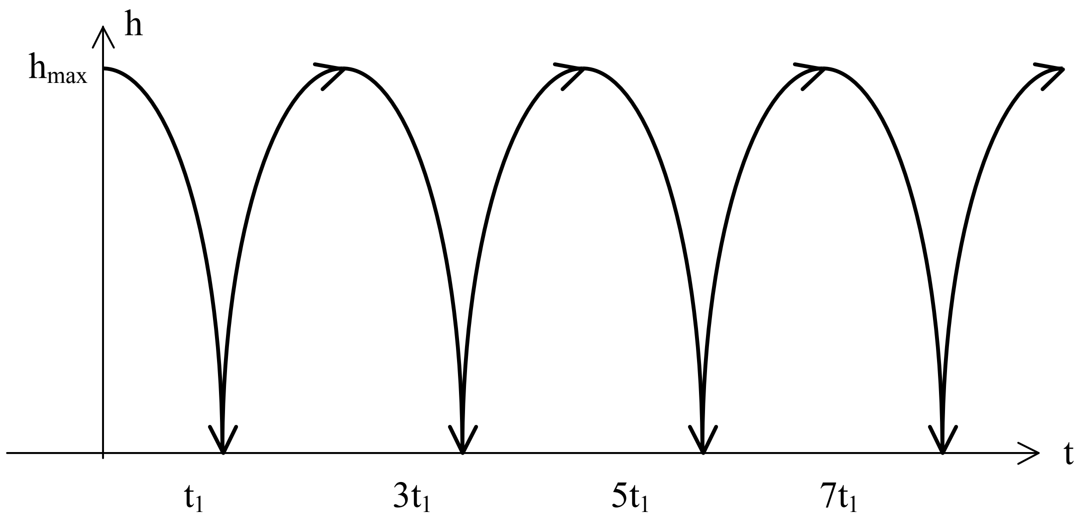
Energy II: The Dissipation of Energy

1: Potential – kinetic energy transfer

a. Ideal bouncing superball

Imagine an *ideal* superball which, upon falling to a hard flat surface at ground level, rebounds with an upward velocity exactly equal in magnitude to the downwards velocity it had just before hitting the surface (**Fig. 1, 2**). Thus its kinetic energy just after the bounce is the same as just before. If we now assume that while falling only terrestrial gravity is acting on the ball (no air resistance), then it will rise after the bounce, losing kinetic energy as it climbs while its gravitational potential energy increases with height in exactly the opposite of the sequence followed when it originally fell to the surface. Indeed, upon reaching the original height of the drop, the kinetic energy will be exhausted, the gravitational potential energy will be



$h_{max} = (1/2)gt_1^2$

Fig. 1: Height vs time graph of bouncing ideal superball.

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maximized and the scene will be set for the original drop and rebound to exactly repeat. This cyclic process will then go on *forever* – if, as stated, we assume that terrestrial gravity is the only force acting on the superball (except at the moment of bounce, of course).

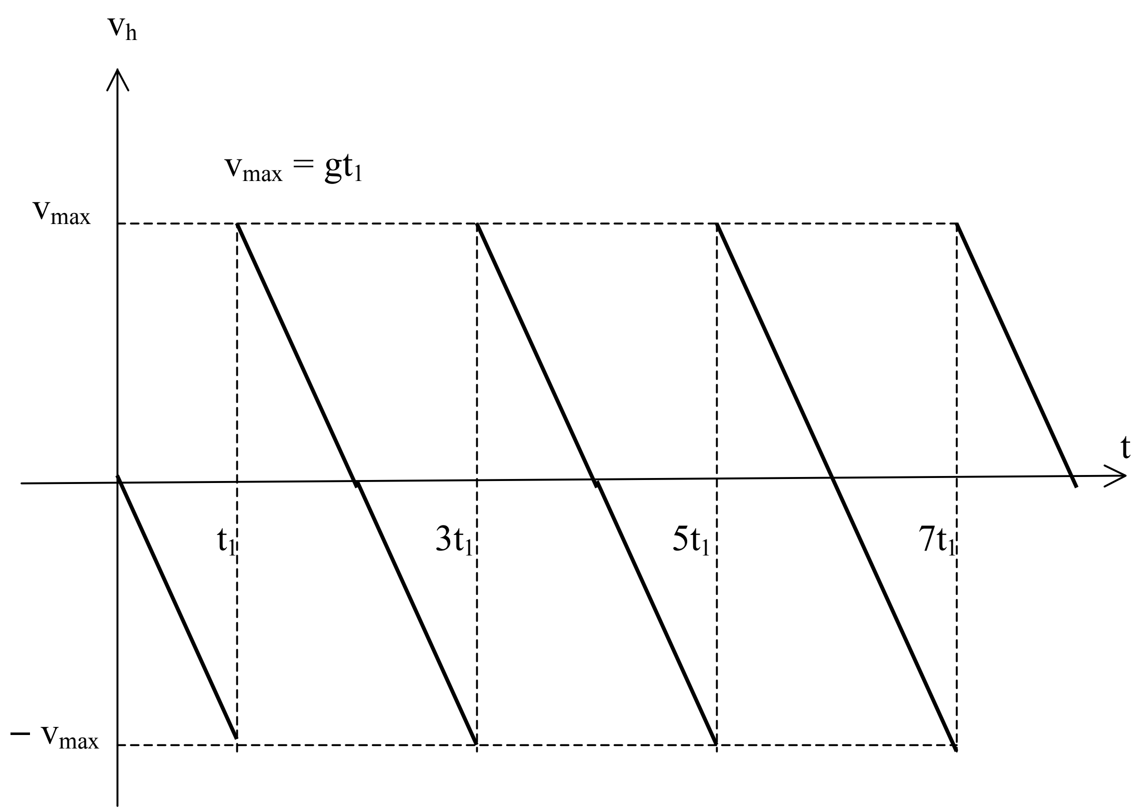


Fig. 2: Vertical velocity vs time graph of bouncing superball.

During any portion of the process, as we saw in **Energy I**, the work done by gravity on the ball is just the negative change in the gravitational potential energy,

$$W_{grav} = -\Delta E_{grav} = -\Delta(mgh)\ . \qquad (1.1)$$

On the other hand, gravity being the only force acting, this is also the resultant force doing resultant work on the superball and, by the work-energy theorem we have,

$$W_{grav} = W_{res} = \Delta K = \Delta((1/2)mv^2)\ . \qquad (1.2)$$

Putting these two results together we have

$$-\Delta E_{grav} = \Delta K \qquad \text{or} \qquad 0 = \Delta[K + E_{grav}] \qquad (1.3a)$$

or

$$K + E_{grav} = (1/2)mv^2 + mgh = \text{constant} , \qquad (1.3b)$$

the sum of the kinetic energy and the gravitational potential energy is conserved (**Fig. 3, 4**). At the top of the cyclic motion, where

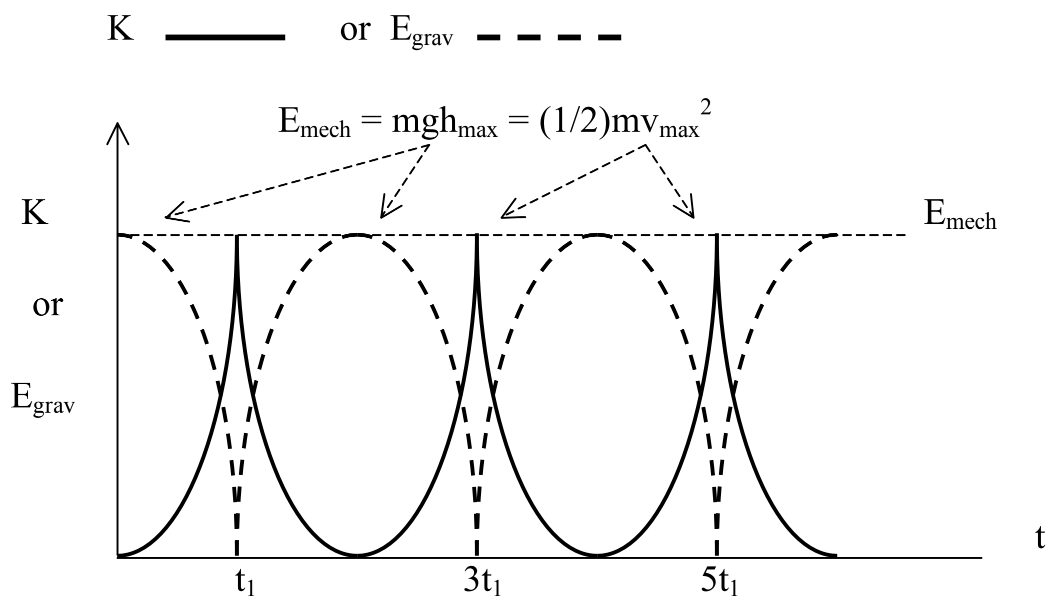


Fig. 3: Kinetic and potential energy vs time graphs for bouncing superball.

$h = h_{max}$, the ball is momentarily at rest with no kinetic energy. At the bottom of the cycle, *just* before and *just* after the bounce, the ball has h = 0, no potential energy and $v = v_{max}$. Since the sum of the two energies is constant we must have,

$$0 + mgh_{max} = (1/2)mv^2 + mgh = (1/2)mv_{max}^2 + 0. \qquad (1.4)$$

This last result is usefull even if we don't have a superball since it tells us what the speed of even an ordinary ball, falling due to gravity alone, will be when it reaches the ground if it is dropped from a height, h_{max}, i.e.,

$$gh_{max} = (1/2)v_{max}^2 , \qquad (1.5a)$$

or

$$v_{max} = (2gh_{max})^{1/2}. \qquad (1.5b)$$

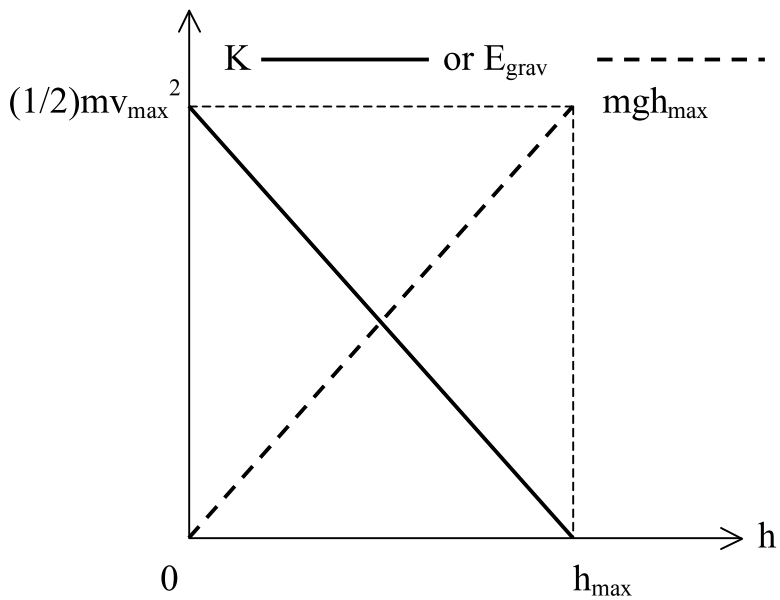


Fig. 4: Kinetic and potential energy vs height graphs for bouncing superball

The quantity,

$$E_{mech} = K + E_{grav} \quad , \tag{1.6}$$

Is called the **mechanical energy** of the falling, bouncing superball

b. Mass and spring in ideal smooth horizontal channel

Consider an object of mass, m, attached to one end of a spring with stiffness constant, k, with the other end fixed at a wall and all lying in a horizontal channel or groove that is so smooth as to offer no resistance to horizontal motion within the groove of the object or the stretching and compressing of the spring (**Fig. 5**). If the object is then pulled in the direction away from the wall, thereby stretching the spring, and then released, the stressed spring will accelerate the object back towards the wall until the spring reaches its relaxed length, L, again. But then, due to inertia, the object will keep on moving towards the wall, compressing the spring, which will in turn resist the compression and decelerate the object until it, the object, stops. The stop will only be momentary as the compressed spring will now re-accelerate the object in the opposite direction, away from the wall. Then, after once again passing the relaxed length and stretching, the spring will decelerate the

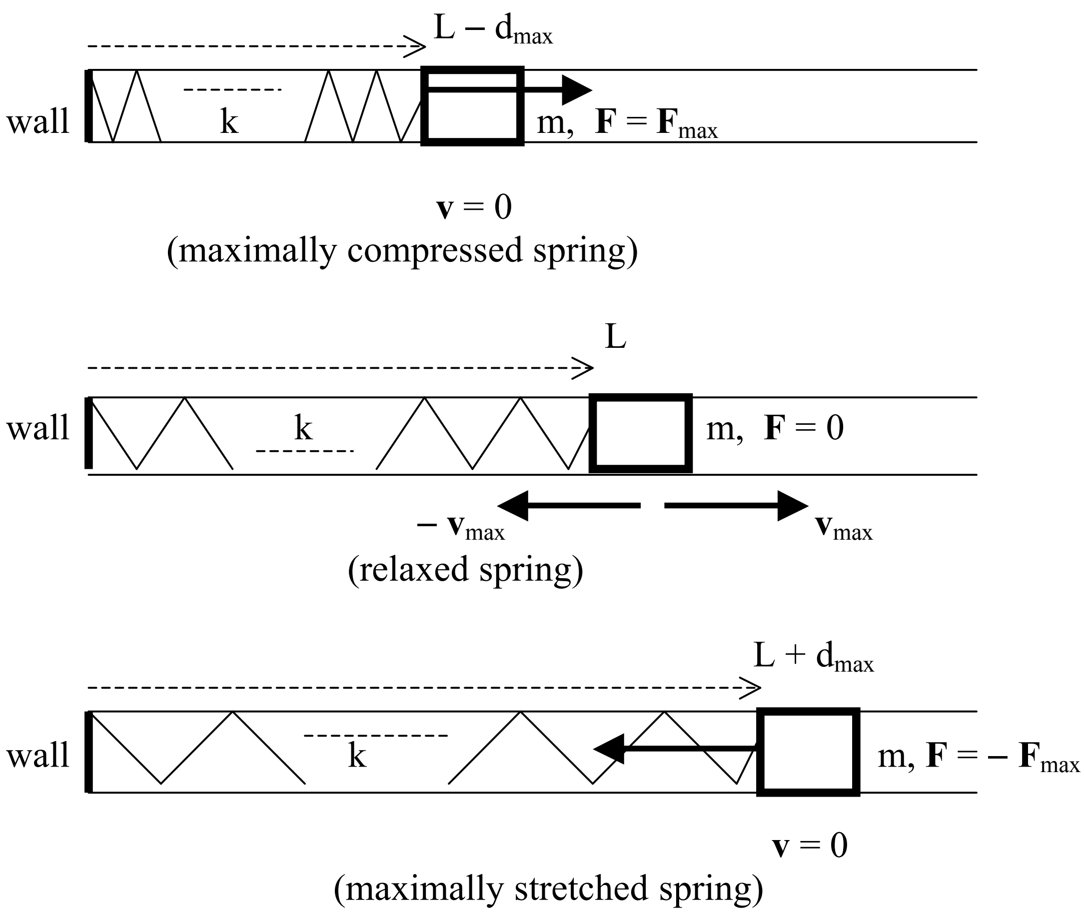


Fig. 5: Looking down upon three stages in oscillating motion of mass, m, attached to spring, k, sliding in groove.

object, bringing it to momentary rest again at the very place it was initially released. The cycle of motions will begin again and repeat endlessly (**Fig. 6**).

With this arrangement the resultant force on the object is just due to the stressing of the spring (gravity pulls down on the object but the surface of the groove pushes up with the same magnitude of force, thus canceling the gravitational force). If the length of the spring is changed from its relaxed length, L, by an amount, d, ($d > 0$ means stretching while $d < 0$ means compressing) where $|d| << L$, then, as we saw in **Energy I**, the elastic potential energy is, $E_{spring} = (1/2)kd^2$. The work done by the spring through any interval is just the negative of the change in the potential energy,

$$W_{spring} = -\,\Delta E_{spring} = -\,\Delta((1/2)kd^2). \qquad (1.7)$$

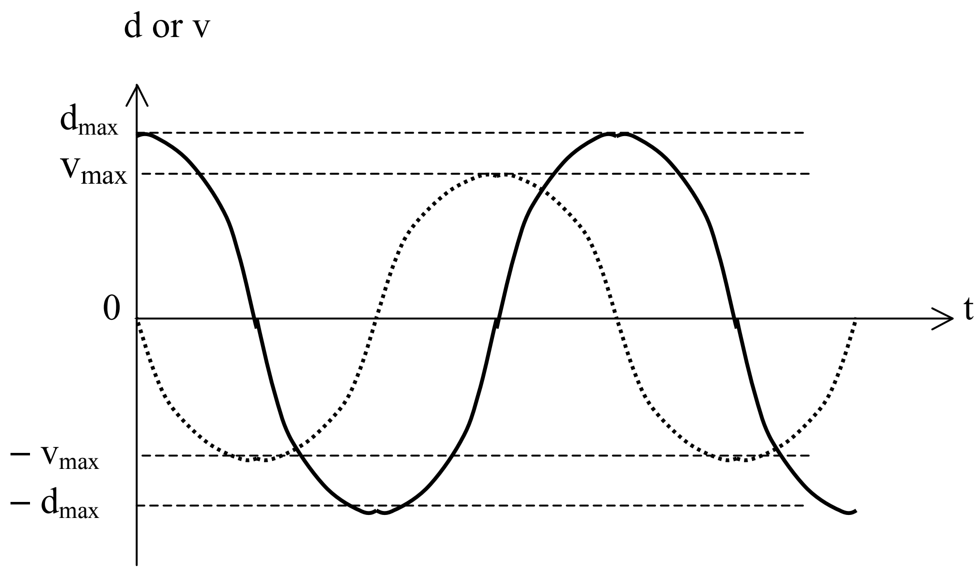


Fig. 6: Graphs of the position vs time and velocity vs time of the object attached to the spring.

That work, being the resultant work done on the object, equals the change in the kinetic energy of the object,

$$W_{spring} = W_{res} = \Delta K = \Delta((1/2)mv^2)\ , \qquad (1.8)$$

and so, as before, we have,

$$-\,\Delta E_{spring} = \Delta K \qquad \text{or} \qquad \Delta[K + E_{spring}] = 0 \qquad (1.9a)$$

or

$$K + E_{spring} = (1/2)mv^2 + (1/2)kd^2 = E_{mech} = \text{constant}\ . \qquad (1.9b)$$

At the midpoint of the oscillation, where d = 0, the speed of the object and its kinetic energy are maximum while the potential energy is zero. At the endpoints of the oscillation, where the stretching or compressing of the spring and its potential energy are maximum, the object momentarily stops moving and has zero kinetic energy (**Fig. 7**), i.e.,

$$(1/2)mv_{max}^2 + 0 = (1/2)mv^2 + (1/2)kd^2 = 0 + (1/2)kd_{max}^2\ . \qquad (1.10)$$

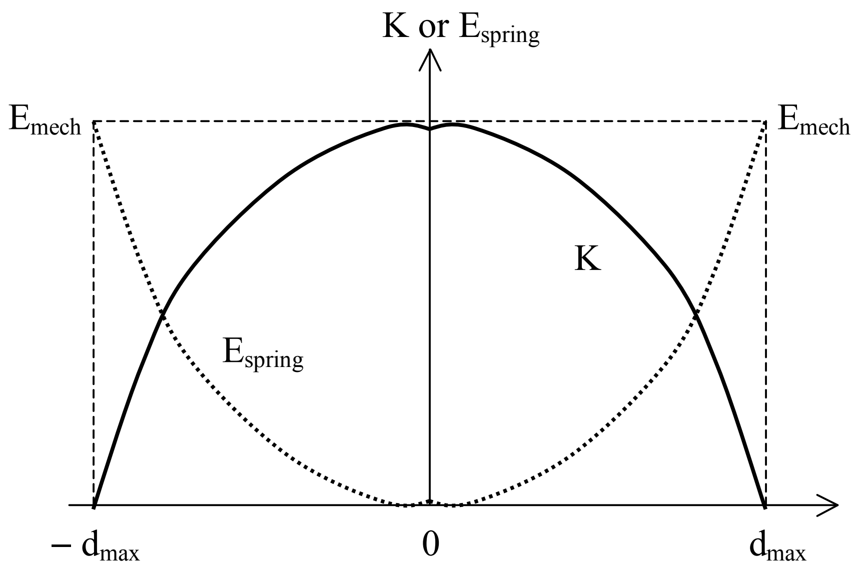


Fig. 7: Graphs of K or E_{spring} vs d for the sliding object attached to a spring.

From this we can deduce that if the initial stretch of the spring from which the object was released was, d_{max}, then the object will sail passed the midpoint of the oscillation with speed,

$$v_{max} = (k/m)^{1/2} d_{max} \ . \qquad (1.11)$$

In the previous superball case the force was constant and always in the same direction, except for the moment of bounce, and the extremes of energies alternated at the endpoints of the motion. Here the force rises and falls in strength and reverses direction through the midpoint of the motion and the extremes of energy alternate between the midpoint and the endpoints. Nevertheless, in both cases the energy passes back and forth between kinetic and potential form, cyclically, the sum or total mechanical energy being conserved.

c. Mass hanging vertically from a spring

We now bring gravity into the mass on a spring system by *hanging the spring vertically* with the mass hanging from the spring. There are now two forces acting on the moving object, gravity and the spring force. Both do work on the object and the resultant force, doing the resultant work, is the sum of the two. In any interval of the rising and falling motion the work done by gravity is, as before,

$$W_{grav} = -\,\Delta E_{grav} = -\,\Delta(mgh)\ . \qquad (1.12)$$

The work done by the spring, assuming the spring is relaxed when the object is at the height, h_0, is,

$$W_{spring} = -\,\Delta E_{spring} = -\,\Delta(\,(1/2)k(h - h_0)^2\,). \qquad (1.13)$$

The resultant work done is the sum of these, or,

$$-\,\Delta E_{grav} - \Delta E_{spring} = W_{grav} + W_{spring} = W_{res} = \Delta K\ . \qquad (1.14a)$$

Therefore (**Fig. 8**),

$$\Delta[K + E_{grav} + E_{spring}] = 0, \qquad (1.14b)$$

or

$$E_{mech} = K + E_{grav} + E_{spring}$$

$$= (1/2)mv^2 + mgh + (1/2)k(h - h_0)^2 = \text{constant}\ . \qquad (1.14c)$$

In this situation let's start the motion by raising the object and compressing the spring a bit so that initially both forces are downwards on the object. (1) Upon releasing the object it accelerates downwards, picking up kinetic energy from both the gravitational and spring potential energies which are decreasing. (2) Upon passing the height, h_0, the object still accelerates downwards for a bit but the acceleration is diminishing as the spring starts to pull up and both the kinetic energy and the spring energy are increasing at the expense of the gravitational energy. (3) Shortly the spring is stretched enough to balance the gravitational force and below that height the object is slowing down and the spring energy is growing at the expense of both the gravitational energy and the kinetic energy. (4) When the kinetic energy reaches zero the object stops and starts back up with kinetic energy and gravitational energy now increasing at the expense of the spring energy.

This continues until the balance point between the two forces is reached again, (5) above which the gravitational energy draws from both the spring and kinetic energies until the spring relaxation height of h_0 is reached. (6) After that, both the spring and gravitational energies draw from the kinetic energy until the release height is regained and the cycle begins again, to repeat endlessly.

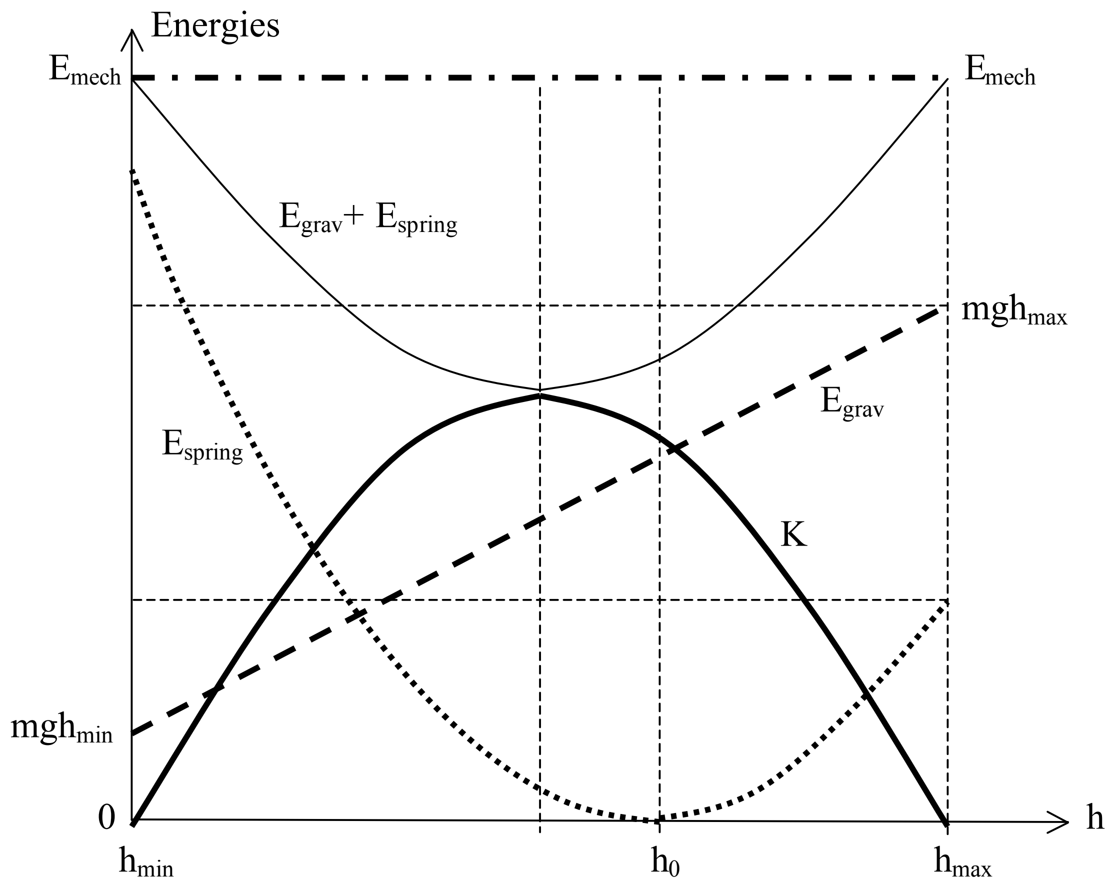


Fig. 8: Energy curves vs height for the object hanging from a spring. E_{mech} _ . _ . _ , E_{grav} _ _ _ _ , E_{spring} .... , $E_{grav}+E_{spring}$ ______ , K ______

d. Earth orbiting the Sun

As was mentioned briefly in **Energy I**, if massive objects are much farther apart than their own dimensions, then their gravitational attraction behaves similarly to the electrical attraction between positively and negatively charged particles. This means that the equal and opposite forces on them, which tend to pull them together, have a magnitude that is proportional to the product of their masses, m_1 and m_2, and inversely proportional to the square of the distance, r, between them, i.e.,

$$F_{grav} = G\, m_1 m_2 / r^2 , \qquad (1.15)$$

to be compared with,

$$F_{elec} = \kappa\, |q_1 q_2| / r^2. \qquad (1.16)$$

It follows from this similarity that this gravitational force will do work that is the negative change of a potential energy that behaves similarly to the potential energy of electrostatic attraction, i.e.,

$$E_{grav} = -\, G\, m_1 m_2 / r, \qquad (1.17)$$

To be compared with,

$$E_{elec} = -\, \kappa\, |q_1 q_2| / r. \qquad (1.18)$$

In the case of the Earth orbiting the Sun both the Sun and the Earth are accelerated, because of the equal and opposite forces acting on them. But the Sun is so much more massive than the Earth (~ 333,000 times more) that we can regard the Sun as stationary as a good approximation. In any case the Sun is more perturbed by Jupiter's gravity than by the Earth's. This means that the constant mechanical energy of the Earth-Sun system is essentially the sum of the kinetic energy of the Earth, K_E , and the potential energy, E_{grav} ,

$$E_{mech} = K_E + E_{grav} \qquad (1.19)$$

If the Earth moved in a circle around the Sun then the distance from the Sun would be constant and both the potential and kinetic energy would be constant. *In such a circular orbit there would be no exchange between kinetic and potential energies at all.* If you're puzzled that this seems to imply that the Sun's gravity would not do any work on the Earth, well, that's exactly correct! In a circular orbit the Sun's attractive force is always perpendicular to the Earth's motion and so no work is done. But this doesn't prevent the force from holding the Earth in orbit.

In fact the Earth doesn't move in a perfect circle around the Sun but, more precisely in a slightly eccentric ellipse (**Fig. 9**). The orbit is such that Earth is a little closer to the Sun during the Northern Hemisphere winter and slightly farther away during the summer.

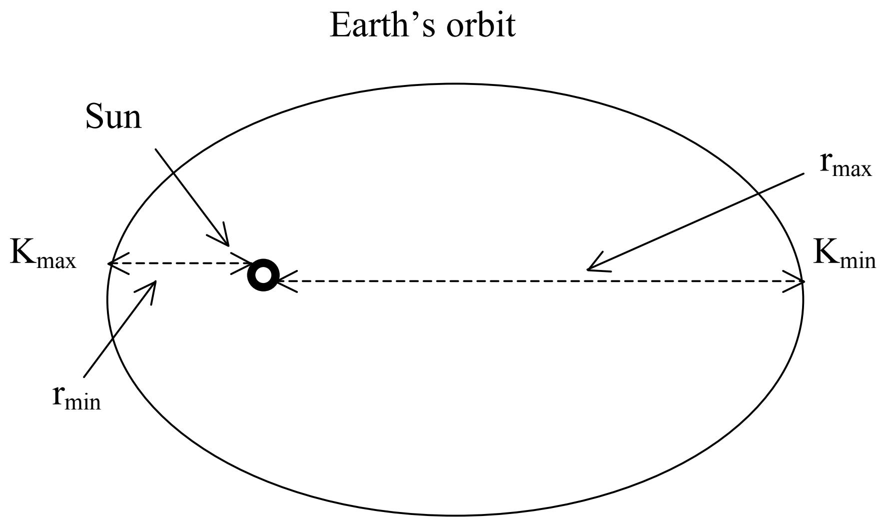


Fig. 9: Earth's elliptical orbit (greatly exaggerated) around the Sun.

Consequently, the potential energy is a little more negative during our winter and a little less negative during our summer. This means the Earth's kinetic energy is a little higher during our winter and a little lower in the summer. We zip around the Sun a little faster in winter than in summer. Is this why the December, January and February period is two days shorter than the June, July and August period?

Finally, an even better approximation to a constant mechanical energy would be achieved by including the kinetic energies of the Earth's large Moon and that of Earth's rotational motion and the gravitational potential energies of the Sun-Moon pair and the Earth-Moon pair,

$$E_{mech} = K_E + K_{ER} + K_M + V_{SE} + V_{SM} + V_{EM} , \qquad (1.20)$$

where we have used the commonly employed symbol, V, to denote a two body potential energy. This exemplifies the construction of mechanical energies for systems with several moving objects.

So, are the preceding examples instances of the *universal principle* of the **conservation of energy**? Unfortunately, NO!

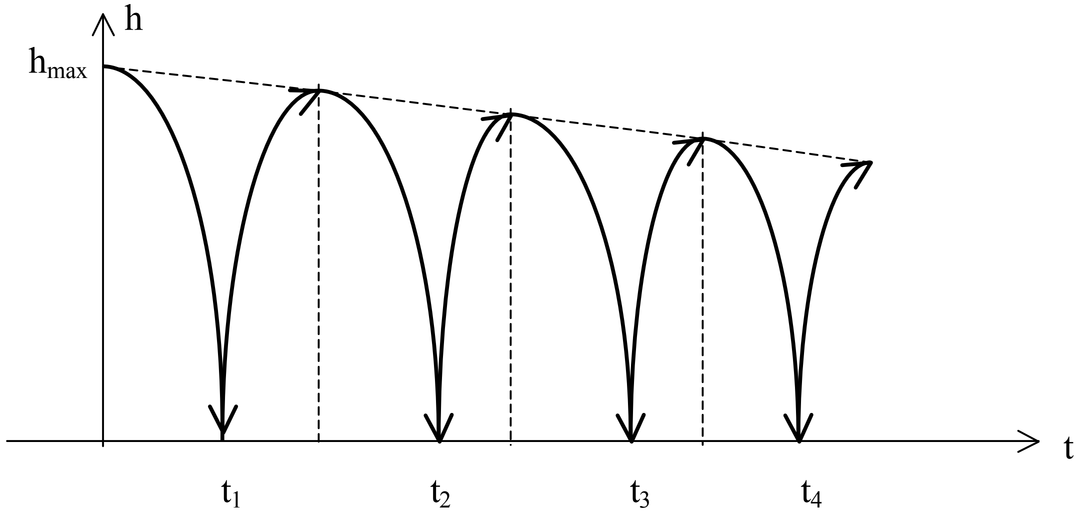
2. The *universal loss* of mechanical energy!

For *all of the above* are, at best, approximations. That's why we spoke of an *ideal* superball and a *smooth* channel. Real balls don't bounce in quite the way we described and all real channels are not perfectly smooth. Even in the case of the object hanging from the spring we ignored the effect of motion through the air and the internal effect of the flexing of the spring. Finally, even in the case of the Earth orbiting the Sun and the Moon orbiting the Earth, we will see that the so-called **tidal effects** of Solar, Terrestrial and Lunar gravity undermine the constancy of E_{mech}.

a. Bouncing real balls

Even with a *real* so-called superball each bounce would result in the ball leaving the ground with slightly less kinetic energy than it had just before the bounce. The difference with ordinary balls is that the drop in kinetic energy for a superball is not nearly as high as with ordinary balls. Furthermore, while the ball is falling and then, after a bounce, rising, it's moving through the air and must push the air out of its way. This produces a reaction force of the air on the ball which is always directed in opposition to the ball's motion and always does negative work. Such a force does work that just grows negatively as the path it acts over lengthens and so it has no potential energy associated with it. The negative work done by the resistance means that while falling the resultant work done is less than with gravity alone and so the ball's kinetic energy doesn't increase quite so much as it otherwise would. After the bounce the negative work of air resistance means that the resultant work is even more negative during the rise than otherwise and the ball loses kinetic energy, while rising, faster than it gained kinetic energy while falling **(Fig. 10, 11**).

So much for air resistance during falling and rising. But why does the ball lose kinetic energy *on the bounce*? When the ball strikes the ground it compresses to some degree depending on its stiffness, very much like a spring. This compression builds up elastic forces which absorb the kinetic energy of the ball into elastic potential energy. After all the downward motion has stopped the elastic forces begin to re-extend the ball and accelerate it upward. At the same time, however, both during the compression and the re-extension, the internal parts of the ball are resisting



$$h_{max} < (1/2)gt_1^{\,2}\,, \quad (t_{n+1} - t_n) < (t_n - t_{n-1}) \qquad (2.1)$$

Fig. 10: Height vs time graph of (almost) *real* bouncing superball.

$$v_{max} < gt_1\,, \qquad (t_{n+1} - t_n) < (t_n - t_{n-1}) \qquad (2.2)$$

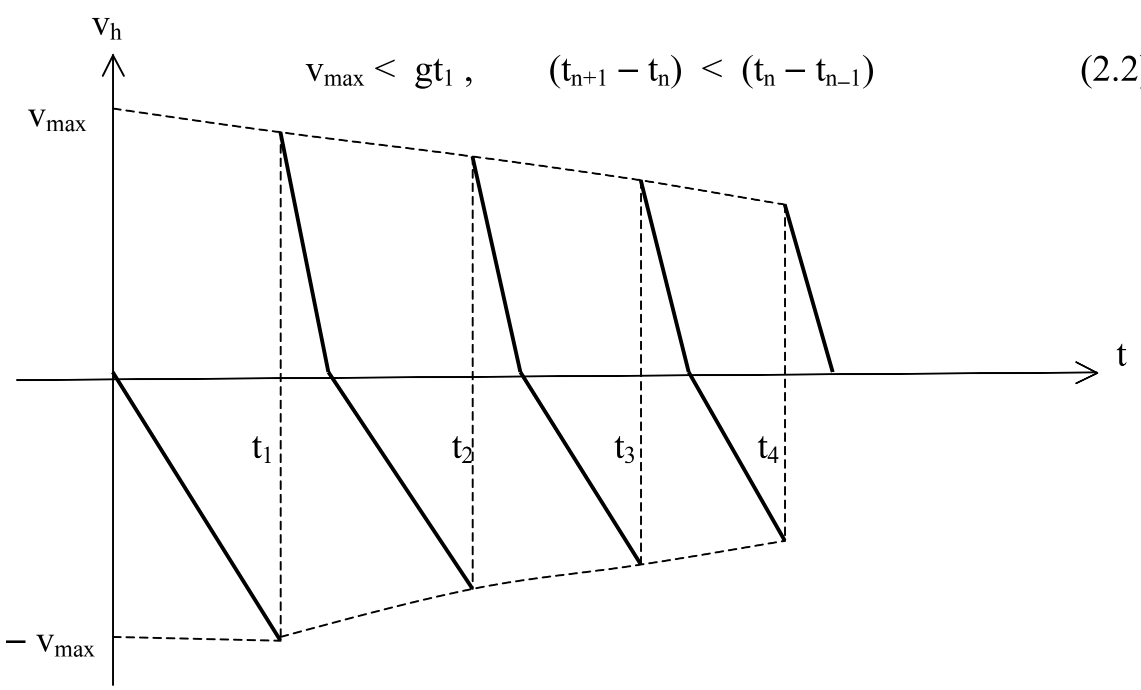


Fig. 11: Vertical velocity vs time graph of (almost) *real* bouncing superball
A similar modification would have to be made on **Fig. 3** for a *real* bouncing superball.

the distortion of its shape. These internal forces of resistance, a kind of internal friction, do only negative work on the ball, just like air resistance, and keep the elastic forces from re-imparting all the original kinetic energy back to the ball.

Consequently, after each bounce the ball fails to rise as high as it previously fell from and, gradually, it bounces less and less, loses more and more of its kinetic energy and comes to rest. The original mechanical energy is gone!

b. Oscillations in real channels

Similar things happen to objects attached to springs and sliding back and forth in a horizontal channel. When any two material surfaces, in contact, move over each other a resistance force, the *original* **friction**, comes into play and opposes the relative motion, always doing negative work on the relatively moving objects. The smoothness of a channel refers to how small the resistance is, but it's always present to some degree. And again, since it always does negative work, there is no potential energy associated with it that can be added to the definition of mechanical energy to keep the latter constant.

Besides the channel friction there is also the resistance due to motion through the air that we saw in the previous example and an internal resistance to the stretching and compressing of the spring, analogous to the bouncing ball's internal resistance to flexing. All of these resistance forces just drain away the mechanical energy of the system and the sequence of oscillatory motions of the object and spring steadily diminish in amplitude (**Fig. 12**) until coming to rest at (or near) the relaxation point for the spring. The original mechanical energy is gone!

c. Real objects hanging from springs

Here there is no channel friction but the air resistance and the internal resistance of the spring to flexing have the same effect as in the previous examples.

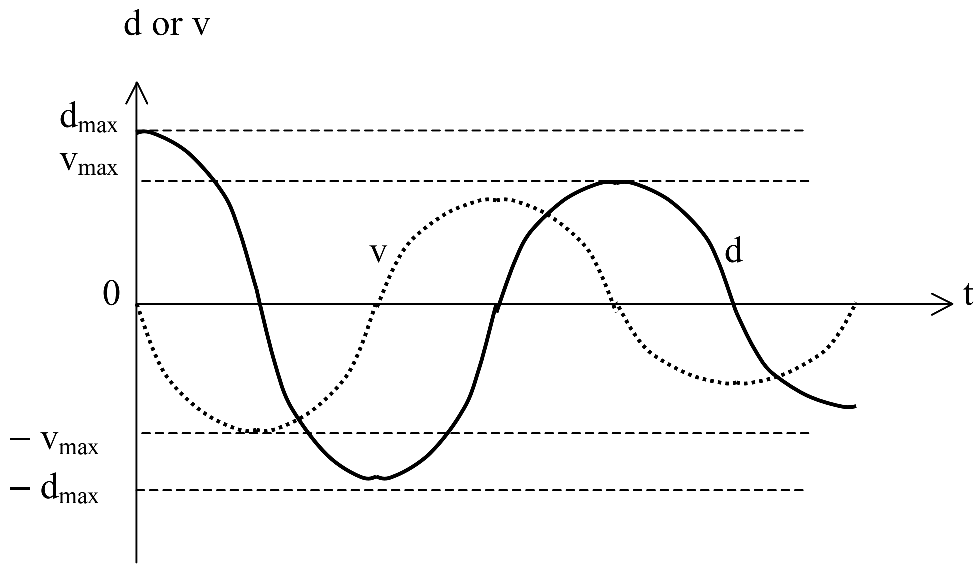


Fig. 12: Graphs of the position vs time and velocity vs time of the (almost) *real* object attached to the spring.

d. The Sun – Earth – Moon system

No air resistance impedes the motion of the Earth or the Moon, although there is a bit of space dust out there, but the Earth's daily rotation produces an analogue of surface friction due to the tidal effects of the Moon's and Sun's gravity on the Earth's oceans.

Tidal effects refers to the fact that for any two of these bodies the gravitational force from one is slightly stronger on the near side of the other than on the far side and the direction of the force is slightly different at the leading edge than at the trailing edge of the other (**Fig. 13**). A dramatic example of this phenomena was provided by the tidal effects of Jupiter's gravity pulling the comet, Shoemaker-Levy 9, apart into many fragments before it (they) collided with Jupiter. In the Earth-Moon system the cumulative dissipative effect is much slower to arise than in the previous examples but in all the cases the basic result is the same; *the mechanical energy of the system does not remain constant but decreases with time. The mechanical energy is gradually lost!* The dominant cumulative effect of this friction is to diminish the rotational kinetic energy of the Earth, very slowly lengthening our days.

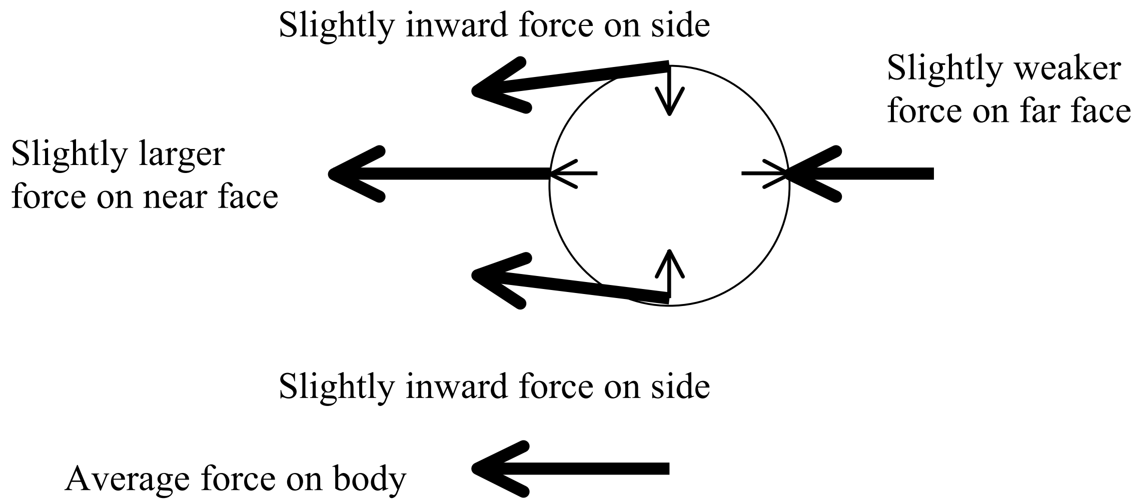


Fig. 13: Tidal forces (small arrows) on spherical body in gravitational field of distant body. Each large force on the body is the sum of the average force and the contiguous tidal force.

Besides this surface friction on the Earth there is internal resistance to flexing that gravitational tidal effects produce inside both the Earth and the Moon as the Moon orbits the Earth at slightly varying distances and the Earth similarly orbits the Sun. Slowly but surely these effects sap the total mechanical energy of the S-E-M system and, in the *very* long run, the Moon would eventually drift closer to the Earth and probably be pulled apart into rocky rings around the Earth similar to Saturn's rings and the Earth would slowly drift closer and closer to the Sun and - - - -. But not to worry, the Sun is expected to expand and *at least* incinerate the entire Earth long before the previous scenario plays out.

3. Dissipative Forces

Forces which always do negative work on the bodies they act upon, i.e., which always oppose the motion that gives rise to them, are called **dissipative forces** because they dissipate, or diminish the mechanical energy composed of potential energies and kinetic energies. These are the forces of **sliding friction, fluid resistance, viscosity**, etc. If a number of forces act within and upon a physical system and the mechanical energy of the system is E_{mech}, then in any time interval during which the dissipative forces do a total amount of work, W_{diss} , which will always be negative, we have,

$$\Delta E_{mech} = W_{diss} . \tag{3.1}$$

At first glance we might wish to be rid of dissipative forces, and, through efforts at lubrication and streamlining and fine polishing we do try to diminish unwanted dissipative forces. But the common examples of such forces of **static friction** and **rolling friction** are absolutely essential for our ability to move around, whether walking or driving. Without them all efforts to get from A to B would be worse than negotiating on smooth, wet ice. We'd also have a much harder time holding things in our hands; everything would feel somewhat like a bar of wet soap in a shower.

Sliding friction is indispensable in bringing our vehicles to smooth stops in reasonable distances via our brakes, dissipating the kinetic energy of our vehicles in the process. Viscosity in our shock absorbers helps to keep our rides comparatively smooth by damping out the oscillations after road bumps. Without fluid resistance, in particular air resistance, we would run from the smallest shower as from the torrential downpour! Raindrops form at considerable altitudes and, in the process of falling, continuously accelerate until reaching **terminal velocity**. That's the velocity at which the force of air resistance on the raindrop balances the downward force of gravity, the drops' weight. Without air resistance the drops would reach the ground traveling much faster than they actually do and would be quite stinging (even dangerous) upon striking the skin.

Still, the price for these dissipative forces, which can be useful, is, apparently, the persistent dissipation of the mechanical energy of the world!