photon concept is firmly entrenched. But it has also become clear that the photons are not so much *in* the light waves as they are an *aspect of* light that is *complementary* to the wave aspect. More on *that* later. See the **Appendix** for a technological application of the photon aspect of X-rays.

4. Matter waves: In 1924 a French aristocrat, Prince Louis de Broglie⁴, was writing a PhD dissertation in which he was trying, among other things, to obtain a deeper understanding of the quantization rules that Bohr had used to account for the light spectra emitted from atoms. Impressed by Einstein's photon concept he conjectured that perhaps atomic electrons could be better understood if, besides their particle nature, they also possessed a wave-like aspect.

If the electrons orbiting an atom were 'acompanied' by an harmonic wave then perhaps the orbits permitted by Bohr's rules were just those in which one could fit an integral number of wavelengths on the circumference of the orbit (**Fig. 2.7**). A simple calculation led to the result that if this requirement were going to yield Bohr orbits with the quantized angular momentum

$$J_z = n (h/2\pi),$$

Then the magnitude of **momentum** of the orbiting electron had to be related to the wavelength of the 'accompanying' wave by,

$$m v = h / \lambda.$$

This was neat because de Broglie had noticed that Einstein's relativity theory required that if Einstein's photons had an energy of $h f$ then they also had to have a momentum of h / λ .

The reason de Broglie thought this explained things was that if the wavelength did not have this value, then as the wave propagated around the circumference of the orbit it would destructively interfere with itself like a snake eating its tail and disappear. It was a flimsy argument but these were desperate times!

In fact, de Broglie's examining committee didn't want to accept his

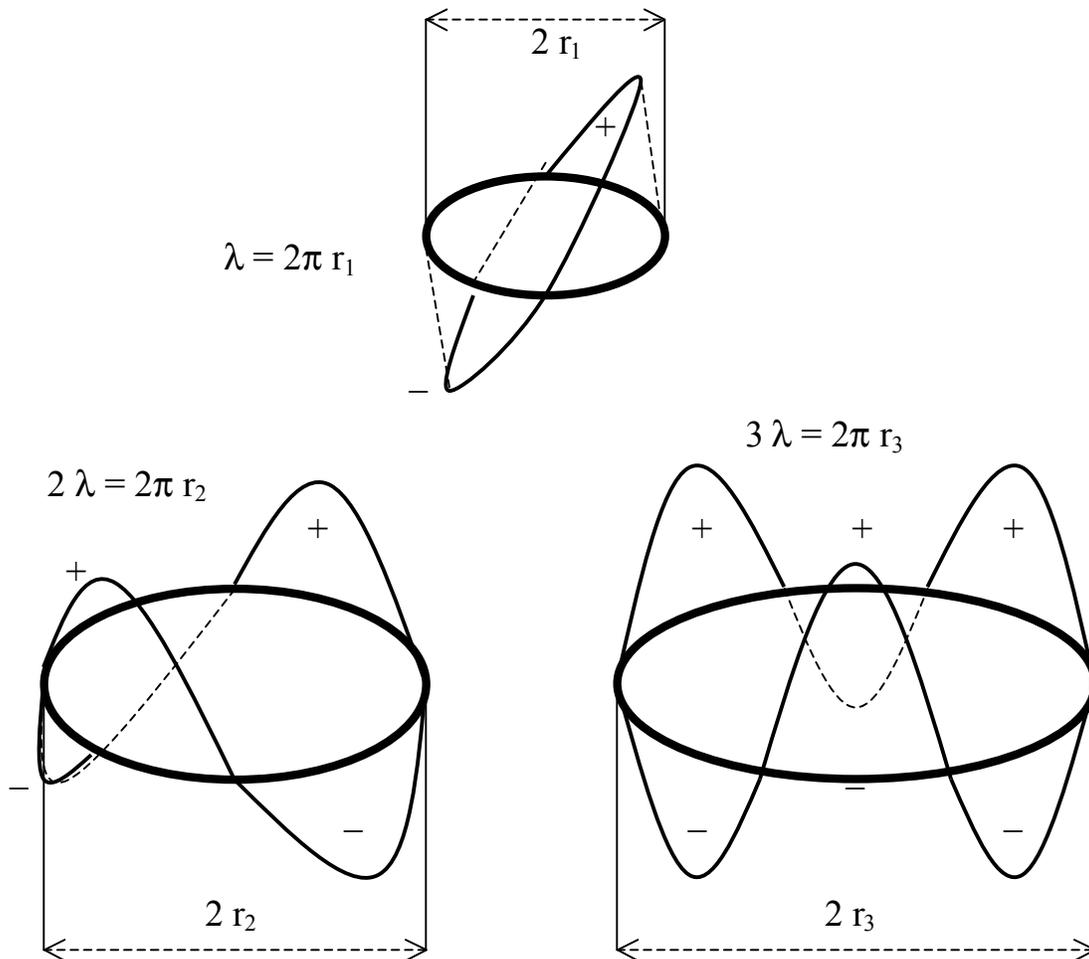
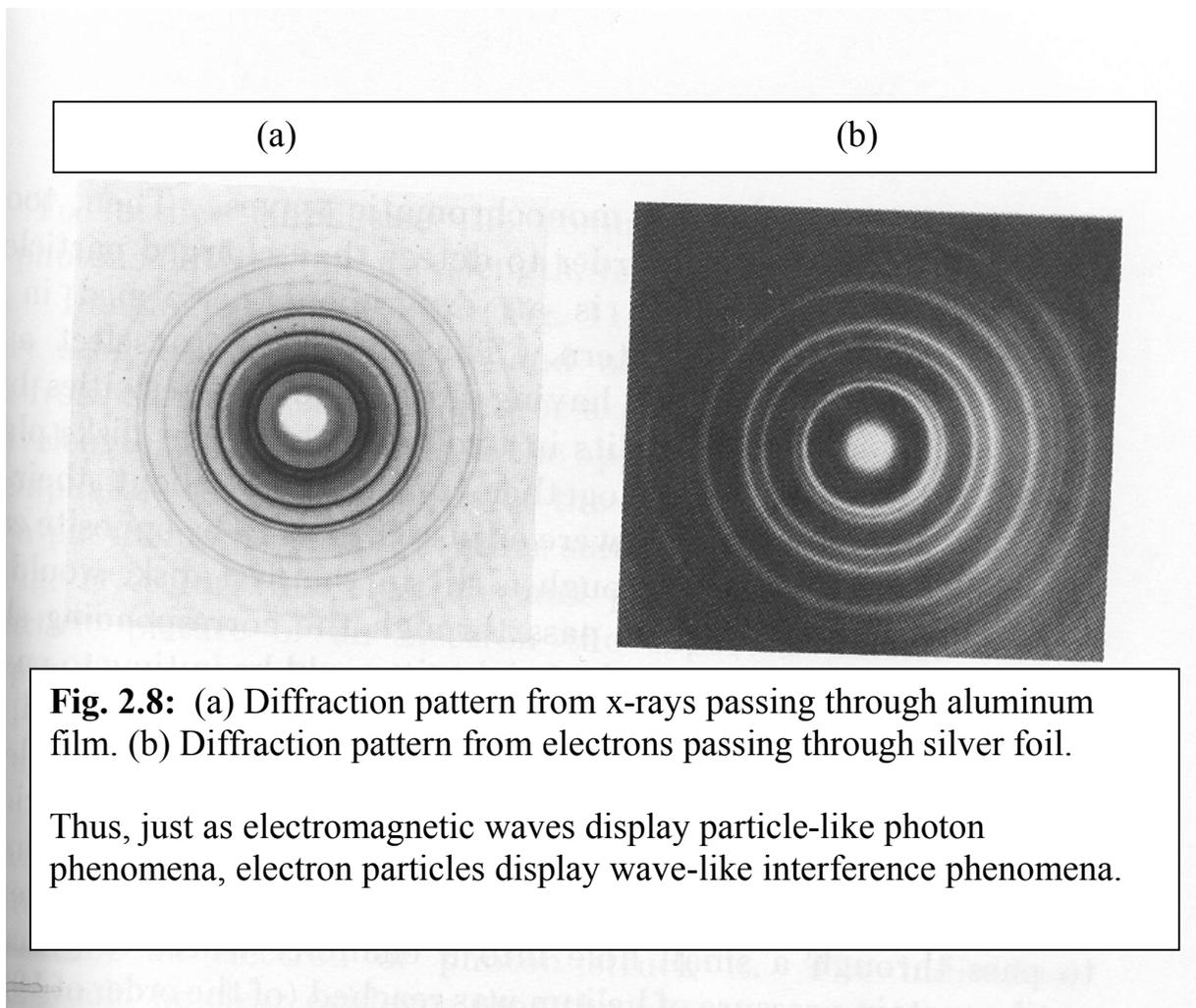


Fig. 2.7: De Broglie waves fitted onto Bohr orbits

dissertation with this argument in it. But Einstein happened to be visiting at the time and he looked at it and said it just might not be so crazy! Indeed, he soon realized that de Broglie's idea could explain the anomalously small contribution that electrons make to the heat capacity of metals, a long standing problem.

By 1927 and repeatedly after that, Davison and Germer⁵, had displayed interference phenomena in the scattering of electrons from crystals (**Fig. 2.8**). Since then interference phenomena have been demonstrated for waves of neutrons and whole atoms. Thus the wave aspect of matter has been firmly established. This means that our former puzzlement as to how a lone photon knows where to go in weak interfering light waves now applies to a lone particle of matter in weak interfering matter waves as well!



Once again, as with photons, the conceptual and interpretation problem with this apparent interference of ‘particles’ was apparent. This time, however, it was more visceral. Electrons had been accepted as tiny material particles for some time by 1927. Unlike the photon *hypothesis*, which some could still hold doubts about, the interference of electrons seemed downright paradoxical! How in the world could the number of non-interacting electrons arriving at a particular area of a detecting screen be *reduced* by opening *additional* paths by which the electrons can reach the area (**Fig. 2.9**).

Meanwhile, in 1926, Erwin Schroedinger⁶ obtained a fully developed mathematical account of the motion of the electron waves and the generalization to *arbitrary* material systems was not long in following.

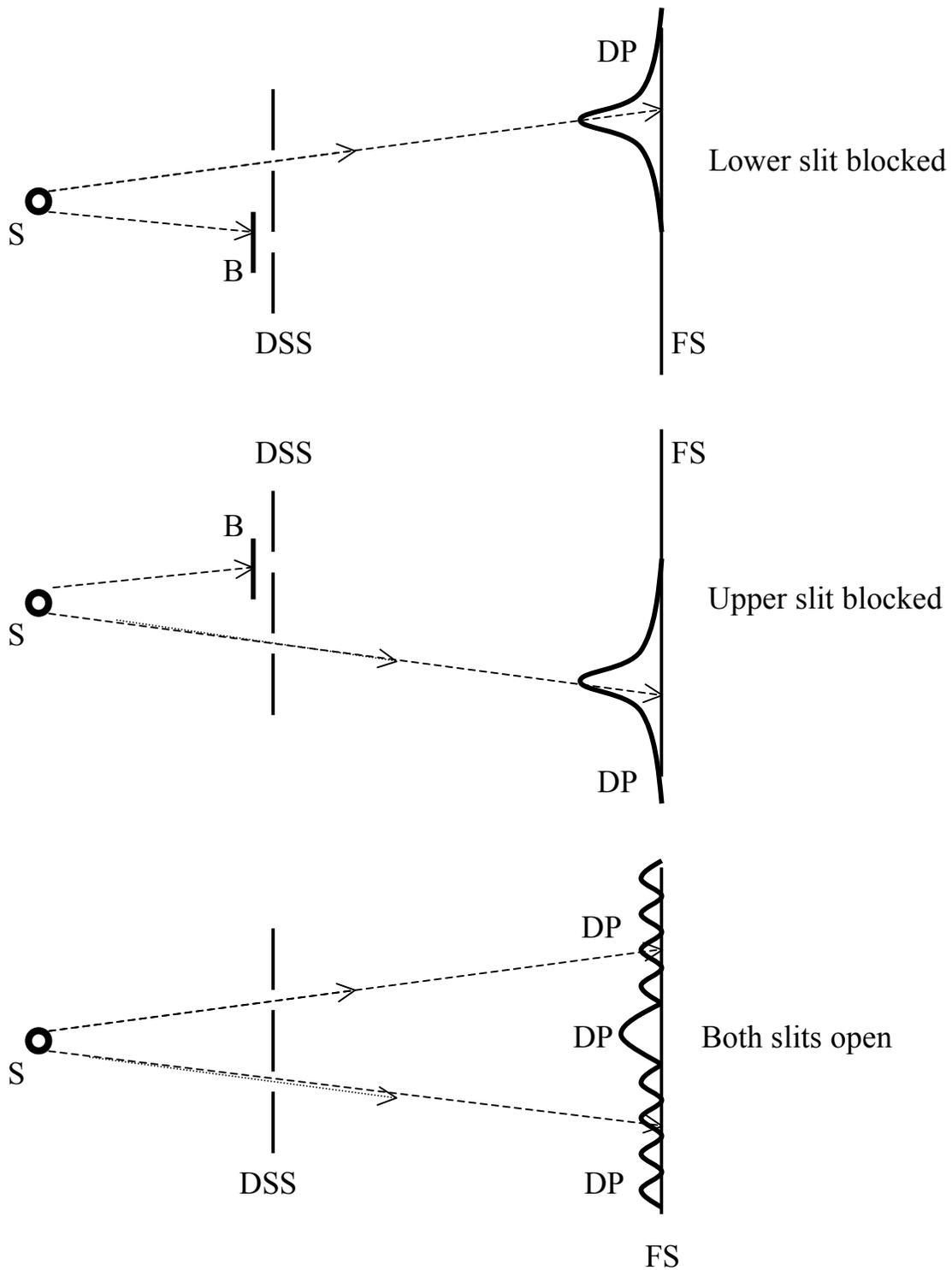


Fig. 2.9: Electron distribution pattern (DP) on final screen (FS) after leaving source (S) and passing (above) through one slit of a double slit screen (DSS) using a barrier (B) or passing (below) through both slits.

But the new **wave mechanics** of Schroedinger had some novel paradoxical features of its own. In particular, in describing where the particle could be found when the total energy was specified, it allowed the particle to be found where the kinetic energy should be negative! (**Fig. 2.10**)

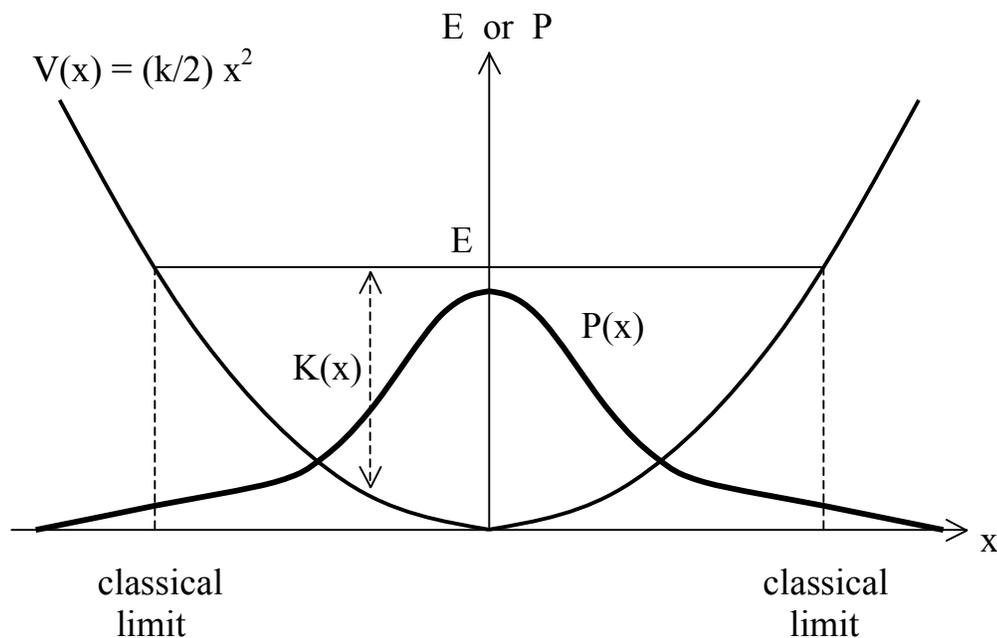


Fig. 2.10: Wave-particle duality ‘paradox’. The potential energy, $V(x)$, is non-negative parabolic, the kinetic energy, $K(x)$, is inherently non-negative and the total energy, E , is constant. Classically, the oscillating particle is confined between the classical limits. According to de Broglie – Schroedinger wave mechanics the particle can be found beyond the classical limits where the kinetic energy would be *negative*!

Footnotes

1. Joseph Fourier (1768 – 1830)
2. Geoffrey I. Taylor (1886 – 1975)
3. Louis de Broglie (1892 – 1987)
4. Arthur H. Compton (1892 – 1967)
5. Clinton Davison (1881 – 1958) and Lester Germer (1896 – 1971)
6. Erwin Schroedinger (1887 – 1961)

Appendix A: Computerized (absorption, axial) tomography (CAT) scans

Narrow X-ray beams are passed through a thin section of the body in 'all' directions of a 360° range. Absorption of the X-ray energy (photons) by the body is measured by detectors arrayed around the opposite side of the body. The appropriate portion of the body is passed axially through the device to sequentially irradiate many sections. The energy absorption data is transformed into a computer generated image of body organs by data analysis software.