Math Appendix 3: De Broglie waves and Bohr orbits

The rule that Bohr originally used to determine his atomic orbits was that the angular momentum of the orbit was an integer multiple of Planck's constant, h , divided by 2π. If the orbit radius is r and the ordinary momentum of the electron is, $p = m v$, then the angular momentum, relative to the nucleus, is given by,

$$L = m v r = p r = n h / 2\pi . \qquad (1)$$

As indicated in **Math Appendix 2** this was eventually replaced by the rule that the action, A , the product of the time for one complete cycle of motion and the average value of the product of the momentum and the velocity, $T < p v >$, was an integer multiple of h. For a circular Bohr orbit this yields,

$$A = T < p v > = p (v T) = p (2\pi r) = n h . \qquad (2)$$

That last equality is equivalent to Bohr's original assumption. Now de Broglie says the rule should be that an integer number of his wavelengths, λ, of matter waves must exactly fit into the circumference of the orbit, i.e.,

$$2\pi r = n \lambda . \qquad (3)$$

But the wavelengths are related to the momenta of the electron by,

$$p = h / \lambda \qquad \text{or} \qquad \lambda = h / p. \qquad (4)$$

So we have,

$$2\pi r = n \lambda = n h / p. \qquad (5)$$

But $p r = n h / 2\pi$ from (1) and $p (2\pi r) = n h$ from (2) and

$2\pi r = n h / p$ from (5) are just three different ways of saying the same thing!