Class II: Time, space and motion before Einstein

1. Time intervals

Our intuitions about time are very deep. We don't change them easily! That's as true for me, the professional physicist, as it is for you. How did they get so deep if they're so wrong? The answer, I think, is that for the entire realm of ordinary human experience the STR correction is, quantitatively, insignificant and the deep intuition is noticeably simpler. Long before history, indeed, long before anything one could call human civilization, I would guess, our forbears semi-consciously noticed and automatically internalized that assessments of time passage between meetings (how many seasons, how many moons, how many days, - -) were, with corrections for forgetting and confusion, the same for all involved parties. Only in spiritual experiences did time seem to acquire a very individual character. Outside of the Spirit World time was not personal, it passed the same for all. After all the millennia required to reach the beginnings of science in the ancient world, this deep intuition could barely be accessed for conscious recognition and only with the Greeks did the West's long puzzlement over the ultimate nature of objective time begin. But its objectivity, i.e., invariance, was a given.

Much later, when science had long since become a thriving concern and mathematics was systematically being used to give quantitative precision to our growing understanding of Nature, the quantitative expression of this intuitive conception of time could be given by the simple equation,

$$\Delta t' = \Delta t \,. \qquad (1.1)$$

The Greek letter, Δ (upper case Delta), is widely used in science to indicate a range or interval of values of a quantity and here it is indicating the interval of time separating two events. The superscript, '(prime), on the left of the equal sign indicates measurement of the time interval between the events of interest by a different observer or set of clocks than on the right. The equal sign tells us that both observers, whoever they are and whatever their circumstances, must get the same result if they measure correctly. The understanding represented by this equation permeated all of pre-Relativistic physical science to such a degree that it was rarely ever written down.

2. Space intervals

Distance was not and is not so simple. Suppose I'm in a seat and I have a ball in my hand. I gently throw the ball upwards, about a foot, and catch it when it falls back down at the same place where it first left my hand. This takes about 1/2 sec, i.e., the time interval between the ball leaving my hand and returning to my hand is about 1/2 sec. The distance between those events is zero! Sure, the ball went up a foot and then back down a foot, but the *net* distance covered by the ball between leaving my hand and returning to it is zero. *Relative to me*, the two events occur at the same place and have zero distance between them.

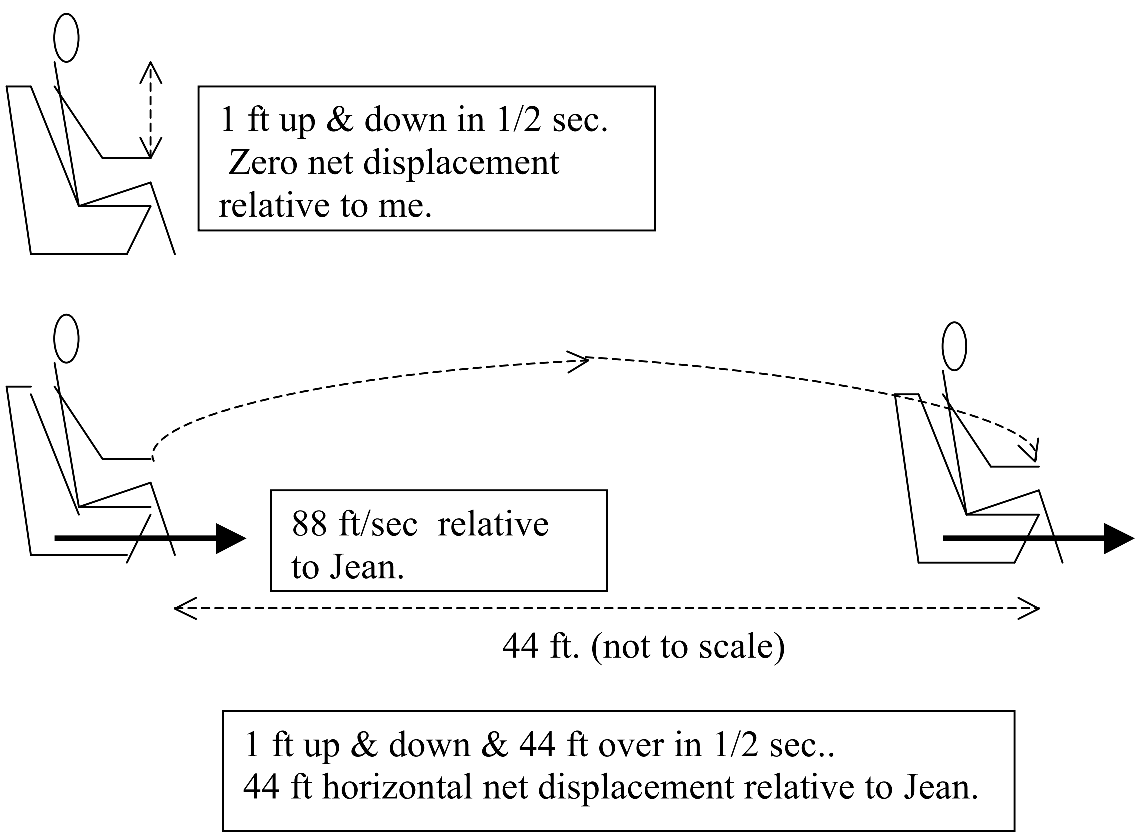

Fig. 2.1a: A moving ball in a moving car. Ordinary relativity of displacements that take time

We now take notice of the fact that the seat I'm sitting in is the passenger seat of a car tooling down the road at 60 mph = 88 ft/sec (~ 29 m/sec). *Relative to Jean*, standing by the side of the road as I pass and lob the ball, the ball's net motion is about 44 ft down the road. We know it's 44 ft because, according to our previous equation, the time between my lobbing and catching the ball is 1/2 sec. for Jean just as it is for me, and in 1/2 sec the car and me and the ball will move down the road 44 ft. (**Fig. 2.1a**)

So a zero distance interval, or **displacement** as it's called, for me is a 44 ft long horizontal displacement for Jean. Unlike time, distances or displacements can change from observer to observer *if the end-points occur at different times*.

Suppose I drop the ball to the floor of my vehicle rather than lob it up and down. If there's enough vertical height in the passenger compartment, I can have it fall through 4 ft, which will again take about 1/2 sec. This time, *relative to me*, or, to be less anthropomorphic, *relative to my car*, the two

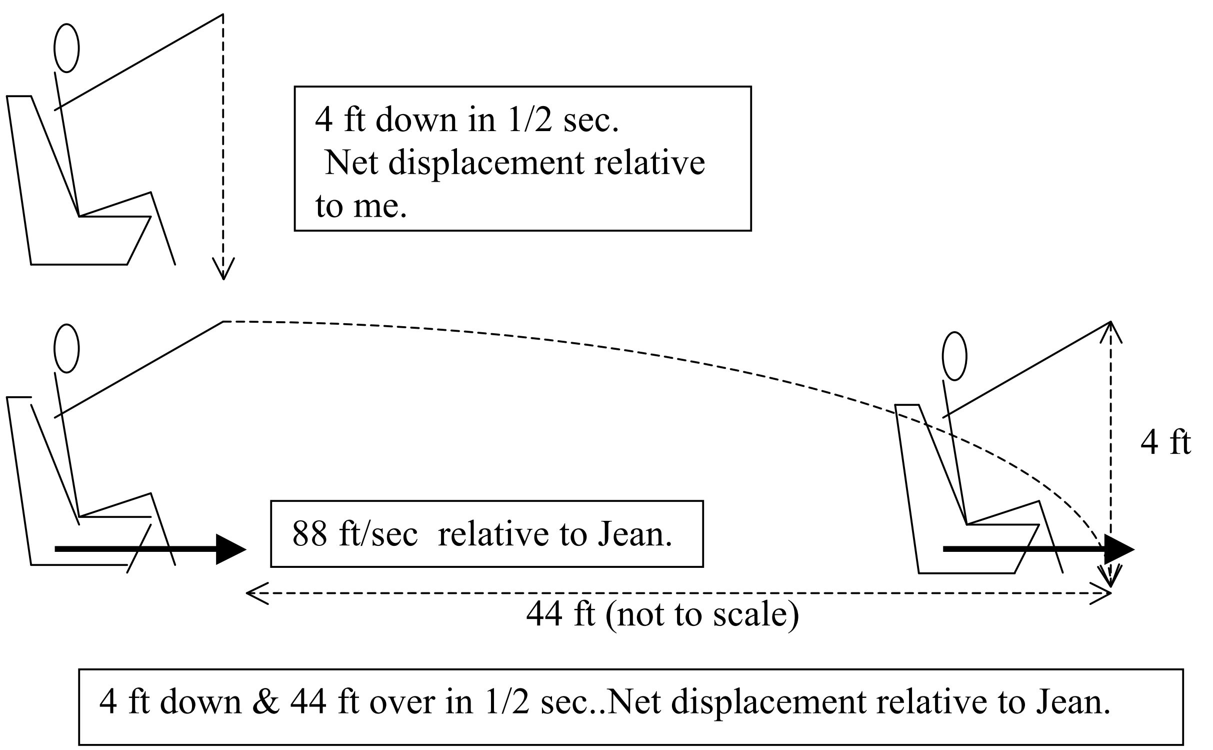

Fig. 2.1b: A moving ball in a moving car. Ordinary relativity of displacements that take time

events, the ball leaving my hand and the ball hitting the floor, are separated by a vertical downward displacement of 4 ft and a time interval of 1/2 sec.

Relative to Jean, or, *relative to the road*, those two events are separated by a displacement of 44 ft horizontally down the road and 4 ft vertically downward and a time interval of 1/2 sec. (**Fig. 2.1b**).

How can we be sure that the 4 ft vertical drop relative to me or my car is also a 4 ft vertical drop relative to Jean or the road? Again this is a conviction based upon a long history of experience and, in this case, it is a conviction that survives the STR revolution!

The conviction was already old when people could notice (If they were so inclined) that if the deck of a boat, with crew on board, was so many ft above the water when the boat was moored, it was the same number of ft above water when under way (unless newly laden with goods for the voyage). Ratchet forward in time to Edgerton's ultra fast photographs of bullets in flight to find the diameters of the bullets in flight to be the same as when held in ones hand (assess the diameter in flight by comparison with the diameter of the apple the bullet has struck in the photo)(**Fig. 2.2**).

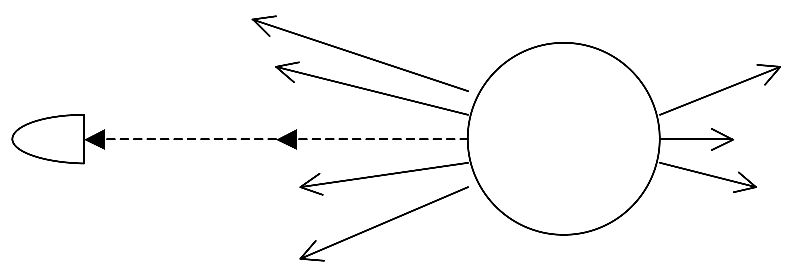

Fig. 2.2: Schematic of famous Edgerton photo of bullet in flight after passing through apple

Beyond this, note that if Jean and I each hold a vertical yardstick in such a way as to guarantee that the bottoms of our yardsticks just graze each other as we pass, then it would be very surprising if the top of one of our yardsticks was lower than the other as they passed. For what would distinguish between my yardstick or Jean's to be the shorter one? Lest someone suggest that I'm moving relative to the very large Earth and Jean isn't, let us take the situation into outer space, not so far fetched these days, and consider us drifting passed each other while holding yardsticks perpendicular to our relative motion. NOW what distinguishes between us? From considerations like this as well as accumulated experience we conclude that for two observers or two **frames of reference** in motion relative to each other, distances or lengths of spatial intervals perpendicular to the relative motion have the same values for each observer or frame of reference. In terms of equations, for quantitative precision, if the direction of relative motion is called the x direction and the two mutually perpendicular directions are y and z, then we have,

$$\Delta y' = \Delta y \qquad \text{and} \qquad \Delta z' = \Delta z \,. \qquad (2.1)$$

But what about $\Delta x'$ and Δx? We've already said more than enough to make it clear they will not be the same. Our examples had $\Delta x = 0$ and $\Delta x' = 44$ ft. What's the general relationship?

Suppose my friend is in the back seat of the car and I lob the ball back to him and he catches it at the same level I threw it, $\Delta y = 0$, 6 ft behind me, $\Delta x = -6$ ft and, again, 1/2 sec later, $\Delta t = 1/2$ sec. Jean observes that each seat, rigidly attached to the car, passes her at a speed of v = 88 ft/sec and that the separation between the moments when the front and back seat pass her is just over 1/16 sec. So she agrees that my friend is 6 ft behind me and assesses the ball to have moved 44 ft down the road and – 6ft from front seat to back seat between being thrown and being caught. So $\Delta x' = -6$ ft + 44 ft = 38 ft. The – 6 ft is just my – 6 ft and the 44 ft comes from traveling 88 ft/sec for 1/2 sec, i.e.,

$$44 \text{ ft} = (88 \text{ ft/sec}) \times 1/2 \text{ sec} = v\, \Delta t \,.$$

Abstracting from the numerical details of our example we 'see' that the general relationship is,

$$\Delta x' = \Delta x + v\, \Delta t \,. \qquad (2.2a)$$

We also notice, with a little help from our first equation that,

$$\Delta x = \Delta x' - v\, \Delta t'. \qquad (2.2b)$$

This makes sense since just as my **velocity** relative to Jean is 88 ft/sec *down the road*, so Jean's velocity relative to me is 88 ft/sec *up the road*, i.e., – 88 ft/sec. If you object that Jean isn't moving up the road, I respond with, “Hey! Relative to me, even the *road* is moving at 88 ft/sec up the road.” “Up the road” just denotes a direction.

3. Vectors (Arrows)

One restrictive aspect of the preceding discussion is its formulation in terms of reference frames that choose their x coordinate axes along the direction of their relative motion and the remaining perpendicular y and z coordinate axes respectively parallel. It is instructive to see an alternative description that is independent of the assignment of coordinates.

Let E_1 and E_2 be highly localized events (Ideally these would be point-instant events that occur at single points of space at precise instants of time. But real events aren’t like that and so we just require them each to be confined to regions of space and intervals of time that are smaller than the precision in the space and time intervals between them, i.e., “highly localized”.). The events may occur at different times or be (approximately) simultaneous. Denoting the time interval between them as Δt, we have $\Delta t > 0$ if E_1 is earlier than E_2 and $\Delta t < 0$ if E_1 is later than E_2. Because of what we said above, (1.1), everyone will agree on the value of Δt.

Let Boris (B) and Claudius (C) be two observers in motion relative to one another. Each observer can represent the spatial interval between E_1 and E_2 by an arrow from one event to the other. The direction of the arrow is as important as its length in indicating the spatial relationship between the events. To be specific let’s assume that $\Delta t \geq 0$ and that the arrow goes from E_1 to E_2. We will denote the arrow by $\mathbf{\Delta x}$. But we’re talking as though there was just one arrow. In fact, except in special cases, observers B and C will represent the spatial relationship between the events by quite different arrows. They will see the events as having quite different spatial relationships.

C will say that B treats the location of E_1 as though it moves along with B until E_2 occurs and then B draws an arrow from the “moved” location of E_1 to the location of E_2. C, on the other hand, draws the arrow from the “actual” location of E_1 to that of E_2 (**Fig. 3.1**).

B will retort that it is C who treats the location of E_1 as though it moved along with C while waiting for E_2 and then connected the “moved” locations with an arrow, while B connected the “actual” locations.

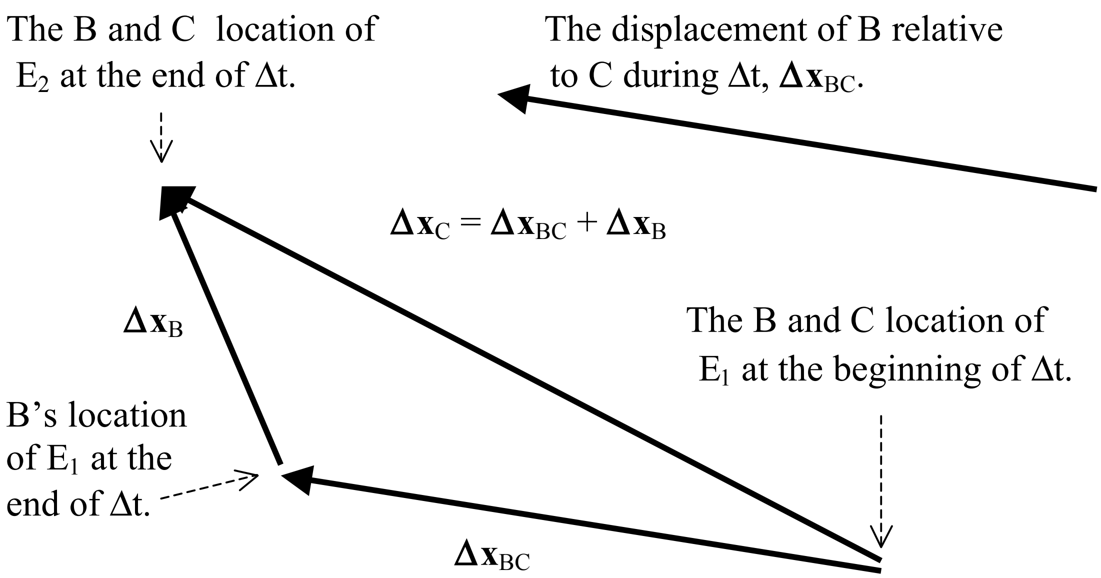

Fig. 3.1: The relationship of the spatial intervals between two events (E_1 and E_2) with respect to two relatively moving observers (B and C). C’s perspective.

Remember the previous examples where Jean watches me lob or drop a ball. From Jean’s perspective, my assessment of where the initial event (the ball leaving my hand) occurred moves down the road with me as we wait for the final event (the ball lands).

Suppose the change in position of B relative to C (and thus measured by C) during Δt is given by an arrow, $\mathbf{\Delta x}_{BC}$ (**Fig. 3.1**). That is also the displacement that C will judge B to have imposed on the location of E_1 in order to fix the location of E_1 relative to B during Δt. Then the arrow from the E_1 location according to C to the E_2 location according to C, $\mathbf{\Delta x}_{C}$, indicates the same net displacement as first displacing from the E_1 location according to C to the E_1 location according to B, $\mathbf{\Delta x}_{BC}$, and then from the E_1 location according to B to the E_2 location according to B, $\mathbf{\Delta x}_{B}$, i.e.(See **Appendix** for the elements of Arrow Algebra),

$$\mathbf{\Delta x}_C = \mathbf{\Delta x}_{BC} + \mathbf{\Delta x}_B \ . \tag{3.1a}$$

This discussion has been carried out primarily from the perspective of C. The corresponding discussion from the perspective of B would lead to,

$$\mathbf{\Delta x}_B = \mathbf{\Delta x}_{CB} + \mathbf{\Delta x}_C \ . \tag{3.1b}$$

If these results are to be compatible we must have,

$$\mathbf{\Delta x}_{CB} = -\,\mathbf{\Delta x}_{BC} \ , \tag{3.2}$$

and this makes sense in that we expect that B would observe C to move in the opposite direction and by the same distance as C would observe B to move.

Fortunately we don't have to decide who is right because, in an important sense, they both are. Each observer, having recorded the location of E_1, upon its occurrence, regards that location as fixed relative to him as he waits for E_2 to occur. But if the observers are moving relative to each other, a location fixed with respect to either of them will be moving with respect to the other. This practice of each observer regarding the spatial location of a momentary event as fixed with respect to the observer in question defines the **frame of reference** associated with each observer.

Ultimately, the reason for this attitude on the part of B and C is that ever since Copernicus, Galileo and Newton we have recognized that there is no single, fundamentally preferred perspective for referring the locations of events. My space is as good as your space, and yours is as good as mine, for assessing locations. The spaces of B and C are equally good for assessing locations of events. This means that, there being no fundamental basis for preferring one of these assessments over another, we can only make choices between them based on convenience and must learn the relationships between them for when we want to change our choice.

The technical name for arrows used to represent changes in position is **displacements**. The technical name for arrows used to represent any quantity that has both a magnitude and a direction is **vectors**. But they're all just arrows.

a. velocities and displacements

During the time interval, Δt, the **average velocity** of B relative to C is defined by the equation,

$$\mathbf{V}_{BC,\,av} := \mathbf{\Delta x}_{BC} / \Delta t\,, \quad \text{or} \quad \mathbf{\Delta x}_{BC} = \mathbf{V}_{BC,\,av}\, \Delta t\,, \qquad (3.3)$$

where dividing or multiplying a physical arrow, i.e., an arrow with dimensional units, by a physical number (with dimensional units) results in a new arrow of a physically different kind, e. g., from a displacement to a velocity.

Similarly, $\mathbf{V}_{CB,\,av} = \mathbf{\Delta x}_{CB} / \Delta t$, and since $\mathbf{\Delta x}_{CB} = -\,\mathbf{\Delta x}_{BC}$, we have,

$$\mathbf{V}_{CB,av} = -\,\mathbf{V}_{BC,\,av}\,. \qquad (3.4)$$

If B and C are not relatively accelerated (the case of greatest importance to us, as we shall see), then $\mathbf{V}_{BC,\,av}$ is constant, i.e., $\mathbf{V}_{BC,\,av} = \mathbf{V}_{BC}$, independent of time, and, from (3.1, 2, 3), we have,

$$\mathbf{\Delta x}_{C} = \mathbf{\Delta x}_{B} + \mathbf{V}_{BC}\, \Delta t\,, \qquad (3.5a)$$

and

$$\mathbf{\Delta x}_{B} = \mathbf{\Delta x}_{C} + \mathbf{V}_{CB}\, \Delta t = \mathbf{\Delta x}_{C} - \mathbf{V}_{BC}\, \Delta t\,. \qquad (3.5b)$$

These relationships tell us two things of immediate importance; First, if the events, E_1 and E_2 are simultaneous, i.e., $\Delta t = 0$, then,

$$\mathbf{\Delta x}_{C} = \mathbf{\Delta x}_{B}\,, \qquad (3.6)$$

and our observers agree on the spatial relationship between simultaneous events. Second, if $\Delta t > 0$ and some object, A, actually moves from E_1 to E_2, then by dividing the displacement relationship through by Δt we find that the average velocity of that object relative to B and C satisfies,

$$\mathbf{V}_{AC,\, av} = \mathbf{V}_{AB,\, av} + \mathbf{V}_{BC} \ . \qquad (3.7)$$

Since this holds regardless of the size of **Δ**t, by considering arbitrarily *small* time intervals, the relation must also hold for the **instantaneous velocities**,

$$\mathbf{V}_{AC} = \mathbf{V}_{AB} + \mathbf{V}_{BC} \ . \qquad (3.8)$$

In fact, this last relationship holds even between reference frames that are in relative acceleration.

4. Inertial Reference Frames

The vector relations, (3.1), between displacements, and (3.8), between instantaneous velocities, hold for *any* observers or reference frames, B and C, when time intervals are invariant as in (1.1). The vector relations, (3.5), on the other hand, hold *only* when the relative velocity between B and C is constant, i.e., no relative acceleration.

Special status is possessed by non-accelerating observers and reference frames in both pre-Einstein physics and Einstein's STR.

The reason accelerated reference frames are less desirable is that Galileo and Newton discovered strong evidence that accelerations are the clear signs of the presence of (unbalanced, physical) causal agencies in nature. *That's what causal agencies do – they tend to change velocities – they tend to accelerate*. Consequently, if an accelerating perspective (reference frame) is used to assess locations and motions, etc., it will mask and distort the accelerations due to the causal agencies. While Aristotle and most of his contemporaries thought that the causal agencies of Nature determined the proper locations and velocities of things, Copernicus, Galileo and Newton showed us that it was accelerations that they really determined.

This discovery was refined by Newton who characterized the natural perseverance of un accelerated motion as **inertia** and who subsumed under the concept of **force** the notion of physical cause and environmental influence.

So if one wants a perspective from which all of nature's causes or forces will manifest themselves effectively one should adopt a non-accelerated perspective, i.e., a uniformly moving reference frame – and anyone will do.

But how does one tell when a reference frame is accelerated or not since only relative motion can be observed?! *First*, if the observed relative motion is accelerated, then at least one of the systems related by the motion is, itself, accelerated. *Second*, we can *feel* accelerated motion because we can feel forces inside us (except for those we have grown so used to we no longer notice them) and we can often tell when forces are acting on objects. Forces can squeeze and/or stretch objects or make them spin, etc. But we can't *always* tell and, ultimately, there is a deep "chicken and egg" problem here that has *never been adequately resolved*. Can we really *identify* un accelerated motion and then observe unbalanced forces from such non-accelerated perspectives or do we simply *declare* certain motions to be un accelerated and define forces as anything that accelerates relative to the declared perspectives?

In fact, what we resort to in this dilemma is the following:

(1) We *assume the possibility* of unaccelerated motion and the *equivalence*, for the purpose of describing the basic structure and laws of nature, of *any* reference frame that is not accelerated but is moving uniformly. We call such reference frames **inertial**.

(2) In practice we try to make better and better approximations to the identification of inertial reference frames. For many common purposes the surface of the Earth is good enough (Jean on the road and, therefore, me in my car) . But the Earth does rotate (which is accelerated motion, a ~ 0.15 ft/sec^2, equatorial) and so the center of the Earth with x,y and z axes fixed with respect to distant stars is much better. But the Earth does orbit the Sun (a ~ 0.027 ft/sec^2) and so the center of the Sun with similarly fixed axis directions is better yet. But the Sun does orbit the Galaxy (a ~) and so - - -.

Historically the assumption of the equivalence of all inertial frames was made without much fanfare from Newton's day through to the latter half of the nineteenth century, when it was seriously challenged by Maxwell's new theory of electromagnetic phenomena which predicted a fundamental *velocity* in nature. But Einstein, as we will see, regarded this challenge as a consequence of an erroneous interpretation of Maxwell's theory and he reasserted the assumption of *the equivalence of all inertial frames* in his 1905 STR paper. Ever since then the assumption has been called the **principle of relativity**.

As a consequence of the velocity composition relation, (3.8), and the principle of relativity we immediately deduce two results directly violated by Einstein's hypothesis concerning light.

(1) *Relative velocities of moving entities can not be invariant under a transformation between relatively moving inertial observers.*

(2) *There can be no limit of principle on the magnitude or direction of a relative velocity since, by repeated transformations between equivalent inertial observers any relative velocity can be reduced to zero or enlarged and rotated to any degree*.

In particular a pulse of light could not possibly have the same speed relative to all inertial frames! When we reflect on how our conclusions rested on the invariance of time intervals, asserted in (1.1), it is not surprising that Einstein's hypothesis concerning light must contradict that invariance.

But, as we shall see, when one contradicts the invariance of time intervals due to the invariance of vacuum light speed, it is only the beginning. A sizeable list of equally counterintuitive consequences immediately follow. Rather than be cowed by this, Einstein quietly showed that it was all quite internally consistent and that no existing evidence refuted the results. Within a short time corroborations started coming in and, by our time the essential correctness of STR is not only extensively corroborated but indispensable to the functioning of many aspects of our technology.

Even so, STR was not the last word. It was not compatible with Newton's theory of gravity and Einstein immediately began to try to build a relativistic theory of gravity. The effort would require ten years and several people came very close to beating Einstein to the finish line.

In 1916 Einstein published his relativistic theory of gravity, called the **General Theory of Relativity**, and in it the notion of inertial reference frames was finally given up, but a vestige of the spirit of the idea was retained within any sufficiently small region of space-time. More importantly, the invariance of the 'local' vacuum speed of light is also retained. Perhaps I will give a CALL course on GTR in eight years.

More troubling to Einstein was the emergence of a mature form of **Quantum Mechanics** (QM) in 1926 – 28, the basis for physical theories of the molecular, atomic, nuclear and sub-nuclear levels of the physical world. The abandonment of determinism (the idea that the complete physical condition of the universe at one time precisely determines [in principle] its complete physical condition at any other time), which had held throughout Newton's physics, Maxwell's physics and STR and GTR, was one of several aspects of QM that were never acceptable to Einstein.

Eventually people learned how to effectively unify STR and QM but a unified theory that incorporates GTR and QM still eludes us, perhaps, in part, because QM still retains inertial reference frames and the principle of relativity.

Appendix: Elements of the Algebra of Vectors (Arrows)

A: Addition and multiplication

To conveniently express the spatial relationships between the differing perspectives or reference frames it is useful to employ an Algebra of Arrows. We've already seen how arrows are useful in describing spatial intervals between events. Such intervals have magnitude and direction. So do arrows! Many important physical concepts involve magnitudes and directions; displacements, velocities, accelerations, momenta, forces, etc. So do arrows! Arrows representing different physical concepts are said to be arrows of different *kinds*. So arrows are useful. So we learn to manipulate arrows. Here are the rules of the game.

Let **A** and **B** be any two arrows of the same kind. The magnitudes (lengths) of **A** and **B** will be denoted by A or |**A**| and B or |**B**|, respectively. If we now double the magnitude of **A** while keeping its direction fixed, we call that arrow, 2**A**. If we reverse the direction of **B** and shrink its magnitude to 1/3 its former value, we call that arrow, −(1/3)**B**. In general:

(1): For any arrow, **C**, and any number, λ, with absolute value, $|\lambda|$, the magnitude of λ**C** is $|\lambda|$ times the magnitude of **C**, i.e., $|\lambda|$C or $|\lambda||$**C**$|$, and the direction of λ**C** is the same as **C** if λ is positive and is the reverse of **C** if λ is negative.

(2): For the purposes of the Algebra of Arrows it doesn't matter *where* an arrow is. All that counts is its magnitude and its direction. So two arrows, **A** and **B**, of the same magnitude and direction which differ only in where they are located are said to be equal, **A** = **B**.

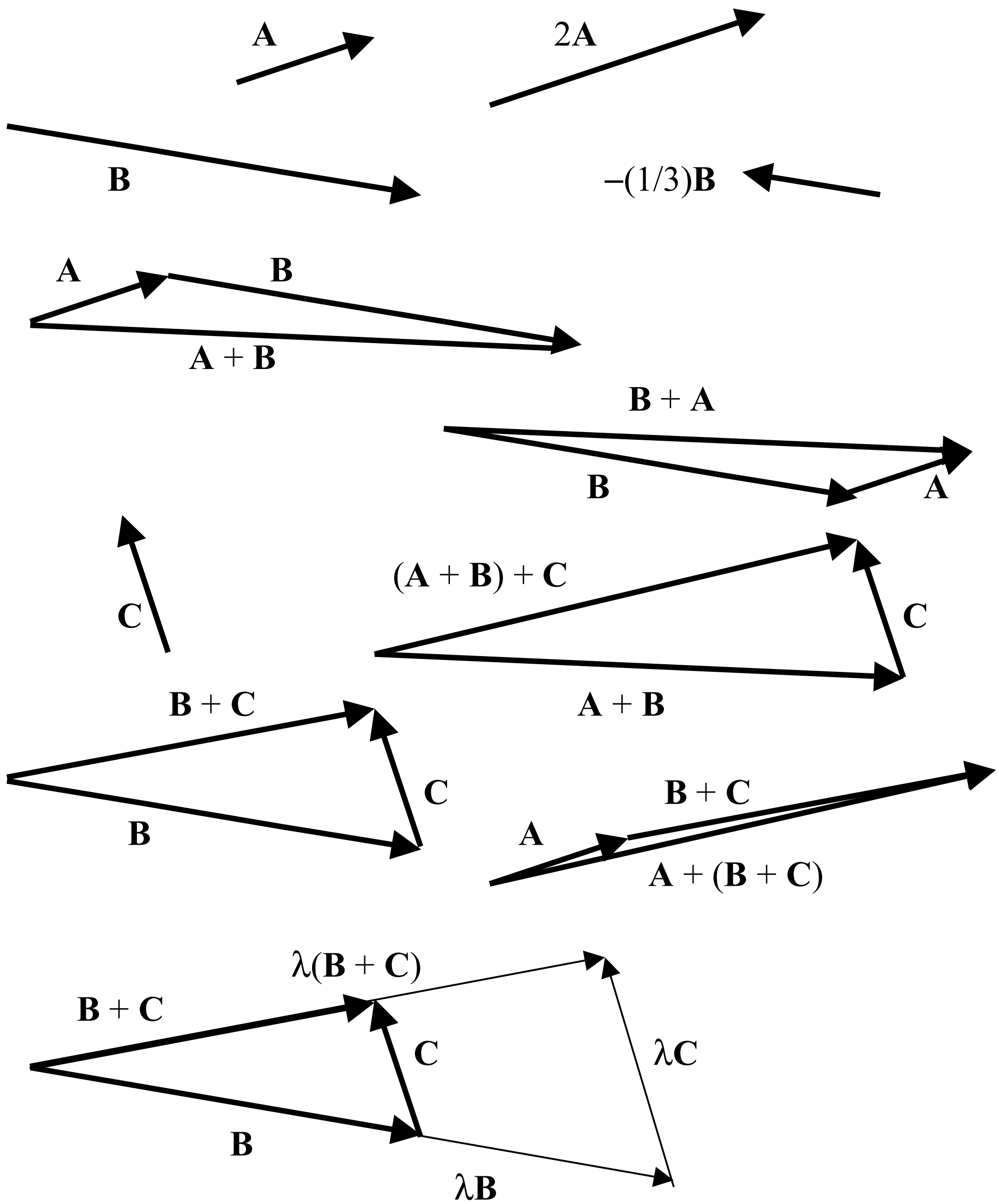

Fig. A.1: The Elements of the Algebra of Arrows

(3): Let **A** and **B** be any two arrows of the same kind. Move **B**, without changing its magnitude or direction, so that its tail touches the head of **A**. Then the arrow from the tail of **A** to the head of the moved **B** is, by definition, **A** + **B**. Also **A** + (–**B**) := **A** – **B**. It then follows that: (**Fig. A.1**)

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}, \qquad \mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}, \qquad \text{(A.1a,b)}$$

$$\lambda(\mathbf{A} + \mathbf{B}) = \lambda\mathbf{A} + \lambda\mathbf{B}, \qquad (\alpha + \beta)\mathbf{A} = \alpha\mathbf{A} + \beta\mathbf{A}, \qquad \text{(A.2a,b)}$$

$$(\alpha\beta)\mathbf{A} = \alpha(\beta\mathbf{A}). \qquad \text{(A.2c)}$$

B: Components of Arrows

Let **A** be an arrow and L be an infinitely long straight line with a definite direction. Consider the perpendicular projection of **A** onto L (**Fig. A.2**). Call the projection, $\mathbf{A}_L$. The **geometric component** of **A** in the direction of L is $\mathbf{A}_L$, the algebraic component of **A** in the direction of L, denoted by A_L, is a number equal to $|\mathbf{A}_L|$ (the magnitude of $\mathbf{A}_L$) if the direction of $\mathbf{A}_L$ is the same as the direction of L and equal to $-|\mathbf{A}_L|$ if the direction of $\mathbf{A}_L$ is the reverse of the direction of L. It then turns out that for any three arrows, **A**, **B**, and **C** satisfying the relationships, $\mathbf{A} = \lambda\mathbf{B}$ or $\mathbf{A} + \mathbf{B} = \mathbf{C}$, and any directed line L, the components satisfy,

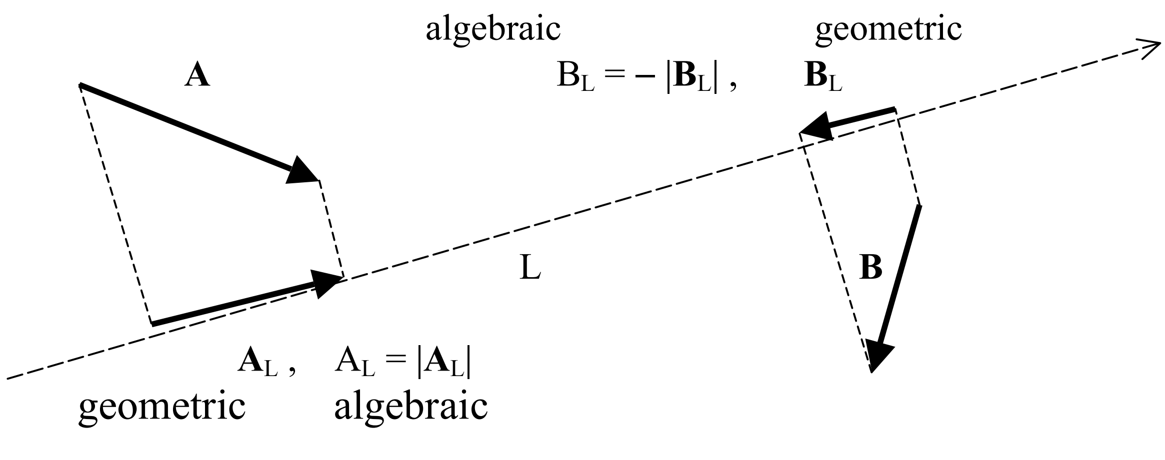

Fig. A.2: The geometric and algebraic components of arrows relative to a direction, L. The geometric component is, itself, an arrow. The algebraic component is a number.

$$\mathbf{A}_L = \lambda\, \mathbf{B}_L \qquad \text{and} \qquad A_L = \lambda B_L, \qquad\qquad (A.3a)$$

or

$$\mathbf{A}_L + \mathbf{B}_L = \mathbf{C}_L \qquad \text{and} \qquad A_L + B_L = C_L, \qquad\qquad (A.3b)$$

respectively.

C: Examples

(Eg. 1) The equations (2.1) and (2.2) on pp.5 and 6, respectively, are the component relationships corresponding to the vector relationships, (3.5), on p. 13. The component directions are, for (2.1), perpendicular to the relative velocity, $\mathbf{V}_{BC}$, between the observers and, for (2.2), parallel to that relative velocity.

(Eg. 2) We are all familiar with being passed on the highway, while doing 65 mph, by an eighteen wheeler doing 70 mph. The passing takes a little while, which can be unnerving if it takes place during a snowstorm in heavy traffic. The reason it takes much longer than it would for the eighteen wheeler to pass a car that was stopped on the berm is that, with respect to us doing 65, the eighteen wheeler is only moving at

$$5 \text{ mph} = 70 \text{ mph} - 65 \text{ mph} ,$$

or 7.33 ft/sec.

This is an example of the equation, $V_{AC,\,B} = V_{AB,\,B} + V_{BC,\,B}$, where A is the eighteen wheeler, C is the car and B is the road ahead with the velocity of the road, B, relative to the car, C, being $-$ 65 mph.

If the eighteen wheeler is 60 ft long it will take about a tense 11 sec for the rig to pass us.

(Eg. 3) Similarly, suppose we are tooling down a long straight stretch of back country two lane blacktop at 50 mph and, way down the road, we notice two cars heading our way – one in the opposing lane doing 50 mph like us and the other passing the first in our lane doing 70 mph. We tensely and alertly watch for the second car to complete the pass and move out of our lane for he’s closing on us at an alarming

$$120 \text{ mph} = 70 \text{ mph} + 50 \text{ mph} ,$$

or 176 ft/sec! Upon reaching us his passing is completed in less than 1/5 sec.

Again, this is an example of $V_{AC, B} = V_{AB, B} + V_{BC, B}$ with the same assignments as before except that now A is the oncoming passing vehicle. The values are $V_{BC, B} = -$ 50 mph, $V_{AB, B} = -$ 70 mph and $V_{AC, B} = -$ 120 mph.

These tense episodes have been assessed by either subtracting or adding the speeds of the participating vehicles and that is what simple deductions from our previous transformation equation, (3.8), or our component equation, (A.3b), will tell us to do.