# Class I: Introductory survey

## 1. Introduction

In the first half of 1905 the twenty five year old Albert Einstein, just two years from receiving his doctorate and employed to examine patent applications for the Swiss patent office in Berne, published four research papers on very different subjects in physics. Each of them was seminal in its domain and three of them were revolutionary for the subsequent history of the science. For this reason the year is known as Einstein's *Annus Mirabilis*.

Such miracle years had and have occurred for others both in science and the arts, but, in physics at least, no other such occurrence is comparable except for Isaac Newton's miracle year of 1666 when, escaping the plague in London at his family home in Woolsthorpe, he discovered the mathematics of the Calculus, the physics of Universal Gravitation and the spectral composition of light. These discoveries accelerated the already moving scientific revolution and, subsequently, led, in Newton's hand, to a systematization of all known physical science that would continue to grow, without fundamental modification, for over two hundred years.

The beginning of the 20$^{th}$ century would see, for the first time, substantial modifications of those foundations from two directions and the three revolutionary papers of Einstein's miracle year would make a basic contribution to one of those directions and, almost single handedly, create the other. It is that latter, almost single handed modification of the foundations of physical science that we will be discussing.

The topics covered by Einstein's papers were:

1. The interpretation of the jittery motion of plant pollen grains suspended in liquids ('Brownian motion' after the 19$^{th}$ cent. botanist, Brown, who first observed it) as being a consequence of collisions of individual molecules of the liquid with the pollen grains. Einstein showed how to use measurements of the motion of the pollen grains to determine properties of the molecules.

The very existence of atoms and molecules was controversial at the time because, while the hypotheses of their existence and properties were quite useful for understanding phenomena, particularly chemical phenomena, there was no previous *direct* physical evidence for their existence. Einstein's paper on Brownian motion was very instrumental in swaying the scientific community to accept the reality of atoms and molecules. The paper also introduced methods of analysis which influenced the subsequent development of Statistical Theory.

2. The interpretation of the measurements concerning the emission of electrons from metallic surfaces exposed to ultra-violet light as indicating a particle-like structure to the incident light. This directly contradicted the universally accepted wave theory of light which had been first established on both theoretical and experimental grounds in the late 18$^{th}$ century and firmly corroborated in the late 19$^{th}$ century with the recognition that light was an electromagnetic wave. At first ignored, this paper would play an important role in the development of the Quantum Theory and give rise to our recognition of the **photon** as the particle-like aspect of light. Einstein subsequently won the Nobel prize for this paper.

3. The demonstration that one could resolve the problems surrounding the long-term search for the so-called Luminiferous Aether, or just ether (the presumed medium that supported the propagation of electromagnetic waves, "What was waving?") by replacing the whole concept with a reanalysis of what one means by distances and time intervals. The basis of the analysis was the acceptance of two principles. First, the conservative retention, in the face of then recent arguments to the contrary, of the equivalence of the perspectives, for describing the laws of nature, offered by any one of a set of so-called **inertial reference frames**. This principle, which we will discuss in detail later, was called the **principle of relativity**. Second, the seemingly paradoxical but experimentally corroborated **invariance of the vacuum speed of light**. The theory resulting from these principles came to be called the **Special Theory of Relativity** (STR), our topic in this course.

4. The derivation from the preceding STR that mass and energy were not measures of two qualitatively distinct properties (as seemed axiomatically clear from the very introduction of the concept of energy in the 18$^{th}$ century) but were, instead, convertible into one another and, basically, different measures of the same thing. This was

an essential recognition leading to the later discovery of nuclear energy and our understanding of the source of energy in stars. It is represented by the most famous equation of physical science, $E = m c^2$. We will discuss this later in the course.

After this tremendous outburst of creativity Einstein would struggle for ten more years to extend his Special Theory to accommodate gravitational phenomena, while at the same time playing an increasing role as the leading figure among physicists as his pervasive reworking of the foundations of the science gained global recognition. In 1916 he finally published what has come to be known as the **General Theory of Relativity**, and remains to this day a superb theory of the structure and nature of space and time in the presence of gravitating energy-matter on all but the smallest, sub-nuclear scales. This theory has revolutionized Astronomy and Cosmology. We will not discuss the General Theory in this course except in passing.

Meanwhile, Einstein was becoming more and more dissatisfied with the development of Quantum Theory, which he had done much, even beyond his photon idea, to bring about. One problem was Quantum Theory's apparent treatment of some manifestations of probability in nature as being fundamental and not just a consequence of human ignorance of details, i.e., the abandonment of determinism. This led to his famous quote that "I can not believe in a God that plays dice!". An even deeper problem, which Einstein recognized before anyone else, was that Quantum Theory abandoned the long standing notion that the properties of a composite system are determined by the properties of the constituents of the system. In fact Quantum Theory could prohibit definite properties of the one in the presence of definite properties of the other. Interpreting this as indicating that Quantum Theory was basically incomplete, Einstein fought a valiant and extremely instructive battle with most of his colleagues, but to no avail. Refusing to concede, Einstein gradually became more and more isolated from the mainstream of physics research to the point that long before his death he was almost completely ignored.

Many different views are held about the estrangement of the later Einstein from the mainstream of the science he had advanced so much. My view is that, like most of us, he held deep convictions that were difficult to divest himself of. Those convictions were quite compatible with the innovations he introduced and probably even contributed to his seemingly unerring judgement up to about 1916. After that his preconceptions became obstacles to following and contributing to the problems that were ripe for solution. For whatever reasons he could not or would not be swayed.

In the history of science it is not unusual for a major contributor to lose the capacity for seminal innovation with increasing age. It is unusual, however, for such a contributor to move into isolation from and alienation to the primary developments in the field.

**2. Overview of the Special Theory of Relativity (STR)**

Just what were the so-called revolutionary assertions of the Special Theory of Relativity (STR)?

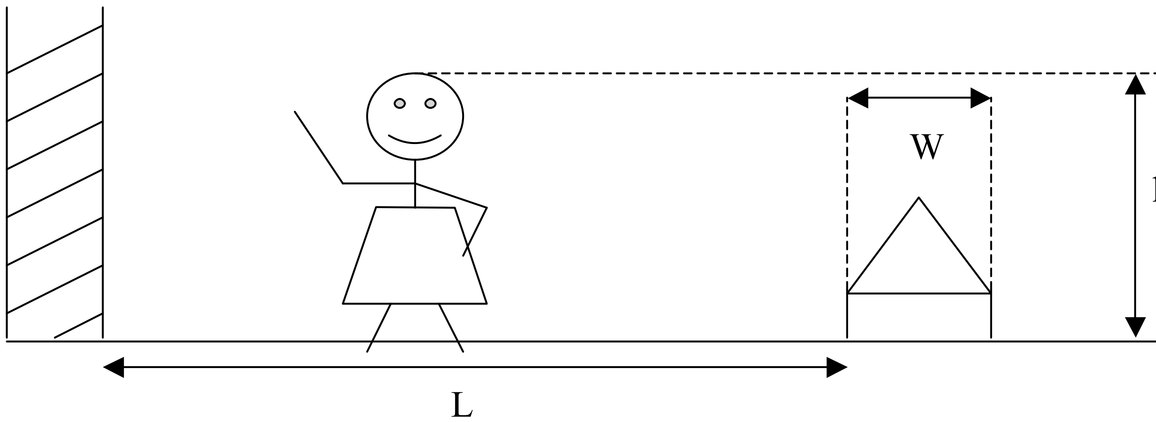
The first was one of the *assumptions* of the theory, that the vacuum speed of light was invariant. Invariant of what? Invariant of the motion of the source of the light or the observer of the light! In other words, if you measured the vacuum speed of a light beam, or a light pulse, with respect to yourself, you would get the same value regardless of whether you were moving towards the light source or away from it or whether the light source were moving towards you or away from you, and no matter at what speed. Now , if you understand what this means (and we'll spell it out later), this sounds just crazy – and it's one of the assumptions! But Einstein was convinced that then recent theory and experimental data forced it on us.

OK, but why call it a theory of *relativity* when it begins by assuming that something we normally regard as relative (the vacuum speed of light with respect to the measurer), isn't? Because the consequences that follow from this *invariance* assumption and the *principle of relativity* that we mentioned above force a number of things that we normally think of as absolute or invariant to be, in fact, relative, i.e., dependent on states of motion and reference frame perspective.

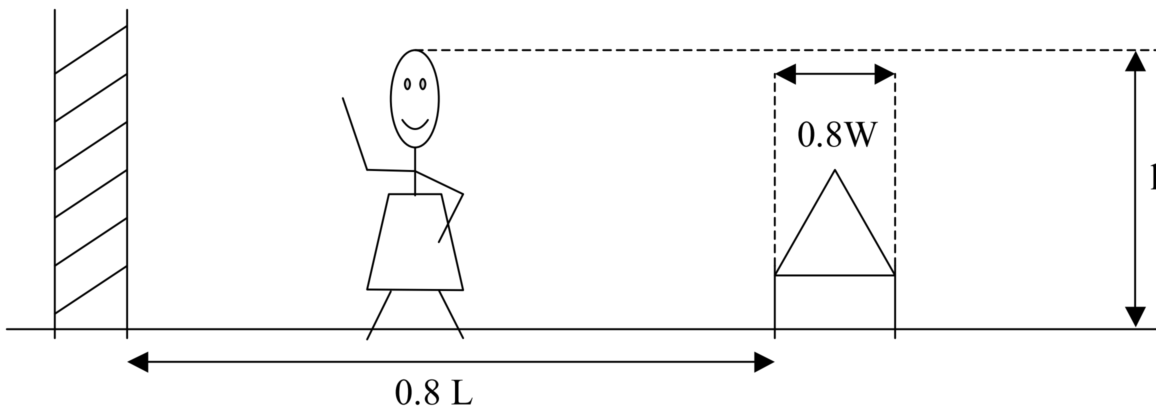
For instance, the amount of time that passes between two consecutive meetings of two observers! In our normal conception of things, time is time, and if my wife, Jean, and I get together and then go our separate ways for awhile before our next meeting, the amount of time between our meetings from my perspective is the same as from Jean's perspective. I'm not talking here about our emotional reaction to the time that has passed between meetings, whether Jean missed me more than I missed Jean. I'm talking about the so-called objective passage of time – how many hours, minutes and seconds between our meetings. According to Einstein that can be different for Jean and I and will depend on how we each were moving between the meetings. In most cases the difference will be too small to measure, let alone notice, but in extreme cases the difference can be enormous! So, according to STR, we have the **relativity of time lapse**.

Connected with this is the **relativity of simultaneity**. According to STR if two events, *separated by some distance*, are judged by some observer, say me, to be simultaneous, then another observer, say Jean, in motion with respect to me, may judge them to not be simultaneous. Again, this doesn't refer to common occurrences such as if I hear the thunder claps from two equidistant lightning strikes at the same time and Jean is moving towards the site of one strike and away from the other she may hear one thunder clap before the other. That sort of difference in sensory experience could be corrected for, or so we always presumed, so that Jean and I would agree on whether the thunder claps really were simultaneous or not. What STR says is that even after making all the necessary corrections for Jean's position and motion with respect to me she will *judge* that the events were not simultaneous for her – and, furthermore, *I will agree that they're not simultaneous for her even though they are simultaneous for me!* Again, this is an extremely small effect in ordinary circumstances but can be huge in extreme cases. So, according to STR, **the simultaneity of spatially separated events is relative**.

Similarly, we have the **relativity of spatial dimensions**. Suppose I'm driving a 20 ft long vehicle down the road and Jean is standing by the side of the road with the best high speed measuring equipment money can buy. As I zip by, according to STR, Jean's measurement of the length of my vehicle will be a shade less than 20 ft. For ordinary speeds, of course, money might NOT be able to buy sufficiently precise equipment to detect the shrinkage. But if I were doing 6/10 of the speed of light (!), Jean (now having to be very quick on the trigger) would measure the length of my vehicle to be only 16 ft.! At 8/10 light speed she'd get 12 ft. I wouldn't notice the cramped quarters because, by Jean's measurements, I would have shrunk in the direction of motion by the same amount! Furthermore, if I measure Jean as we pass, I would find her to be skinnier and the telegraph poles along the road to be closer than normal by the same amount (**Fig. 2.1**). So, according to STR, **the linear dimensions of moving objects are relative**. As we will see later, this relativity of spatial dimensions is closely related to the relativity of simultaneity, in the sense that the latter requires the former.




Passing Jean, a pole and a trash can at ordinary speeds.




Passing Jean, a pole and a trash can at 0.6 the speed of light (rough approx.).

**Fig. 2.1:** Relativistic dimensions of moving objects; this is not how things would 'look' to me, this is how they would be!

Having introduced some quantitative aspects in this example of the relativity of spatial dimensions we can also provide an example for the relativity of time lapse which, to many, seems virtually inconceivable.

Suppose Jean and I each have two incredibly precise clocks that can be programmed to digitally read out times for the occurrence of events in their immediate vicinity to within less than a nanosecond (ns), i.e., less than a billionth of a second. Jean synchronizes her two clocks, $J_1$ and $J_2$, and places them on the side of the road, separated by about 11'10.5'', so that I, speeding down the road at 0.6 light speed, will encounter $J_1$ first. Similarly, I synchronize my two clocks, $G_1$ and $G_2$, and place $G_1$ in the front and $G_2$ in the back of my vehicle separated by about 11'10.5''. We also arrange the clocks so that just as my $G_1$ and Jean's $J_1$ are sweeping by each other, essentially momentarily coincident, they both read out the time 0.0ns.

Now as I come down the road, what assessments does Jean make concerning my clocks? (**Fig. 2.2**) Well, first of all, because of the relativity of simultaneity, she does not judge my two clocks to read out 0.0ns simultaneously, as I do. Instead, she assesses my trailing clock, $G_2$, to always read 7.2ns ahead of my lead clock, $G_1$, and to read 0.0ns when $G_1$ is still about 5'4'' away from $J_1$. Because of length contraction Jean judges $G_1$ to be only 9'6.4'' ahead of $G_2$, not 11'10.5''. When $G_1$ and $J_1$ meet, we both agree they both read 0.0ns and next Jean observes $G_2$, reading 20ns, to sweep by $J_1$, reading 16ns, quickly followed by $G_1$, reading 16ns, sweeping by $J_2$, reading 20ns. So Jean observes her synchronized clocks to tick off 20ns, during which interval, according to Jean, my clocks tick off only 16ns. The sequence of clock coincidences Jean observes is first, $G_1$-$J_1$, then $G_2$-$J_1$, then $G_1$-$J_2$, and last, $G_2$-$J_2$.

From my reference frame the sequence of events are assessed, judged, and observed differently. I observe *Jean's clocks* to be out of synch with her $J_2$ leading her $J_1$ by 7.2ns and reading 0.0ns 9.0ns before $G_1$ and $J_1$ meet. I observe her clocks to be separated by only 9'6.4'' and to be ticking only 16ns to every 20ns on my clocks. Finally I observe the sequence of clock coincidences to be $G_1$-$J_1$, $G_1$-$J_2$, $G_2$-$J_1$, $G_2$-$J_2$, with the second and third coincidences reversed from Jean's observations.

Note that we have no difference concerning the readings on any two clocks at their moment of spatial coincidence.

Next we have the rule for **relativistic velocity composition**. As we will discuss in more detail later, if you're driving down the highway at 60 mph and an eighteen wheeler is passing you at 70 mph, his speed relative to you is 10 mph. An oncoming car in the opposing lanes doing 60 mph has a speed of 120 mph relative to you. STR does not change these results by any noticeable amount. But if you and the trucker and the oncoming car are doing 6/10 and 7/10 and 6/10 the speed of light, respectively (!), then STR says the speeds of the truck and car relative to you are not 1/10 and 12/10 the speed of light, respectively, but rather, 10/58 and 15/17 the speed of light respectively. An increased relative speed for the truck and an important

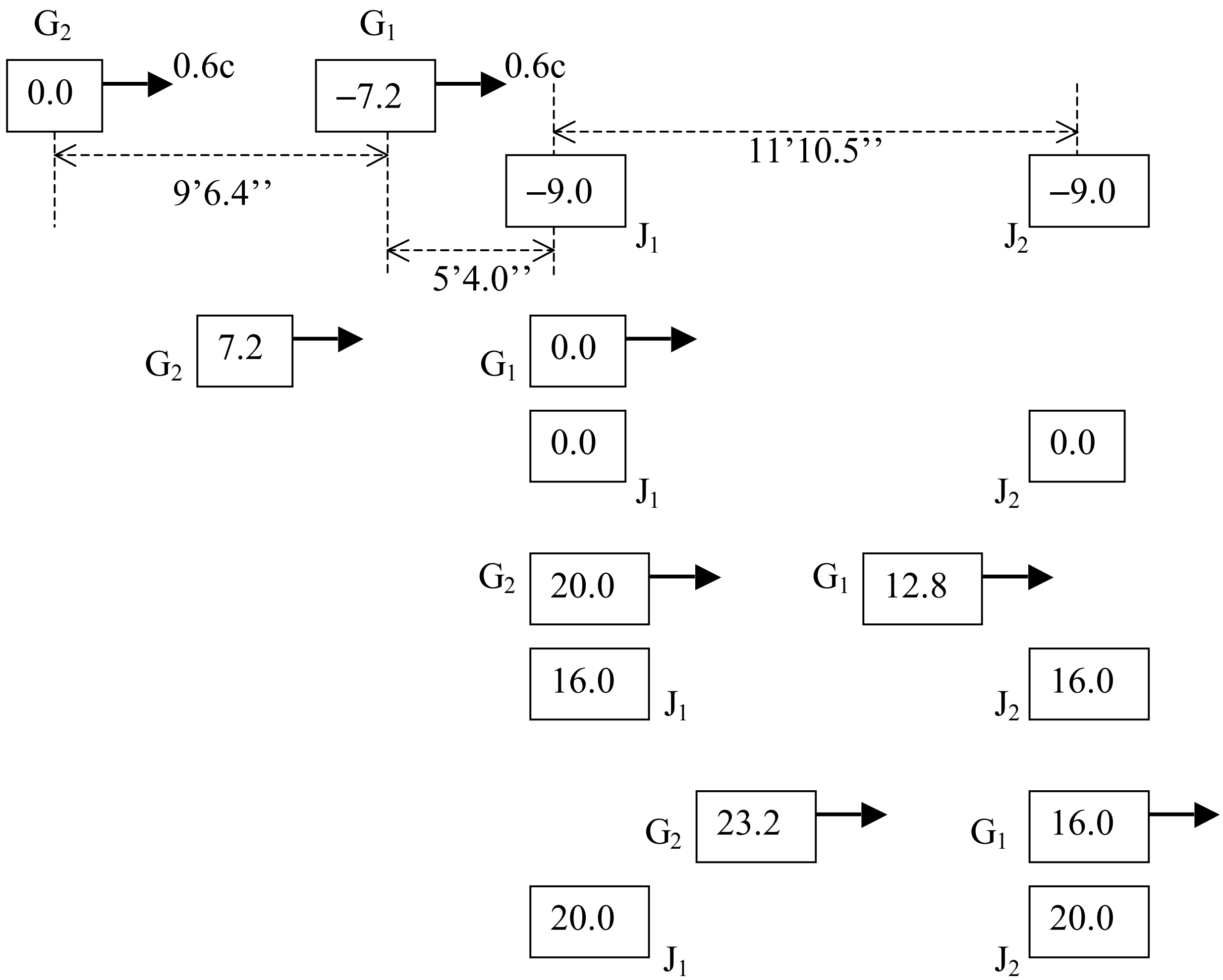



**Fig. 2.2:** Four stages in Jean's perspective on my clocks moving passed her clocks. Jean observes my clocks to be out of synch by 7.2ns and to be closer together than hers by 9'6.4'' vs 11'10.5''. Because of that difference she observes the $G_2$-$J_1$ coincidence to occur before the $G_1$-$J_2$ coincidence. She also observes each of my out of synch clocks to tick 16ns to 20ns on her clocks. My perspective reverses these assignments.

decrease for the oncoming car. In fact the *velocity composition rules* of STR will never allow a speed greater than light to result from combining speeds less than light. This is how the theory achieves consistency with the *basic assumption* of the invariance of the vacuum speed of light.

Finally, it takes force to accelerate objects and the more massive they are, the more force required for a given acceleration. Old Isaac Newton first figured that out. If the force keeps acting and the object picks up more and more speed the force puts energy into the object called **kinetic energy**, which means energy due to motion. Prior to STR, theory said and experiment seemed to confirm that as the object sped up a constant force could maintain a constant acceleration because the mass didn't change. The mass was understood to be an intrinsic property of the object. But, according to STR, as the speed picks up it will require more and more force to maintain a constant acceleration. The mass will increase with speed and this will happen in just such a way as to make *the change in mass be proportional to the kinetic energy acquired*. So the mass of an object is dependent on its' motion and, upon generalizing the analysis from kinetic energy to all kinds of energy, STR tells us that all forms of energy carry a proportional amount of mass. As in the previous paragraphs, these effects are negligible at ordinary speeds, but if - - - - and thus we have **the relativity of mass** and **the equivalence of mass and energy**.

But it's one thing to just *assert* these conclusions of STR and quite another thing to *understand* how Einstein was led to them and why we should accept his conclusions. To prepare ourselves to get a better handle on STR we need to review some of what came before.