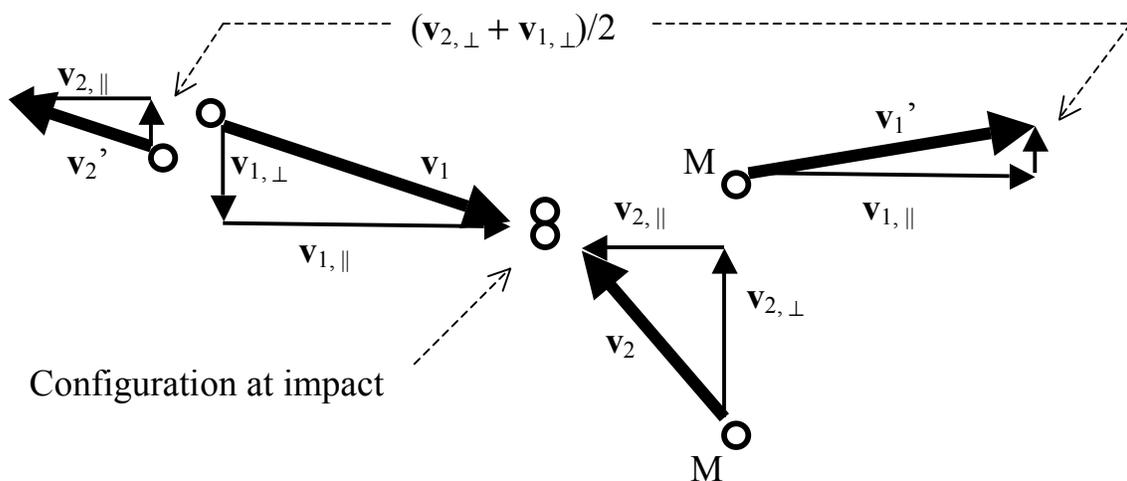


## Energy III: The Conservation of Energy

### 1. Heat as a Conserved Substance

A long development was required before it emerged that a deeper understanding of the universal dissipation of mechanical energy was to be had through the study of **heat** and **temperature**.

Not everyone thought that a deeper understanding was needed. From the work of Newton right through the 18<sup>th</sup> century there persisted a strong minority view that the dissipation of mechanical energy was to be expected. This view held that *matter was composed of absolutely immutable atoms that were not subject to any kind of internal change or deformation*. This meant they were infinitely hard! Thus in any interaction between two such atoms in which the atoms actually touch, i.e., collide, their hardness would instantly stop their relative motion towards each other and their lack of deformation in the collision would produce no force to generate a rebound or relative motion away from each other (**Fig. 1**). Thus the component of their relative velocity in the direction of the line joining their centers at the



**Fig. 1:** Collision between two, identical, infinitely hard, immutable atoms of mass  $M$  and initial velocities,  $\mathbf{v}_1$  and  $\mathbf{v}_2$ . After the collision their velocities are  $\mathbf{v}_1'$  and  $\mathbf{v}_2'$  as indicated. In every instance,  $v_1'^2 + v_2'^2 \leq v_1^2 + v_2^2$ . Thus kinetic energy gradually dissipates in collisions of such 'atoms'.

moment of impact would just cease to exist and, over time, mechanical energy would just dwindle away as the number of collisions between atoms increased.

The opposing minority view, originally championed by Descartes and Leibniz, held that some measure of the ‘quantity of motion’ must be conserved in nature and rejected the argument from infinite hardness if not the whole conception of atoms. At first an internal dispute raged over whether the sum of the products of mass and speed (Cartesian camp) or the sum of the kinetic energies, called *vis viva* at the time, (Liebnizian camp) of all the pieces of matter in an isolated system should be conserved. Eventually this view evolved into the conservation of what we have called mechanical energy which posed a problem in the face of dissipative forces. Perhaps the dissipation was transferring energy to microscopic forms of mechanical energy that were not observationally accessible.

The resolution of the dispute would hinge on the study of heat and temperature.

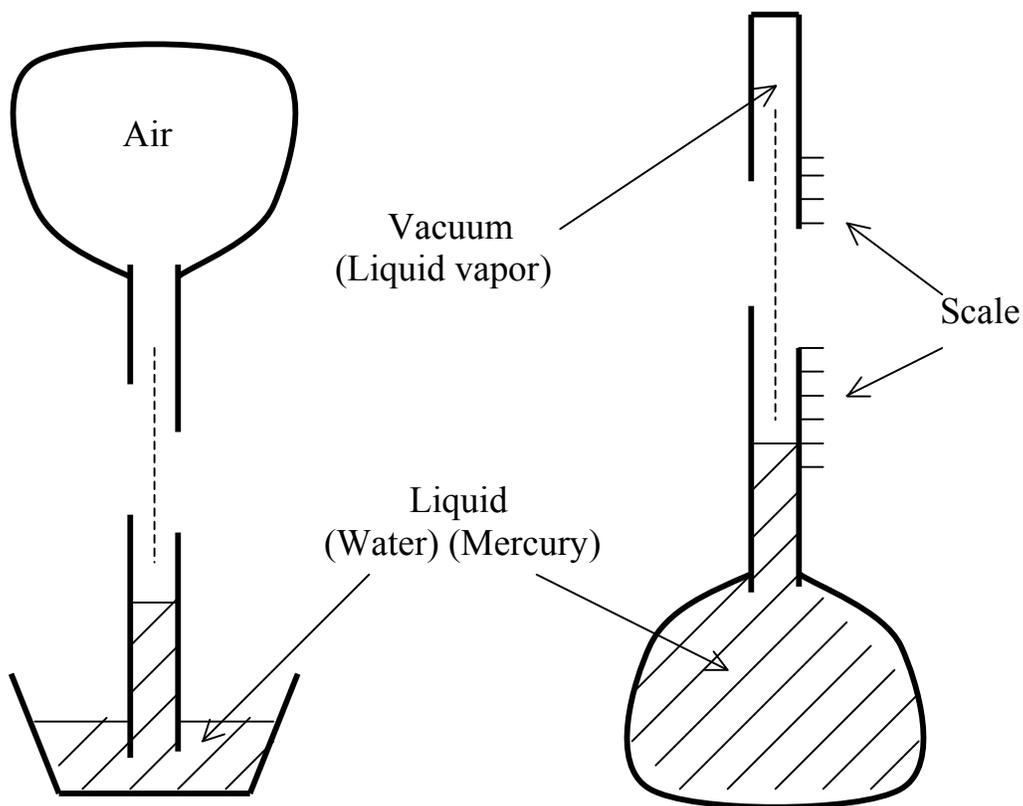
The initial motivation for trying to bring the study of heat and temperature within the domain of natural science was the obvious importance of thermal phenomena in weather and medicine.

### **a. Temperature**

The first devices for indicating changes of temperature and levels of temperature in more or less reproducible ways were introduced in the early 17<sup>th</sup> century. While the indicator was usually a changing level of liquid, often water, in a vertical tube, the devices did not operate on the basis of thermal expansion of the liquid, as we are used to, but rather, on the basis of the thermal expansion at ‘constant’ pressure of the gas (air) trapped in the tube above the liquid (**Fig. 2**). This mode of operation was not initially well understood and often led to confusion by mixing temperature with the effects of changes in atmospheric pressure. Furthermore, the instruments usually had no numerical scale attached, or at least no standardized one, so that only qualitative estimates could be achieved. Nevertheless, with these first **thermoscopes** people in Europe learned such surprises as that winter air could be colder than the snow and ice on the ground, that a mixture of snow and salt was colder than ‘pure’ snow, that summer air could be warmer than pond water, etc.

While uncritical tradition attributes the invention of the **thermometer** (a thermoscope with a scale) to Galileo, the reality is not so clear. He certainly experimented with a well made air thermometer, but quite a few people around the same time had been inventing and developing such devices.

On the other hand, the transition to a sealed liquid thermometer (**Fig. 2**) that worked on the basis of thermal expansion of the liquid is clearly attributable to the Grand Duke of Tuscany, Ferdinand II, sometime between 1641 and 1654.



**Fig. 2:** Early air thermoscope (left) and basic sealed liquid thermometer (right). The vertical tubes (glass) are long and thin compared to the air or liquid holding bulbs.

A piece of good fortune for the development of the science of heat and temperature was that, for the range of temperatures encountered, the

fractional changes in volume,  $\Delta V / V_0$ , of various liquids and solids were very nearly proportional to one another so that each could be expressed as proportional to the change in a single temperature scale. For the Celsius scale we write,

$$\Delta V / V_0 = \kappa_E (T - T_0), \quad (1.1)$$

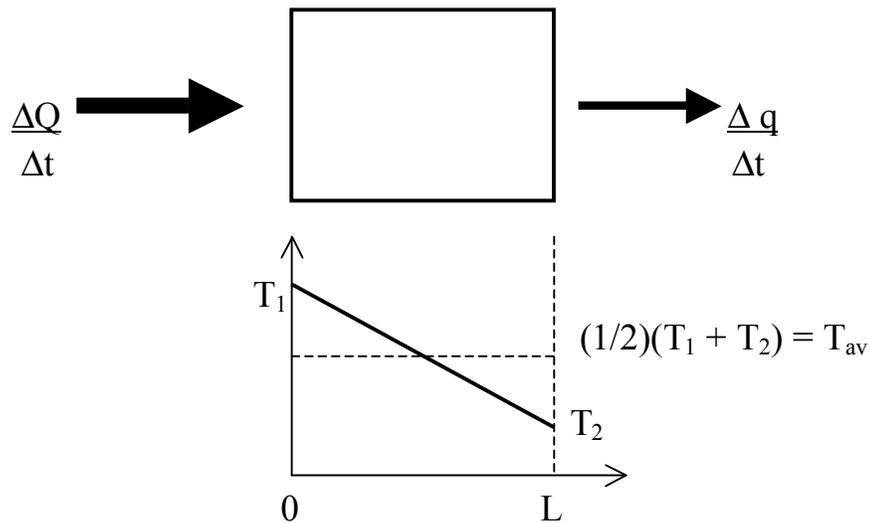
where  $V_0$  is the initial volume,  $T_0$  the initial Celsius temperature and  $\kappa_E$  the coefficient of thermal expansion of the substance. By virtue of introducing linear temperature scales on the thin necks of their thermometers, in which the change in volume,  $\Delta V$ , of the liquid was closely proportional to the change in length,  $\Delta L$ , of the column of liquid in the neck, the indicated change in the temperature scale on the neck was proportional to the actual change in temperature.

But while temperature may measure how ‘hot’ an object or a substance is, the measurement is not always compatible with what we feel when we touch hot or cold things. Nor does it measure how much heat, whatever heat may be, is in a hot or cold object.

The sensory incompatibility is illustrated by the common demonstration of having someone first condition their two hands by immersion of one in very cold water and the other in rather hot water. After awhile the hands are removed and both inserted in a single vessel with room temperature water. To the cold hand the new water feels hot, sometimes painfully so initially. To the hot hand the new water feels cold, really cold! Clearly the conditioned hands are not sensing the temperature of the new water, which is the same for both hands!

The natural conjecture that the hands are sensing the difference of their conditioned temperature and the temperature of the new water could be dispelled by having the hands be immersed in a bucket of room temperature sand, or, for that matter, just held open to the room temperature air. Clearly something else is going on! What the hands are sensing is the rate of heat flow into the cold hand and out of the hot hand. This is what we always sense when we touch things at different temperatures; how fast the heat flows into (hot things) or out of (cold things) our skin. This rate depends on the **thermal conductivity** and, secondarily, the **heat capacity** of what we’re touching (**Fig. 3**). Thermal conductivity refers to the ease with which heat

can move around in a material or an object and the rate at which heat can flow between it and the immediately surrounding material. Heat capacity refers to the amount of heat that must be gained or lost to raise or lower, respectively, the temperature by one unit or degree.



$$(\Delta Q / \Delta t) - (\Delta q / \Delta t) = C (\Delta T_{av} / \Delta t)$$

$$(\Delta q / \Delta t) = \kappa_{TC} (T_1 - T_2) / L$$

**Fig. 3:** Schematic representation of the relationship between heat capacity,  $C$ , and coefficient of thermal conductivity,  $\kappa_{TC}$ .

### b. Heat capacity

Clearly different amounts of heat are required to raise the temperature of different systems by the same amount. A small container of room temperature water held over a fire will reach boiling temperature more quickly than a large container over the same fire. Similarly for lowering the temperature. Initially, however, this awareness led to the competing theories that the amount of heat required for a given temperature change was *universally* proportional to the mass of material or to the volume of material involved. That neither theory was correct was first demonstrated convincingly by George Martine in 1739. He heated equal volumes of

mercury and water over a fire and found the mercury's temperature to rise twice as fast as the water's. Since mercury was thirteen times as dense as water this would mean that equal masses of each would have required thirteen times the volume of water over mercury, in which case the mercury temperature would have climbed 26 times as fast as the water temperature.

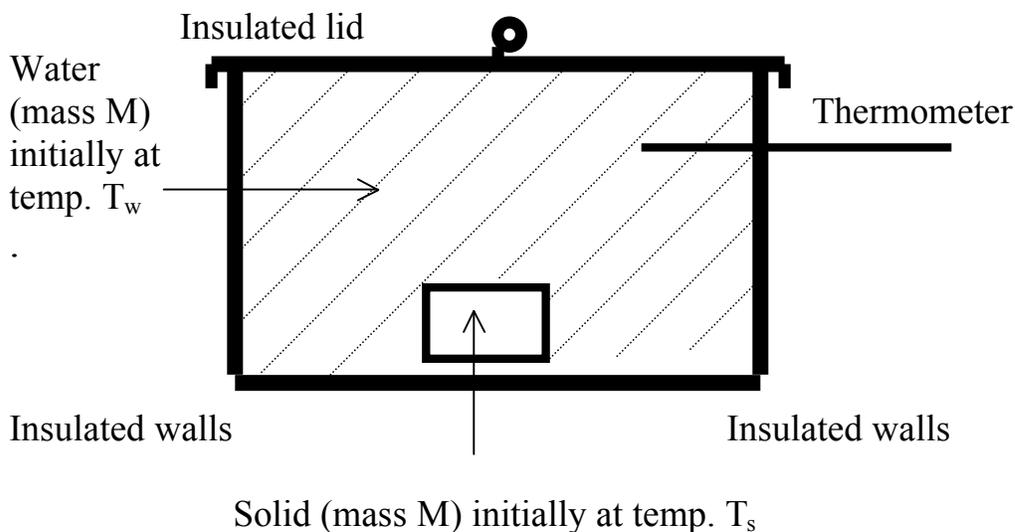
Joseph Black took these results to indicate that different substances had different capacities to hold heat at a given temperature. He introduced the concept that we call **specific heat**, a (near) constant for each substance that measures the amount of heat required to raise the temperature of a unit mass of the substance one degree. If the amount of heat absorbed is denoted by  $Q$ , the mass by  $M$  and the temperature change by  $\Delta T$ , then,

$$Q = c M \Delta T, \quad (1.2)$$

where  $c$  is the specific heat. If  $\Delta T < 0$  then  $Q < 0$  and denotes the loss of heat during cooling. For a given amount of mass,  $M$ , the product,

$$C = c M, \quad (1.3)$$

is called the **heat capacity** of that mass. Black measured the specific heats of various solids (**Fig. 4**) by immersing them in known amounts of water



**Fig. 4:** Schematic apparatus for measuring the specific heat of a solid.

at different initial temperatures and observing the common temperature the combination reached. Suppose a solid, of mass,  $M$ , and at initial temperature  $T_s$ , is immersed in a mass,  $M'$ , of water at initial temperature,  $T_w$ , in an insulated container. The solid and the water will come to the same temperature,  $T$ , as a consequence of the water losing heat to the solid,  $T_w > T_s$ , or the solid losing heat to the water,  $T_s > T_w$ . In either case we have,

$$M' c_w (T_w - T) = M c_s (T - T_s) , \quad (1.4)$$

Which can be solved for  $c_s$  if  $c_w$  is known. But  $c_w$  is used to *define* the unit of heat,

$$c_w = 1.0 \text{ kcal/kgm.}^{\circ}\text{C.} \quad @ \quad 15^{\circ}\text{C} \quad (1.5)$$

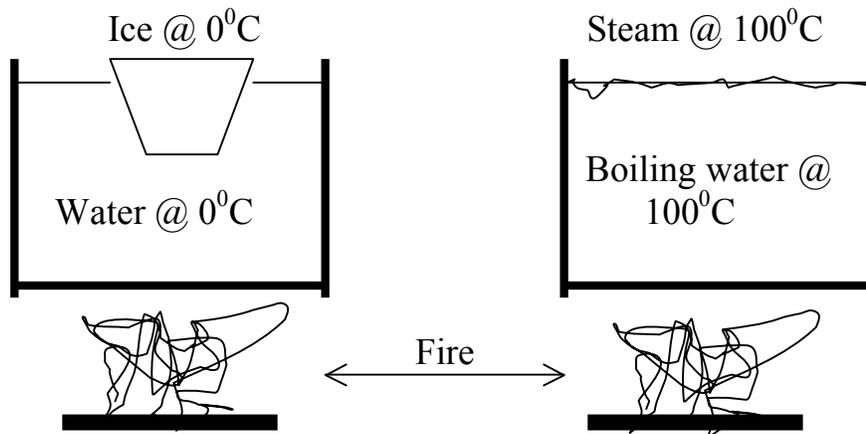
Black measured the specific heats of liquids by mixing known amounts with known amounts of water in similar insulated containers. Most substances have specific heats that are significantly lower than water.

The dominant view of heat at this time, mid to late 18<sup>th</sup> century, was that it was a substance that had no mass or weight, a so-called ‘imponderable’ or ‘subtle fluid’ (because no weight change could be detected upon heating or cooling an object). This raised the problem of *why* different substances required different quantities of heat to reach a given temperature. Black preferred to avoid all theories and just focus on the measurable relationships.

### c. Latent heat

In 1757 Black asked the question, “Why doesn’t snow melt right away as soon as it reaches the melting/freezing temperature?”. He understood clearly that it was a *very good thing* that snow didn’t melt right away since, if it did, every Spring would bring devastating floods. But *why* didn’t this happen? He concluded that all substances contain a certain amount of heat that the thermometer is not sensitive to, that this heat is absorbed or released only at the temperatures for boiling/condensing and for melting/freezing (**Fig. 5**). He called this heat per unit mass, which differed from substance to substance, the **latent heat**, denoted by  $L$ . Each substance had two latent heats, the **latent heat of fusion**,  $L_f$ , (for melting/freezing) and the **latent heat of vaporization**,  $L_v$ , (for boiling/condensing). For water,

$$L_f = 80 \text{ kcal/kgm}, \quad L_v = 540 \text{ kcal/kgm} . \quad (1.6)$$



**Fig. 5:** Latent heat: the water temperature will not rise until (left) all the ice is melted or (right) all the liquid has boiled away. Instead all the heat is going into melting the ice or vaporizing the liquid.

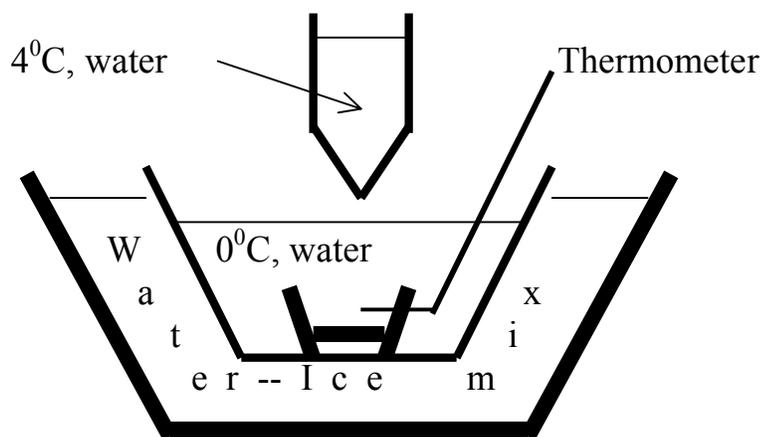
#### d. Heat and friction

There was, among the several minority views concerning heat, one, originating with Newton, that regarded heat as a manifestation of unobserved motion of the microscopic particles of matter and, therefore, as carrying mechanical energy. This view could not win over the champions of heat as a conserved substance, usually called **caloric**, until phenomena that were obviously in conflict with the caloric view acquired overwhelming importance. This did not happen until well into the 19<sup>th</sup> century. But there were champions of the microscopic motion, or energetic view of heat in the late 18<sup>th</sup> century who conceived and performed illuminating experiments that were later regarded as compelling against the caloric theory.

One of the most dramatic experiments was performed by Benjamin Thompson (later Count Rumford), an American colonial who sided with Britain in the Revolutionary war, became an adventurer in Europe and was employed to reorganize the Prussian army. During this latter capacity he noticed the incredible heat generated by the process of boring cannon barrels. The caloric theorists treated this phenomena as due to the boring process ‘squeezing’ the caloric out of the metal. But, if so, then eventually all the caloric would be removed and further boring would no longer

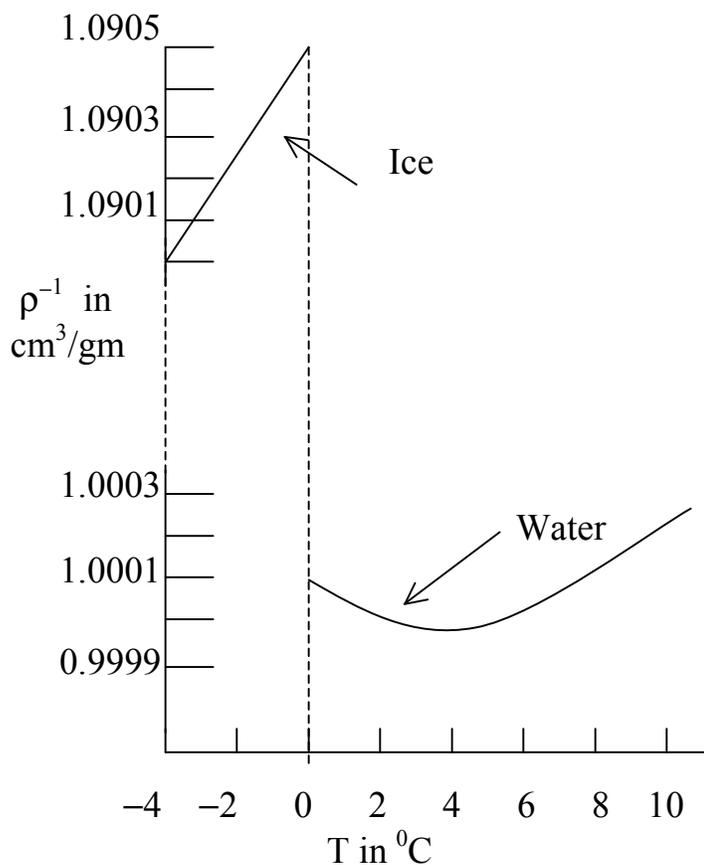
generate heat. Thompson tried to demonstrate the inadequacy of this view by immersing a whole cannon barrel boring arrangement in water and showing that eventually the water becomes hot enough to boil and then continues boiling as long as the boring continues. In the process he attempted to measure how much work was required to generate a given amount of heat, what we now call the **mechanical equivalent of heat**. His value was a bit too high, probably because it was much easier to measure the work done than to keep track of all the heat generated. Of course, eventually the boring had to stop and the champions of caloric simply argued that Thompson had failed to continue long enough to exhaust the vast caloric content of the barrel. The fact that this claim conflicted with the modest measured specific heat of the metal didn't shake the calorists.

Another effort of Thompson to refute caloric theory focused on the caloric view that substances expand when heated and contract when cooled because they absorb and release, respectively, caloric which occupies volume, even if it doesn't seem to have weight. He took his cue from the well known fact that ice floats and, therefore, from Archimedes' principle, water must expand sometime during the cooling process through the freezing transformation. Examining the matter (**Fig. 6**), Thompson found that the density of water is a maximum close to  $4^{\circ}\text{C}$ , below which temperature water expands and continues expanding as it solidifies into ice at  $0^{\circ}\text{C}$ , liberating substantial latent heat in the process (**Fig. 7**).



**Fig. 6:** Thompson's demonstration of the higher density of  $4^{\circ}\text{C}$  water over  $0^{\circ}\text{C}$  water.

To demonstrate the  $4^{\circ}\text{C}$  density maximum he constructed the display of **Fig. 6**. A container of a water-ice mixture surrounds a smaller container of water with an even smaller cork cup in the center. A thermometer inserted through the cork extends above the surface of the water. The water-ice mixture maintains the temperature of the water at  $0^{\circ}\text{C}$  or very slightly above. Thompson then carefully deposits water at  $4^{\circ}\text{C}$  on the center of the surface of the  $0^{\circ}\text{C}$  water. If the latter were more dense than the former, as caloric wisdom dictated, then the  $4^{\circ}\text{C}$  water would spread over the surface and cool to  $0^{\circ}\text{C}$ . In fact *the  $4^{\circ}\text{C}$  water sank directly to the cork cup and raised the thermometer reading to  $4^{\circ}\text{C}$  before cooling to the ambient  $0^{\circ}\text{C}$ .*



**Fig. 7:** The dependence of the reciprocal density of water on temperature near the melting/freezing temperature at atm. pressure.

## 2. Work and Heat

The first clear indication that one could extract work and/or energy from heated systems, whatever heat itself was, did come from the almost universal phenomena of the expansion of matter with the absorption of heat.

We've already noted the thermal expansion of liquids and how that was used in thermometers to measure temperature changes. Solids also display thermal expansion and contraction with the absorption and release of heat, respectively, as we notice when doors and windows get tighter in Summer and the gaps between bridge sections get larger in Winter.

But the more dramatic and really useful thermal expansion occurs with gases and vapors, which, if the temperatures aren't too low and the densities aren't too high, are well described by the **ideal gas law**,

$$P V = N k T . \quad (2.1)$$

Here  $P$  is the pressure,  $V$  the volume,  $T$  the **absolute temperature** (defined by (2.1) and see (2.3) below),  $N$  the number of gas molecules and  $k$ , **Boltzmann's universal constant**. Of course, initially, people had no conception of 'absolute' temperature, could not measure  $N$  and were not even sure there were such things as molecules. So in those days the product,  $N k T$ , would be written as  $n_{\text{mo}} R (T_C + 273^0)$  where  $T_C$  is Celsius temperature and,

$$N k = M \kappa = (M / M_0)(M_0 \kappa) \equiv n_{\text{mo}} R, \quad (2.2)$$

where  $M$  is the mass of the gas and  $\kappa$  is a constant that varies from gas to gas while  $M_0$  is the mass that chemists call a **mole** of the gas (see **Energy IV**), defined such that its' product with  $\kappa$ ,  $R$ , does not vary from gas to gas and is called the **universal gas constant**. The ratio,  $k / \kappa = M / N$ , is the mass of a single molecule of the gas.

Before this section we have only referred to changes in temperature and did not have to worry about any special significance of particular values of temperature. But according to the ideal gas law there is something special about the Celsius temperature of  $-273^0$ . At that temperature the ideal gas

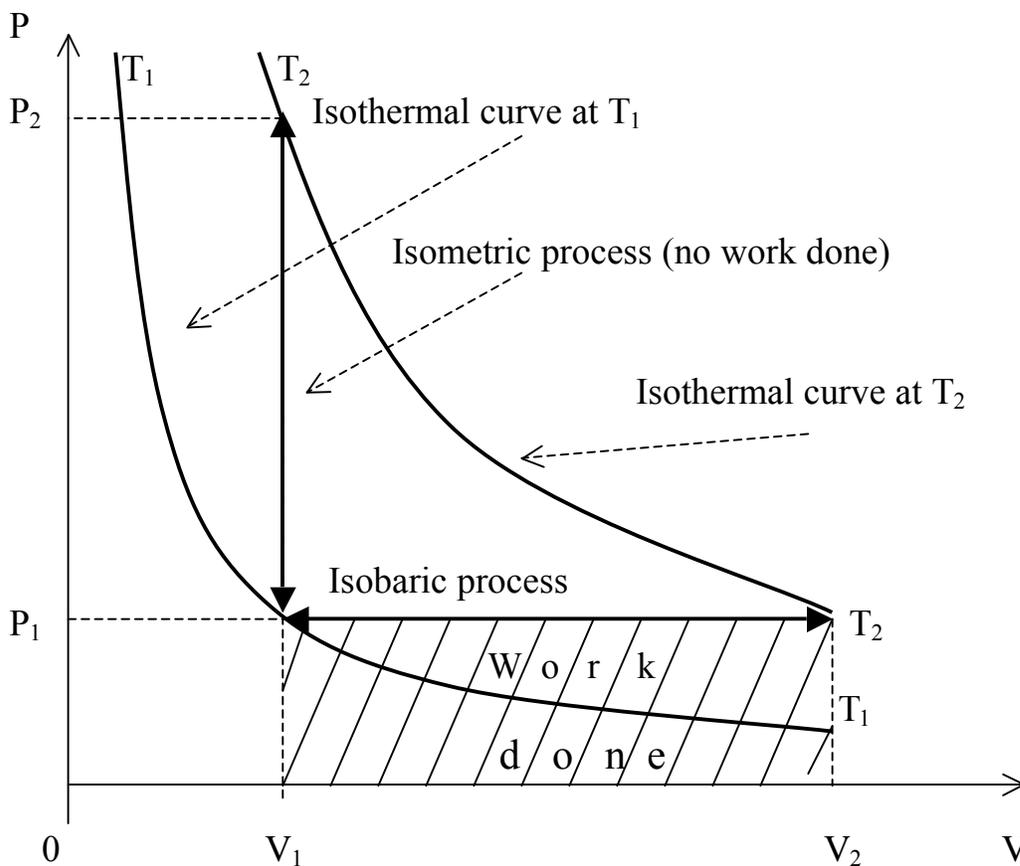
law (if it continued to hold) would require,  $P V = 0$  ! This recognition motivates the introduction of the absolute or **Kelvin** temperature scale defined as,

$$T = T_C + 273^0 . \quad (2.3)$$

The special Kelvin temperature is  $0^0$ ! Henceforth,  $T$  denotes absolute or Kelvin temperature

### a. The heat capacity of gases

A straightforward way for us to see the intimate connection between heat and work or heat and energy is to examine the heat capacity of gases (**Fig. 8**). This is not the way in which the connection was historically established but it did play an important role in suggesting the connection.



**Fig. 8:** P-V diagram for an ideal gas showing isothermal curves and isometric and isobaric processes between the temperatures,  $T_1$  and  $T_2$ .

First suppose we heat a gas while holding the volume of the container constant, a so-called **isometric process** (from  $P_1, V_1, T_1$  to  $P_2, V_1, T_2$  in **Fig. 8**). Strictly speaking this is hard to do since the container will also get hotter and will expand as a consequence. But the expansion of the container due to heating is much less than the expansion the gas would undergo if, for instance, we kept the pressure constant while heating the gas. So, to a good first approximation, we can neglect the expansion of the container. Strictly speaking, if our container is a cylinder with a moveable piston we can depress the piston during heating to compensate for the increasing radius of the cylinder. But this would complicate the analysis a bit, so we'll just ignore the small change in the container volume.

We find the heat required to raise the temperature of the gas by  $\Delta T$  is

$$Q_V = c_V M \Delta T, \quad (2.4)$$

where  $c_V$  is the **specific heat at constant volume**.

Next we heat the gas through the same temperature change,  $\Delta T$ , while holding the pressure constant, a so-called **isobaric process**, and allowing the gas to expand (from  $P_1, V_1, T_1$  to  $P_1, V_2, T_2$ ). This time we find,

$$Q_P = c_P M \Delta T, \quad (2.5)$$

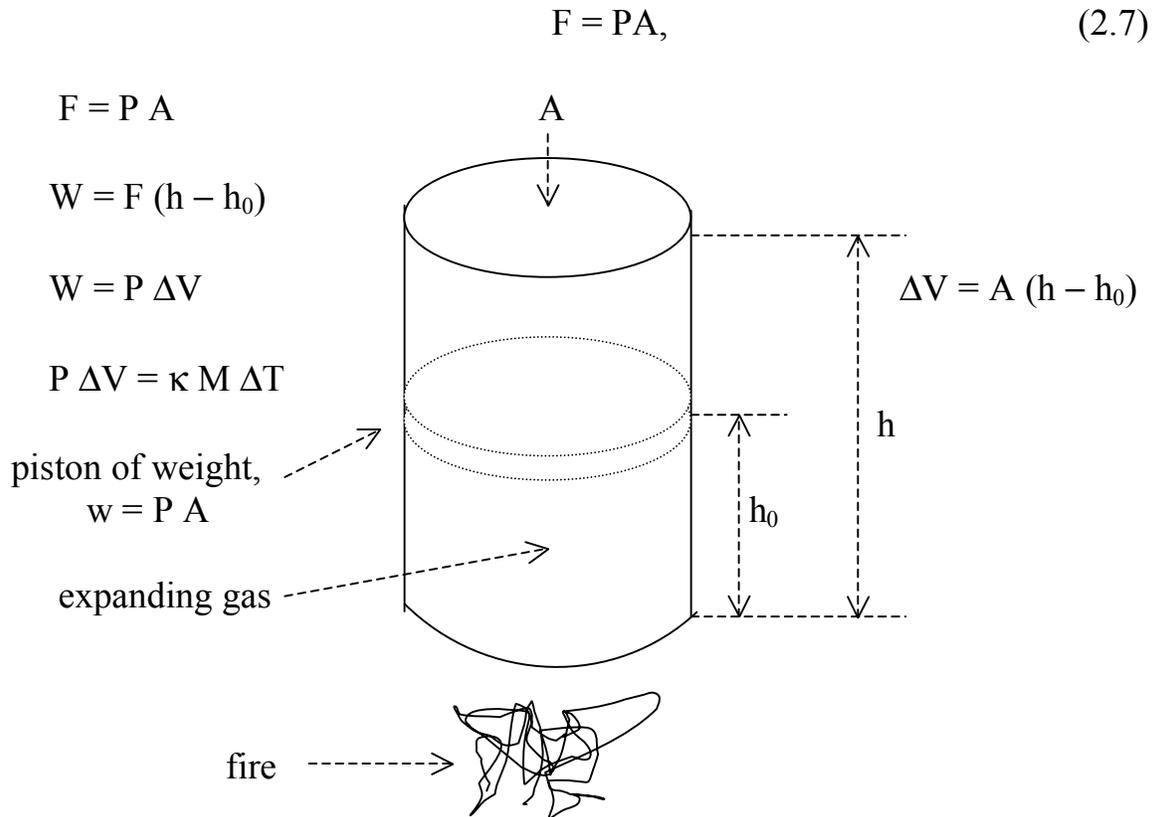
Where  $c_P$  is the **specific heat at constant pressure**. In every case we find,  $c_P > c_V$ . In fact,

$$c_P = c_V + \lambda \kappa, \quad (2.6)$$

where  $\lambda$  is a positive proportionality constant that is *the same for all gases*.

Now why might that be?

Well, when the gas expands during heating under constant pressure the gas is doing work on the raising piston (**Fig. 9**). If the area of the piston is  $A$  and the height of the piston changes from  $h_0$  to  $h$  during the heating then the volume increase is  $\Delta V = A (h - h_0)$ . Since the force on the piston is



**Fig. 9:** Heating an ideal gas at constant pressure which appears to convert some of the heat into work as the gas expands.

the work done on the piston is

$$W = F (h - h_0) = PA (h - h_0) = P \Delta V. \quad (2.8a)$$

But since the pressure is constant,

$$\begin{aligned} W &= P \Delta V = \Delta(P V) = \Delta(\kappa M T) \\ &= \kappa M \Delta T = (1/\lambda)(c_p - c_v) M \Delta T = (1/\lambda)(Q_p - Q_v), \end{aligned} \quad (2.8b)$$

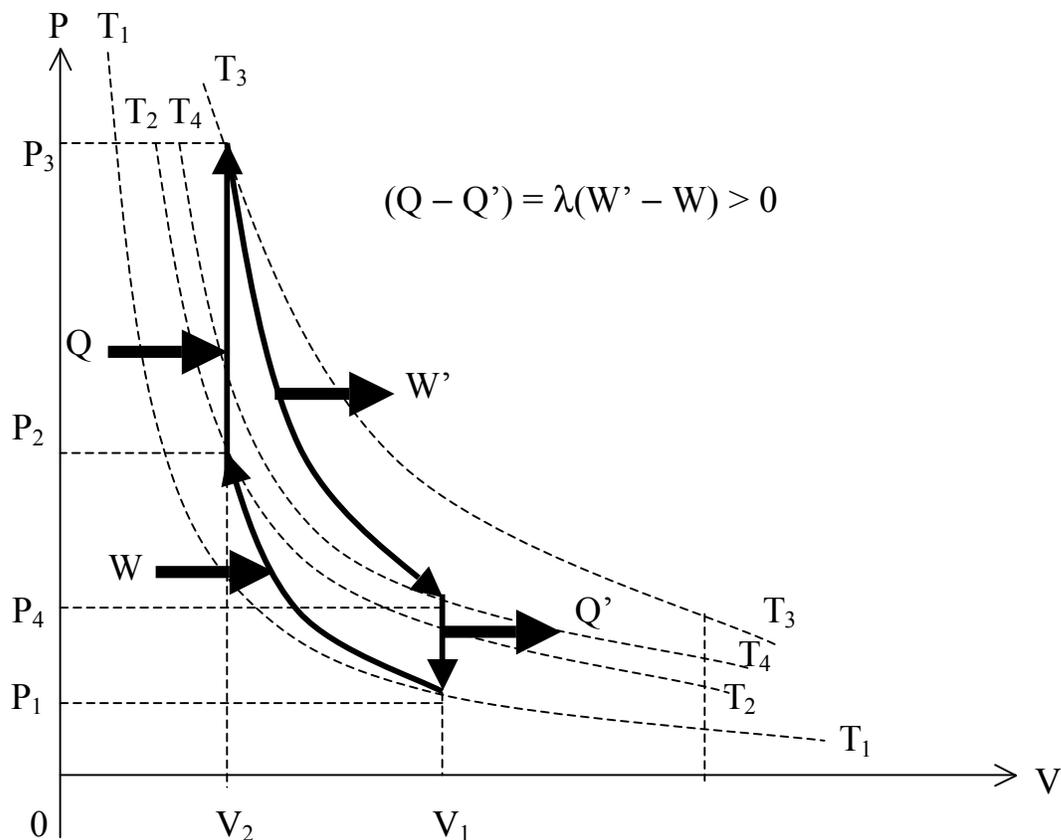
*i.e., the work done is proportional to the difference in the heat absorbed under constant pressure and the heat absorbed under constant volume!*

A reasonable conjecture is that when the volume is held constant and no work can be done, all the heat goes into raising the temperature of the gas by

the amount,  $\Delta T$ , but when the pressure is held constant the extra heat is required to do the work of constant pressure pushing against increasing volume. In other words the extra heat is directly converted into work!

Considerations similar to this played a role in Robert Mayer's claim in 1842 that *heat was a form of energy and the total energy of the world was conserved*.

We can reinforce this interpretation by considering processes in which work is done without heat exchange, so-called **adiabatic processes**, achieved by thermally insulating the container. This is the opposite of heating at constant volume so that no work is done (**Fig. 10**). Now consider a closed cycle in which, starting at volume,  $V_1$  and temperature,  $T_1$ , we first do work,  $W$ , *on the gas* by adiabatic compression to volume,  $V_2$ , and temperature,  $T_2$ . Next we isometrically pump heat,  $Q$ , into the gas, further raising the temperature



**Fig. 10:** Adiabatic – isometric cycle between isothermals at  $T_1$ ,  $T_2$ ,  $T_3$ , and  $T_4$ , where  $T_1 < T_2 < T_3 > T_4$ .

to  $T_3$ . Then we allow the gas to expand adiabatically back to the initial volume,  $V_1$ , cooling to the temperature,  $T_4 > T_1$ , in the process and doing work,  $W'$ . Finally we further cool the gas isometrically to the original temperature,  $T_1$ , by extracting heat,  $Q'$ . In such a cycle  $W' > W$  since the expansion occurred at higher pressures than the compression and  $Q > Q'$  since (as it turns out) the temperature ranges for isometric heating and cooling satisfy,

$$(T_3 - T_2) > (T_4 - T_1). \quad (2.9)$$

Furthermore, it always turns out that the *net* work done *by* the gas,  $W' - W$  is proportional to the *net* heat *absorbed* by the gas,  $Q - Q'$ ,

$$Q - Q' = \lambda(W' - W). \quad (2.10a)$$

If the cycle is arranged so that  $T_2 = T_4$  then we also have the proportionalities,

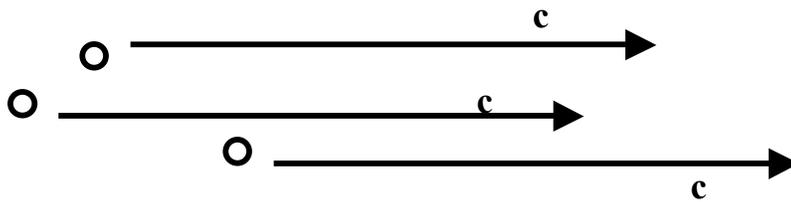
$$Q = \lambda W' \quad \text{and} \quad Q' = \lambda W. \quad (2.10b)$$

Now since the gas returns to its initial state at the end of the cycle we can imagine running the cycle over and over again, each time obtaining a net work done *by the gas* that is proportional to the net heat absorbed. If we run the cycle in reverse we similarly generate a net heat that is proportional to the net work done *on the gas*. The suggestion is very strong that heat and work are just two inter-convertible forms of a common quantity. Since energy is the capacity to do work, heat appears to *have* or to *be* energy!

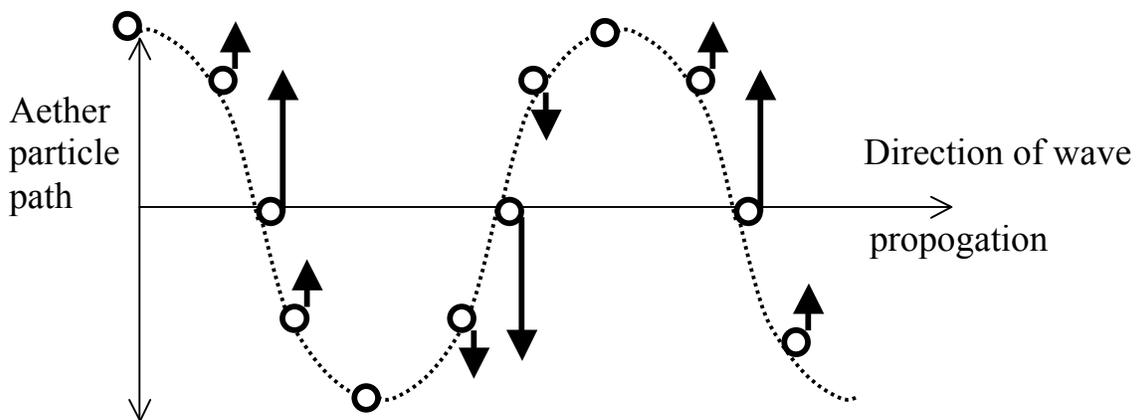
But in the early 1800's when Sadi Carnot, the discoverer of the second law of thermodynamics, was first considering such cycles he still embraced the prevailing theory that heat was a conserved substance, i.e.,  $Q = Q'$ , and work was created from scratch by the cycle, or, if run in reverse, that work was used up to lift heat from lower to higher temperatures. He could persist in these beliefs because calorimetric measurements were rather poor at the time. But his unpublished notes indicate he was starting to realize the convertibility of heat and work into each other before his early death.

### 3. The Conservation of Energy

The development that finally brought about a real decline in support for the caloric theory of heat was the establishment, by 1830, of the wave theory of light. The wave theory had been proposed originally by Huyghens against Newton's particle theory, but the particle theory continued to enjoy support until the experimental and theoretical work of Young and Fresnel in the late 18<sup>th</sup> and early 19<sup>th</sup> century (**Fig. 11**).



Particle theory of light: Light particles (carrying caloric?), with velocity  $c$ , on their way from the Sun to the Earth.



Wave theory of light: Aether particles (and their instantaneous velocities) oscillating up and down as a light wave passes from the Sun to the Earth. No substance is making the trip, but kinetic energy will be transferred.

**Fig. 11:** Comparison of particle and wave theories of light.

Why did the wave theory of light kill support for the caloric theory of heat? If light was a stream of particles it was easy to understand how the particles

coming from the Sun could carry caloric with them. But if light was a wave motion with the particles of the luminiferous aether just bobbing back and forth as the wave passed over them it was harder to accept that a stream of caloric followed the wave all the way from the Sun. In fact, with the acceptance that radiant heat from fire or the Sun was, like light, a wave phenomena, the tide of fashion in theories enjoyed a temporary swing to regarding *all* manifestations of heat, radiant, conductive or convective, as a wave motion.

But that extreme was not sustainable (e.g., conduction and convection are much too slow) and the conception of heat in matter as a form of microscopic motion, and therefore related to mechanical energy, gained ground.

#### **a. The mechanical equivalent of heat**

With Robert Mayer's proposal of this idea and energy conservation in 1842, scientists reconsidered Benjamin Thompson's work as clearly supportive. But both Fourier's 1822 analysis of heat conduction and Carnot's 1824 analysis of heat engine efficiency were important advances that seemed to work very well, and they were both based on the conserved caloric theory.

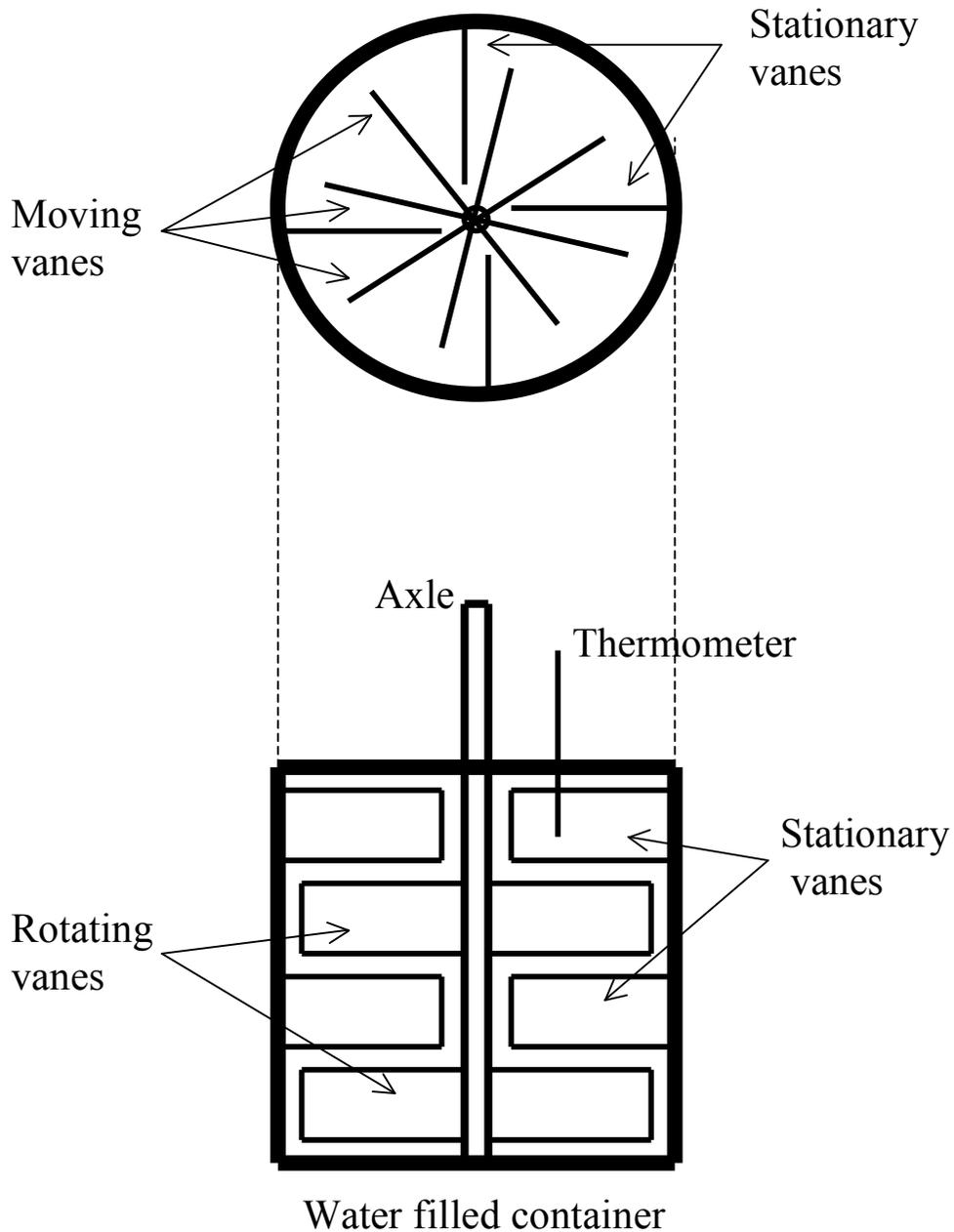
The missing ingredient was a really careful measurement of just how much heat could be generated from a given amount of work and vice versa. In modern terms, how many calories could a watt-second generate and how many watt-seconds could one get from a calorie? The experimental work was conducted during the 1840's by James P. Joule by using falling weights to turn paddles in a container of liquid and compare the temperature rise of the liquid induced by the paddle churning with the loss of gravitational potential energy of the falling weights (**Fig. 12**). To insure that the work done in turning the paddles was only minimally converted into rotational kinetic energy of the liquid and that that minimal amount was quickly damped, Joule staggered the paddles, or turning vanes with stationary vanes attached to the container. In this way large scale flow patterns could not be established and turbulence and eddies in the liquid would quickly dissipate into heat. Joule also took careful account of heat conduction and radiation from the container.

The upshot of this work, with many different liquids, was to establish the correspondence,

$$1.00 \text{ Btu} \sim 772 \text{ ft lbs.} \quad (2.11a)$$

or

$$1.00 \text{ cal} \sim 4.18 \text{ J} = 4.18 \text{ watt-sec.} \quad (2.11b)$$



**Fig. 12:** Joule's insulated container of water with stationary and rotating vanes, the latter driven by falling weights.

But if heat *is* just a form of energy, then the units of heat, the Btu and the calorie, need not be restricted to just *corresponding* to quantities of energy, they can be *identical* to quantities of energy, i.e.,

$$1.00 \text{ Btu} = 772 \text{ ft lbs}, \quad (2.12a)$$

and

$$1.00 \text{ cal} = 4.18 \text{ watt-sec}, \quad (2.12b)$$

or

$$3.6 \text{ Mcal} = 4.18 \text{ kwh}. \quad (2.12c)$$

With these equivalences established, the universal gas constant,  $R$ , has the value,

$$R = 2.0 \text{ cal} / ^\circ\text{K mo.} = 8.3 \text{ J} / ^\circ\text{K mo.} \quad (2.13)$$

Clausius showed that the important caloric based work of Carnot could be reformulated compatibly with the new energy interpretation of heat without losing Carnot's deep insights. This led to the development of the extremely useful field of **Thermodynamics**. Similarly, Fourier's work on heat conduction survived the reinterpretation because, during pure conduction of heat no transformation of the form of energy is taking place.

### **b. energy conservation**

With the publication in 1847 of Helmholtz's "*On The Conservation of Force*" (the continental terminology was not keeping pace with the science; what was meant here was the conservation of energy) the stage was set for the rapid and universal acceptance of energy as the unifying concept of all of physical science and the conservation of energy as a basic, if not *the* most basic principle of physical science. Physical processes came to be seen more and more as primarily involving the transfer from one place or system to another and the transformation from one form to another of energy, while the totality of energy remained unchanged. This generalization went far beyond what the evidence could firmly support, but it seemed, at least, compatible with the evidence.

Problems would arise. Assuming that the known forms of energy were all there were, one could not account for the Sun being older than a few million years, much less time than the geologists were asserting for the age of the Earth or than the Darwinists would require for the evolution of life by Natural Selection. In the early 20<sup>th</sup> century new forms of energy would be

discovered but two episodes would still briefly cast doubt on the conservation of energy. Nevertheless, when applied to less grandiose and more practical situations the employ of the new energy perspective proved very successful and led to rapid advances on many fronts.

### **c. Previous dissipative examples**

In our previous class we noted the loss of mechanical energy in several simple systems due to dissipative forces of resistance. The bouncing ball, the oscillating mass on a spring, the orbiting Earth and its Moon. We can now assert that in each case the lost mechanical energy is transformed into heat energy.

The bouncing ball transfers kinetic energy to the immediately surrounding air and the resulting air currents spread and diminish into heat deposition in the air. The ball, itself, warms slightly as the interior flexing of the ball at each bounce converts kinetic and elastic potential energy into heat within the ball.

The mass on the spring disturbs the surrounding air in the same way as the ball and the flexing of the spring generates interior heat similar to the bounce flexing of the ball.

The gravitational squeezing and stretching due to the Moon and Sun on the waters of the Earth and (much more so in earlier eons) the interior of the Earth, generates heat and diminishes kinetic energy of motion, most noticeably in the lengthening of the day night cycle and, by reciprocal action on the Moon, the lengthening of the lunar month cycle.

In all these and similar cases it is believed that if we could keep track of *all* the forces and *all* the generated heat (an impossible task!) we would find the total energy to be conserved. To the degree that we can monitor these quantities, we do find corroboration of energy conservation.

### **Appendix: Formulating energy conservation**

So how shall we formulate the new understanding? An essential novel concept must be introduced, that of the **internal energy**,  $U$ , of a physical object or system. This represents the energy held inside and distributed throughout the internal microscopic makeup of a material system. In time

one would learn more and more about the detailed form or forms of internal energy, but for the purpose of initially formulating energy conservation, one does not have to declare on that issue.

We now formulate the conservation principle for an arbitrary material system in terms of work done *on* the system,  $W$ , heat absorbed *by* the system,  $Q$ , the observable kinetic energy of the system,  $K$ , and the internal energy of the system,  $U$ . In any process involving the system,

$$W + Q = \Delta K + \Delta U. \quad (\text{A.1})$$

This bears a resemblance to the work-energy theorem we discussed earlier and may be thought of as a generalization of it.

Now some or all of the work done,  $W$ , may come from conservative forces within the system, in which case that part of the work done is equal to the negative change in some total potential energy of the system,  $V$ , i.e.,

$$W = W_{\text{cons}} + W' = -\Delta V + W'. \quad (\text{A.2})$$

This enables us to rewrite the principle as

$$W' + Q = \Delta E_{\text{mech}}, \quad (\text{A.3})$$

Where

$$E_{\text{mech}} = K + V + U \quad (\text{A.4})$$

If the system consists of two parts, 1 and 2, then

$$W_1 + Q_1 = \Delta K_1 + \Delta U_1 \quad \text{and} \quad W_2 + Q_2 = \Delta K_2 + \Delta U_2, \quad (\text{A.5})$$

where

$$W_1 = W_{1,\text{cons}} + W_1', \quad W_2 = W_{2,\text{cons}} + W_2' \quad (\text{A.6})$$

and

$$W_{1,\text{cons}} = -\Delta V_1, \quad W_{2,\text{cons}} = -\Delta V_2. \quad (\text{A.7})$$

Consequently,

$$W_1' + W_2' + Q_1 + Q_2 = \Delta(K_1 + K_2 + V_1 + V_2 + U_1 + U_2). \quad (\text{A.8})$$

Now, in,  $W_1' + W_2'$ , we may find a conservative contribution from equal and opposite forces acting between systems 1 and 2, i.e.,

$$W_1' + W_2' = W_1'' + W_2'' - \Delta V_{12} , \quad (\text{A.9})$$

where  $V_{12}$  is the potential energy the subsystems have relative to one another. Defining,

$$E_{\text{mech}, 12} = K_1 + K_2 + V_1 + V_2 + V_{12} + U_1 + U_2, \quad (\text{A.10})$$

as the *total mechanical energy* of the composite system (1 & 2), we have,

$$W_1'' + W_2'' + Q_1 + Q_2 = \Delta E_{\text{mech}, 12} . \quad (\text{A.11})$$

The heat energies,  $Q_1$  and  $Q_2$  can also be decomposed into quantities absorbed from the external world and a quantity exchanged between the subsystems,

$$Q_1 = Q_1' + Q_{12} \quad \text{and} \quad Q_2 = Q_2' - Q_{12} . \quad (\text{A.12})$$

The conservation equation for the composite system then becomes,

$$W_1'' + W_2'' + Q_1' + Q_2' = \Delta E_{\text{mech}, 12} . \quad (\text{A.13})$$

If the composite system is isolated from the external world so that  $W_1''$ ,  $W_2''$ ,  $Q_1'$  and  $Q_2'$  are all zero then,

$$\Delta E_{\text{mech}, 12} = 0 , \quad (\text{A.14})$$

the total energy of the composite system is conserved. For the Universe as a whole, there is no external world and so,

$$\Delta E_{\text{Univ}} = 0. \quad (\text{A.15})$$