Class VI: $E = M c^2$ and Space – Time

1. $E = M c^2$

OK! So now we know that all energy carries mass and that all mass can be converted into energy, both in accordance with the equation,

$$E = M c^2. \tag{1.1}$$

Strictly speaking, the phraseology of the previous statement is a little misleading. Instead of saying that energy *carries* mass we should say that energy *has* mass, or even that *energy is mass*. Instead of saying that mass can be converted into energy we should say that the highly concentrated form of energy that we call matter (and which we quantify primarily in terms of mass) can be converted into the less concentrated and more useful forms of energy we traditionally have called energy.

It can be helpful to introduce a new, hyphenated term, “mass-energy”, and recognize traditional energy as that form of mass-energy which is sufficiently dilute so as to be comparatively manipulable for moving things around against resisting forces, while matter or mass is that form of mass-energy which is so concentrated as to manifest itself primarily in terms of its inertial, elastic and gravitational properties. Contrary to the earlier view, there is just one kind of thing involved, mass-energy, not two different things. Furthermore, as a matter of principle, all mass-energy can be manipulated and all mass-energy has inertial, elastic and gravitational properties.

People tried this way of talking for awhile. But physicists quickly got used to the notion of a unitary mass-energy and then they dropped the hyphenated word and just used “mass” and “energy” interchangeably with a slight prejudice towards “energy” as being the more fundamental term. In this course I will oscillate between all these different ways of talking about the subject.

So what about the quantitative aspect? How much traditional energy is associated with a given amount of traditional mass?

For 1 kg of mass we have,

$$M c^2 = 1 \text{ kg x } (3 \text{ x } 10^8 \text{ m/s})^2 = 9 \text{ x } 10^{16} \text{ kg m}^2/\text{s}^2$$

$$= 9 \text{ x } 10^{16} \text{ j} = E , \quad (1.2)$$

90 quadrillion joules or 90 million billion joules! Sounds like a lot of energy, but unless we have some feeling for the energy unit, the joule, the statement doesn't convey much. We're all reasonably familiar with the kilowatt-hour as a unit of energy so lets use that. Since a joule is a watt-second, a kilowatt-hour (kwh) is 3.6 million joules. Consequently,

$$(1 \text{ kg}) c^2 = 2.5 \text{ x } 10^{10} \text{ kwh} . \quad (1.3)$$

So, if we could convert 1 kg of matter completely into electrical energy, we would get 25 million megawatt hours worth! Now we're talking!

But are there processes in nature where traditional mass does get converted into useable traditional energy? There are at least three such processes: **radioactive decay** of unstable particles and nuclei, **fission** of heavy nuclei into lighter nuclei and **fusion** of light nuclei into heavier nuclei. To discuss these it is convenient to first introduce mass and energy units that are appropriate for these sub-microscopic entities. The rest masses of a proton, neutron and electron, the particles that comprise ordinary matter, are,

$$m_p = 1.672 \text{ x } 10^{-24} \text{ g} := 1.008 \text{ au} \quad (1.4a)$$

$$m_n = 1.675 \text{ x } 10^{-24} \text{ g.} = 1.010 \text{ au} \quad \text{and} \quad m_e = 5.5 \text{ x } 10^{-4} \text{au} , \quad (1.4b,c)$$

where au stands for **atomic mass unit**.The energy equivalent of 1 au is

$$(1 \text{ au}) c^2 = 1.49 \text{ x } 10^{-10} \text{ j} \simeq 931 \text{ Mev} , \quad (1.5)$$

about 900 million electron volts, which is the kinetic energy acquired by a proton or an electron upon being accelerated through a potential difference of 900 million volts.

Radioactive decay: A free neutron, all by itself, will spontaneously disintegrate into a proton, an electron and a neutrino with a half life of about

1000 seconds. Half life means that if there are very many free neutrons, then half of them will decay within about 1000 seconds and half of the remainder within the next 1000 seconds, etc. But there is no possibility of predicting when any one neutron will decay, so far as anyone knows, and this is the general nature of radioactive decay wherever it occurs. In any case, when the neutron decays we have,

$$n \rightarrow p + e + \nu \, , \tag{1.6}$$

with a corresponding change in rest mass of,

$$m_n = 1.010 \text{ au} \rightarrow m_p + m_e + m_\nu$$

$$= 1.008 \text{ au} + 0.00055 \text{ au} + 0.0 \text{ au} \simeq 1.009 \text{ au}. \tag{1.7}$$

Consequently, 0.001 au, of the neutron's rest mass has been converted into the kinetic energy of the proton, electron and neutrino, about 0.93 Mev.

More important for practical considerations is the energy release from the radioactive decay of heavy elements like uranium. The kinetic energy of the decay products (electrons, alpha particles, gamma rays and neutrinos) in these cases is distributed (by collisions) as heat energy among the neighboring atoms and such processes play a major role in maintaining the high temperature and molten state of the Earth's core. Prior to the discovery of radioactivity and energy-mass equivalence it was very hard to understand why the Earth hadn't cooled much more than it had in the time that geological evidence indicated as the age of the Earth. Gravitational compression during the formation of the Earth would account for the initial heating but without radioactive energy deposition the core should have long ago cooled much more than it has.

Nuclear fission: For the phenomena of nuclear fission an important example is the fission of a uranium 235 nucleus triggered by the absorption of a slow neutron. A typical example of the many possible fission reactions is,

$$n + U^{235} \rightarrow Zr^{94} + Ce^{140} + 2\,n, \tag{1.8}$$

with the kinetic energy of the initial neutron being negligible. The difference between the total rest masses before the reaction and the total rest masses after is about 0.22 au, which has an energy equivalent of about 208 Mev. Besides the substantial liberation of kinetic energy, the two final neutrons can go on to trigger fission in *two* additional uranium nuclei. The rapid repetition of this multiplicative process induces the notorious **chain reaction** and, depending on the concentration of fissionable material (critical mass) and the manipulation of the probability that free neutrons actually get to induce fission (presence of neutron absorbers) one can generate either a controlled and sustained nuclear reaction or a cataclysmic nuclear explosion.

From the previous numbers we can estimate that 200 Mev is roughly equivalent to 10^{-17} kwh. So if we could induce the fission of 235 g of U^{235} (1 mole), we would liberate about 10^{-17} kwh of energy for each of the

$$\sim 6 \text{ x } 10^{23} \sim \text{Avogadro's number}$$

nuclei that had been split. That's a total energy liberation of about,

$$6 \text{ x } 10^{23} \text{ x } 10^{-17} \text{ kwh} = 6 \text{ x } 10^{6} \text{ kwh} = 6000 \text{ Mwh}. \qquad (1.9)$$

Consequently, if one steadily induced fission in only 1 g of U^{235} every 24 hours the power output would be about 1 megawatt. Tapping most of that output for effective distribution and use poses technological challenges, of course.

Thermonuclear fusion: Finally, we come to nuclear fusion in which lighter nuclei convert traditional mass into traditional energy in the process of fusing together to form heavier nuclei. These are the only processes that enable us to understand how stars form and last as long as they do and how they die. The following partial account is an excerp from my other OLLI course, "The New Cosmology", which will be offered next Spring.

"Gravity tends to have the destabilizing effect of pulling matter into regions that are already more dense than their surroundings and pulling matter out of regions that are less dense than their surroundings. As best we can tell, stars form as a consequence of regions within vast clouds of dust and gas (called **stellar nurseries**) condensing under the attractive influence of mutual gravity until the temperature of the condensing material gets so high that thermonuclear [fusion] reactions are triggered. The regions in which the condensing starts are regions that have, as a consequence of density fluctuations, slightly higher than average density to begin with. The temperature rises as the condensing goes on because the particles, atoms and molecules of the dust and gas clouds, as they are pulled closer and closer together, collide with one another more and more frequently and more and more energetically. Temperature, T, is, basically, a measure of the average randomized kinetic energy per particle, $< K_{random} >_{av}$,

$$T \sim < K_{random} >_{av} , \qquad (1.10)$$

and the collisions tend to convert the relatively organized and increasing kinetic energy of the infalling motion into relatively randomized kinetic energy. At first the collapsing material starts to glow as a consequence of the collisions exciting atoms which then create photons of various frequencies as they deexcite. Once the temperature near the center of the collapsing material reaches a few million degrees (Kelvin or Celsius; it doesn't matter which) the nuclear fusion reactions (called **Hydrogen burning**),

$$^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu , \qquad {}^{2}H + {}^{1}H \rightarrow {}^{3}He + \gamma , \qquad (1.11a,b)$$

$$^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H, \qquad (1.11c)$$

(called the proton – proton I sequence)

and, $$^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma , \qquad {}^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu , \qquad (1.12a,b)$$

$$^{7}Li + {}^{1}H \rightarrow {}^{4}He + {}^{4}He , \qquad (1.12c)$$

(called the proton – proton II sequence with the net effect that four protons are converted into one ^{4}He nucleus) ignite and start burning the Hydrogen of the protostar into Helium.

The collapsing material contains about 75% Hydrogen and 24% Helium (this ratio appears to hold throughout the Universe) to begin with and these reactions keep raising the Helium to Hydrogen ratio [within the star]. If the mass of the collapsing material is not much larger than our Sun's mass then, after about 20 million years of collapsing and heating up, the pressure and radiation generated by these processes will stop the gravitational collapse and the star will be stable for about ten billion years. Our Sun is just about halfway through its ten billion year stable regime."

For some time now physicists and engineers have been struggling to develop practical and useful forms of controlled nuclear fusion for power generation here on Earth. If we ever succeed the process should be much cleaner than nuclear fission power production as there appears to be no waste disposal problem.

In the preceding ways we see the importance of the discovery of $E = M c^2$.

2. Space-Time

"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

Herman Minkowski (1908)

We have spent enough time on the arguments of Einstein that led Minkowski to this dramatic pronouncement in the beginning of his 1908 paper, "Space and Time" so that I will not review Minkowski's arguments supporting his view. Instead we will just examine the view itself.

We will employ a space-time diagram in which the three dimensions of space are suppressed into just one. Since we will always need to employ one dimension of our diagrams to represent time and since I can not draw a useful representation of a four dimensional space on a two dimensional sheet, we will settle for representing only one dimension of space graphically.

The diagrams are drawn from the perspective of some inertial reference frame which we call F. The time axis of F will be vertical with the past at the bottom of the diagram and the future at the top. The space axis of F is horizontal. Rightly interpreted the diagrams should enable us to "see" the internal consistency of the relativity of simultaneity, time dilation, length contraction, the limiting nature of vacuum light speed and the invariance of the relativistic interval,

$$\Delta s^2 = c^2 \, \Delta t^2 - \Delta \mathbf{x}^2. \qquad (2.1)$$

On to the first diagram!

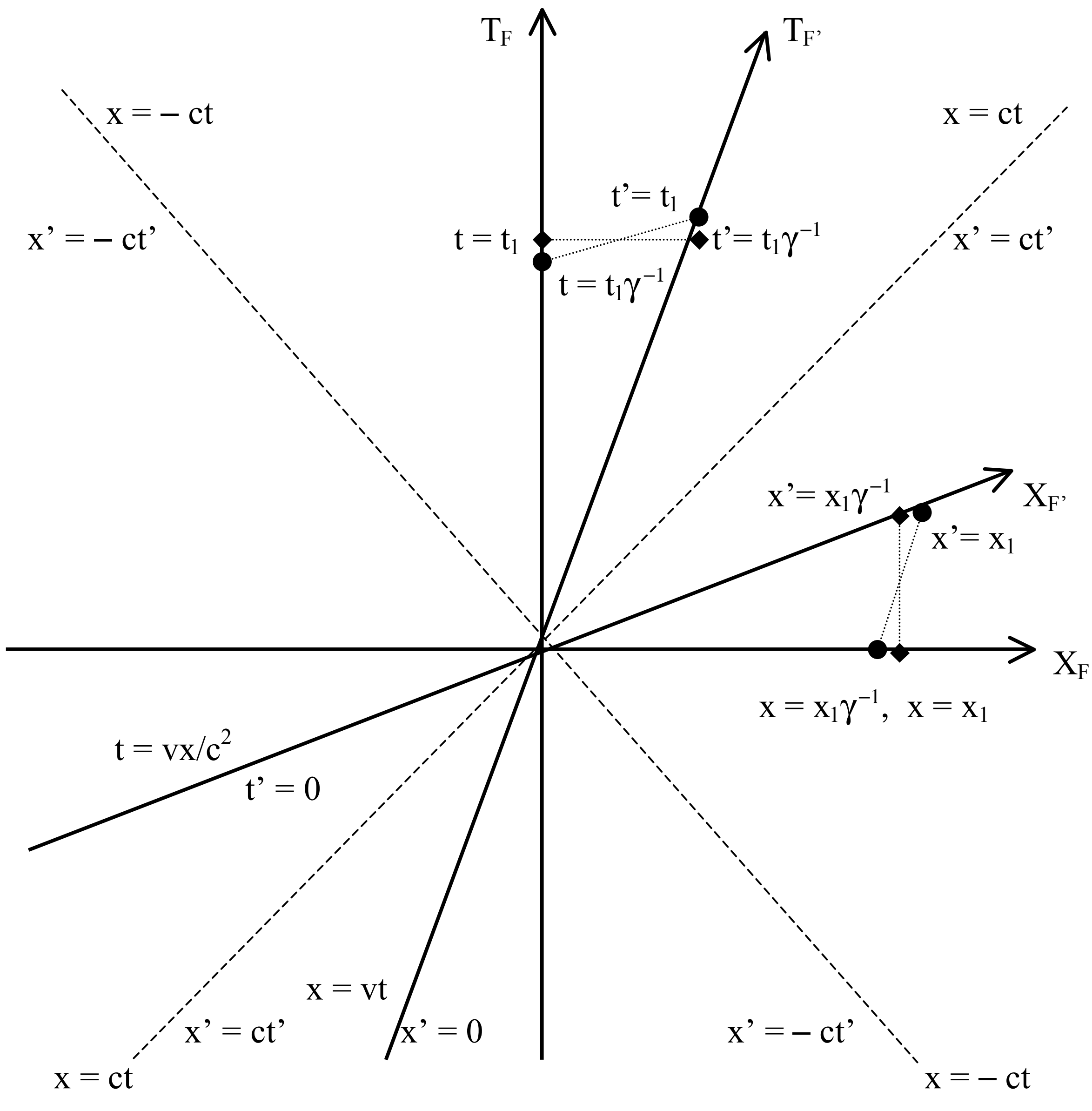

Fig. 2.1: Space-time diagram for the relationship between inertial frames, F and F', where F' moves in the positive x direction with speed, v, relative to F. The relativity of simultaneity, mutual time dilation and mutual length contraction is illustrated. The perspective of the diagram is that of F.

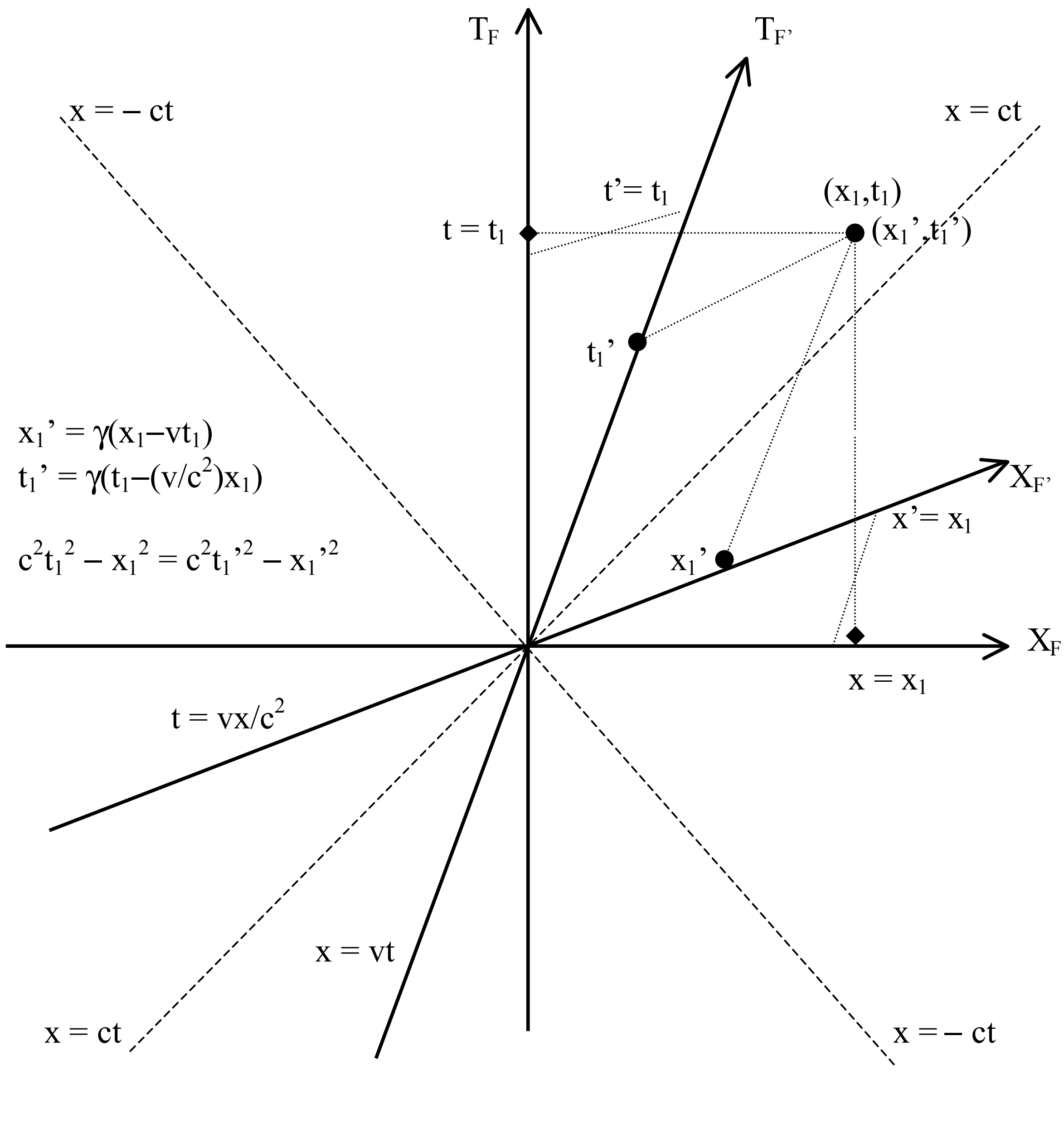

Fig. 2.2: Space-time diagram for the relationship between the same frames, F and F', as above. Illustrated is the Lorentz transformation between the two frames and the invariance of the relativistic interval.