The Physical Forces of Everyday Life: IV.

1. Pressure

Pressure is the distribution of force over an area of a surface. We encounter pressure most frequently as water pressure and air pressure. The former governs the rate of flow from our faucets, shower heads and garden hoses, enables boats to float, influences the rate of percolation of water deep underground through bedrock, limits the depth at which unprotected humans can safely 'swim' and submarines can safely patrol and is, itself, (mostly) determined by the weight of water pressing down on the water below it. Air pressure, as atmospheric pressure, isn't noticed by us unless it changes quickly in weather systems, whereupon it can induce sensations, mostly unpleasant, as our bodies adjust more slowly. But it brings us the wind and sound, enables planes to fly, lifts hot air balloons aloft, contributes to the formation of hurricanes and tornados and is, itself (partly) determined by the weight of air pressing down on the air below it.

But we also encounter pressure as blood pressure, steam pressure in a heating system or boiler, gas pressure in a gas stove or a propane fuel tank, oil pressure maintaining effective lubrication in the engines of our vehicles and myriad other instances.

Carefully speaking, it is not just pressure, per se, that is responsible for most of the listed phenomena. Rather *differences* in pressure, from place to place and/or *changes* in pressure from time to time are responsible. Water rushes out of our faucets because the pressure inside the water pipes is *higher than* at the opening. If pressure didn't *vary with depth* in water or height in the atmosphere there would be no buoyant effect on totally immersed objects. Weather balloons get larger and larger as they rise because the external air pressure is *dropping* as they rise. But they rise at all because at any moment the air pressure at the top of the balloon is *less* than at the bottom. We hear sound as a consequence of *rapid fluctuations* in external air pressure next to our ear drums. But those fluctuations in external pressure result in vibrations of our ear drums because of *differences* between the external pressure and the pressure on the internal side of our ear drums. We feel light headed if we rise *too quickly* from a bent over or sitting position because the hydrostatic pressure *differences* of our blood from head to toe has not *yet* been compensated by our heart and vascular system.

All the instances of pressure we have been considering fall into two very different groups. Pressure in effectively *incompressible* liquids and pressure in very *compressible* gases. Pressure in water and air are excellent examples of these groups. The compressibility of water (under 'standard'* conditions) is 308,000 lbs/in^2. This means that if you subject a mass of water to an *additional* external pressure of 3080 lbs/in^2, the volume of the water will decrease by about 1.0%. Consequently ignoring the compressibility of water is a very good approximation in a wide range of circumstances. By contrast, a given mass of air at any fixed temperature in a 'wide' range very nearly satisfies Boyle's Law that the product of its' pressure and volume is a constant. Double the pressure or the volume and you halve the volume or the pressure respectively. Change one quantity by the factor, A, and the other quantity changes by the factor, 1/A.

Effective incompressibility (as in water), by eliminating density as a variable quantity (you can hardly squeeze more water into a given volume by 'pressing' on it), greatly simplifies the details of pressure phenomena for a substance. The dependence of water pressure on depth is much simpler than the dependence of air pressure on altitude because with air fixed masses of the stuff is squeezed into smaller volumes at sea level than at 10,000 feet up, say.

Accordingly, we begin our detailed consideration of pressure phenomena with water pressure.

2. Water Pressure

One cubic foot of fresh water weighs a tad more than 62 lbs. For sea water it's 64 lbs. As we will see, that's why we float more easily or higher in salty sea water than in fresh water. Now imagine a quiet fresh water lake with a maximum depth of 200 ft, say. Every cubic foot of the water in the lake weighs 62 lbs. The water that comprises the top 1 ft. thick layer pushes down upon and is held up, supported, by the water underneath (**Fig. IV. 1**). So (if we ignore for the moment the atmosphere which is sitting on top of the water) on every square foot of the bottom of that layer the water underneath must push up with a force of 62 lbs. But in a quiet lake with the water not moving, the water is effectively the same everywhere and that 62 lbs must be distributed evenly over the square foot of area. In particular, on each of the 144 square inches that make up the square foot, the force from the water underneath must be 62/144 lbs = 0.43 lbs. We express all this by saying that at every point in the lake at a depth of one foot the pressure is 62 lbs/ft.^{2} = 0.43 lbs/in^2. Notice that pressure is not a force. It's a **force per unit area**.

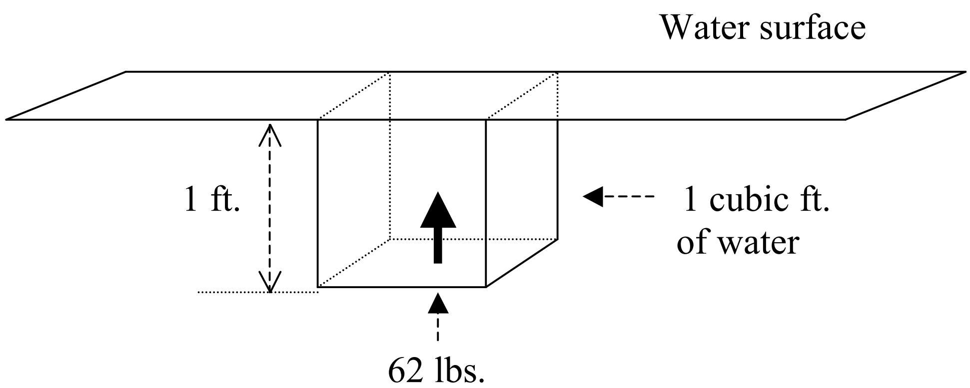


Fig. IV. 1: Water pressure supporting (balancing) water weight

If we had gone through the same argument at a depth of 1/2 ft. every horizontal square foot of area at that depth would only have to support 31 lbs of water. The pressure would turn out to be about 0.22 lbs/in^2. At a depth of 2 ft., 124 lbs of water would need supporting, requiring a pressure of 0.86 lbs/in^2. The drift is clear! The pressure increases proportionally to the depth and for fresh water the equation reads

$$P = [(0.43 \text{ lbs/in}^2)/\text{ft}]\ D$$

Where P is the pressure in lbs/in^2 and D is the depth in ft. At the bottom of our hypothetical 200 ft deep lake the pressure would be 86 lbs/in^2.

But now it's time to stop ignoring the atmosphere sitting on the lake! And it really is *sitting* on the lake. All that air weighs something and the water surface is keeping the air from going lower. Just like still water, on a quiet balmy day the weight of the air is uniformly distributed over the lake surface and is expressed by what we call atmospheric pressure. And remember what it is? A whopping 14.7 lbs/in^2!

So the water at a depth of 1ft. doesn't just support the water above it. It supports the atmosphere as well, and similarly all the way down to the bottom of the lake. So, in fact, the pressure equation should read more like (**Fig. IV. 2**),

$$P = [(0.43\ \text{lbs/in}^2)/\text{ft}]\ D + 14.7\ \text{lbs/in}^2.$$

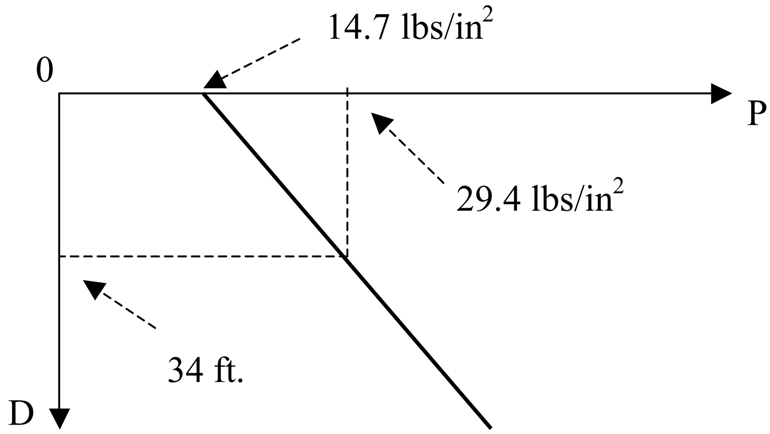


Fig. IV. 2: Graph of linear (straight line) variation of pressure with depth in water according to the equation, $P = [(0.43\ \text{lbs/in}^2)/\text{ft}]\ D + 14.7\ \text{lbs/in}^2$.

For the oceans (which, however, are rarely calm) the corresponding equation is

$$P = [(0.44\ \text{lbs/in}^2)/\text{ft}]\ D + 14.7\text{lbs/in}^2.$$

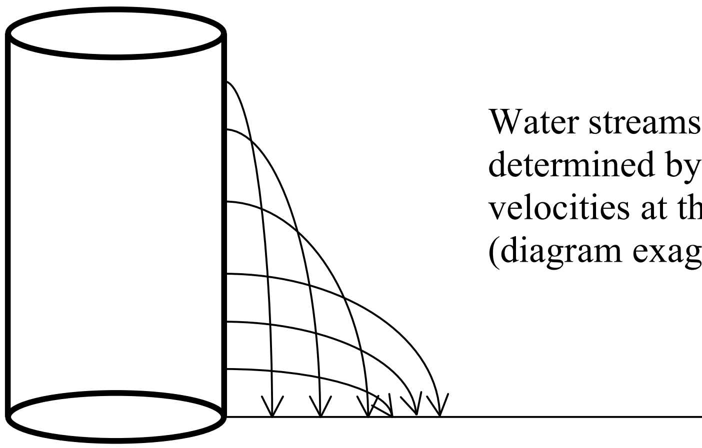
We, of course, are used to the atmospheric 14.7 and we don't feel it. Consequently when we go *down* into the water we only feel the *change* in pressure, the part of the equation depending on D. We can double the 14.7 if we dive to about a 34 ft. depth (**Fig. IV. 2**). So if we felt the atmospheric 14.7 it could feel (pressure wise) like being under 34 ft of water!

The 'average' depth of the worlds oceans is about 2.4 miles and the greatest depth is reached in the Challenger Deep of the Marianas Trench at 6.8 mi., or 35,840 ft. deep! Our pressure equation would indicate an incredible pressure of 15,785 lbs/in^2! At this pressure even the water itself suffers about a 5% volume compression which needs to be considered for a more accurate pressure estimate. Amazingly, living organisms exist there!

Let's go back to the surface.

3. Toricelli's Law

There's an easy way to actually 'see' the variation with depth of the pressure in standing water. Fill a tall container with water and then punch *small* holes in the side of the container at different heights from the bottom. The water will flow out of all these holes at the same time and the speed of the water flowing from each hole will increase with the depth of the hole relative to the height of the water surface (**Fig. IV. 3**).



Water streams reach distances determined by emission velocities at the holes (diagram exaggerated).

Fig. IV. 3: Water streams from small holes in full container at different depths below water surface.

If we ignore the viscosity of the liquid (which is a good approximation for water) we can calculate the speed, v, with which the water comes out of a hole at a depth, D, below the water surface. The equation, called **Toricelli's Law** after the first person to study it, is

$$v^2 = 2\,D\,g\ ,$$

where g represents the acceleration that unopposed gravity produces in all falling objects near the Earth's surface. The appearance of g in the equation reminds us that the water pressure is due to the *weight* of the water. And the weight of the water is due to gravity. A good approximate value of g is (32 ft/sec)/sec.

Notice, according to **Fig. IV. 3**, that even though the emission velocity of the stream increases all the way to the bottom of the container, the horizontal distance the stream can reach is maximal half way down because the lower streams have less time to fall.

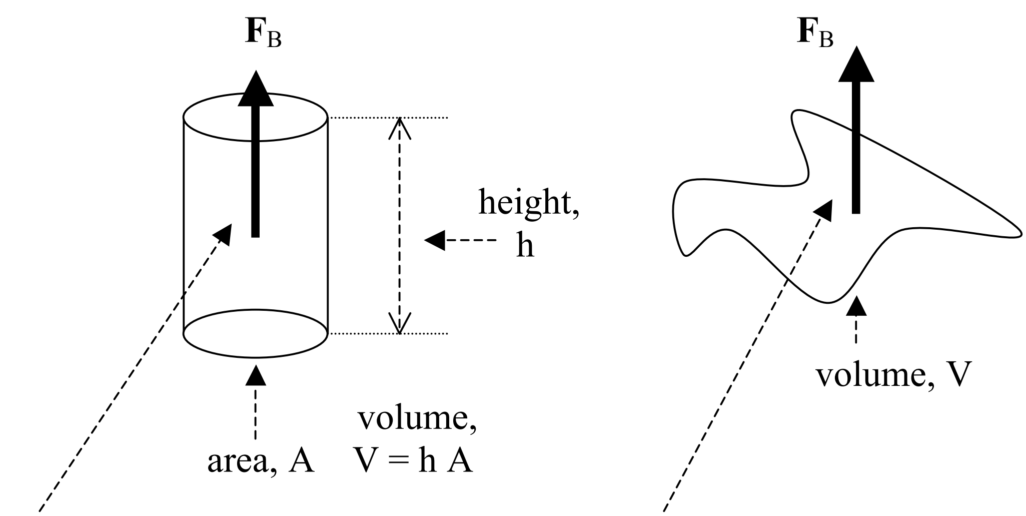
So far all of our discussion has focused on the role of water pressure at any depth in holding *up* the water above that depth. But as the water streaming out of the holes in the container makes clear, the pressure acts sideways as well. Why is that? The answer is simply that we're dealing with fluids and the main feature of fluids is that in response to forces they will flow if possible. If water was just subject to pressure pushing *up* from below and *down* from above and gravity pulling down from within, it would just be squeezed out sideways. But if the sides are obstructed by already present water, then that 'escape' for the squeezed water is prevented by the force of pressure distributed over any vertical surface separating the water on either side of that surface. So, in fluids, *pressure, at any point, acts equally in all directions*. It is the *variation* in pressure from point to point that gives rise to *directed* force due to pressure in a fluid. As we have been discussing, the vertical variation in pressure yields upward forces that balance the weight of the fluid. If the pressure varies horizontally as well, it will produce net horizontal forces that will cause the fluid to accelerate horizontally. We will see examples of that later on.

4. Archimedes principle of buoyancy

We know that air bubbles rise in water. If water pressure didn't increase with depth, they wouldn't. They'd fall instead! The reason being, that the water pressure would push down from above as much as it pushed up from below and the resultant force due to pressure would be zero. Then the weight of the air bubble – and even air bubbles have weight – would make the bubble fall. In fact the water pressure pushes up from below more than down from above. The difference between the push up from below and the push down from above is what we call the force of **buoyancy**. It's a lot more than the weight of the bubble and so the bubble rises – with acceleration even!

A second effect on the bubble due to the pressure change with depth is that as it rises it gets bigger. The squeezing effect of the water pressure is dropping and the air in the bubble expands its' volume in accordance with Boyle's Law (provided the temperature isn't changing). As it expands the discrepancy between the buoyant force and the fixed weight of the air bubble increases, further hastening the rise to the surface. Watch rising bubbles in water closely next time!

Archimedes was the discoverer of the principle of buoyancy , and discovered it in the absence of our understanding of the *general* concept of force. In his day, of course, it was well known that water buoyed objects up. The question was – by how much?! Recognizing that when nothing else is in the water *the water is still buoying itself up by just the right amount*, he concluded that when something is put into the water the buoyant force will be equal to the weight of water that was displaced by putting the object in the water. So if an object weighs less than the water displaced from the occupied volume, it will float. If it weighs more than the water displaced from the occupied volume, it will sink. This simple principle follows from the pressure dependence on depth that we found. It can only be easily seen to do so for simple volume geometries such as vertical cylinders (**Fig. IV. 4**).



$F_B = \Delta P\ A = [(0.43\ \text{lbs/ in}^2)/\ \text{ft}]\ h\ A$, $F_B = [(0.43\ \text{lbs/ in}^2)/\ \text{ft}]\ V$

Fig. IV. 4: Water buoyant force on simple and irregular volumes

5. Hydrostatic 'paradoxes' and the hydraulic lift

We now move our discussion of water pressure to a look at some counterintuitive water pressure comparisons. These are sometimes called **hydrostatic paradoxes**.

To see the sense of the paradoxes consider the next picture, (**Fig. IV. 5**), with each vessel, A, B and C, filled with water to the same height. Then at any given height from the bottom the pressure will be the same in each vessel. Now why is the pressure at the bottom of vessel, B, the same as that in A since there is much less water weight to support in B? How does that same pressure at the bottom of C, spread over the small area, produce enough force to support the weight of all the water above it? Only the pressure at the bottom of A seems 'natural'.

The answer, in the case of B, is that water pressure acts roughly 'upwards' *on the flared surface* near the bottom. Accordingly, that flared surface exerts an equal and opposite force 'downward' *on the water* beneath it and the transmission of this force to the bottom produces the common water pressure across the whole bottom. In the case of C the water pressure at the bottom *isn't* supporting *all* the water weight above it. The water weight above the flared surface is supported by the flared surface.

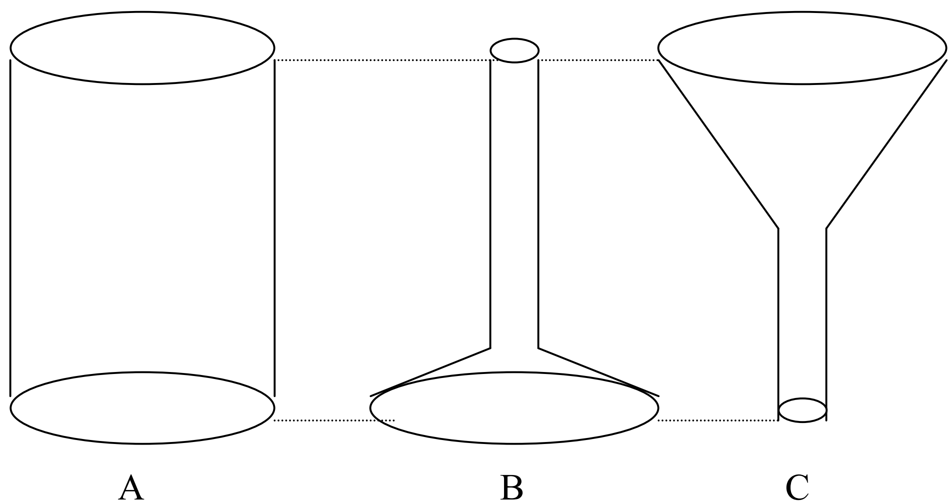


Fig. IV. 5: Hydrostatic pressure 'paradoxes'.

These considerations lead us to our last water pressure example, the hydraulic lift (**Fig. IV. 6**). Here the lquid involved is usually oil, not water, but that's not essential to the operation of the device. Furthermore, the liquid is under considerable additional pressure than weight alone would generate. So the variation of pressure with height is not important. What counts here is that, since the liquid is incompressible and confined, pressure exerted anywhere on it is transmitted undiminished throughout the liquid, just like the pressure exerted on the water by the flared surface in vessel B is transmitted to the bottom. This is an instance of **Pascal's Principle**. So if the pressure at the small area piston is P, the pressure across the large area piston will also be P. But with a sufficiently small area, *a*, of the small piston, the total force exerted is only $f = P\,a$, a force that a human can exert, perhaps with the aid of a pump lever. On the other hand, if the area, A, of the large piston is big enough, the force exerted there is $F = P\,A$, and may be large enough to lift a truck. So a human can lift a truck!

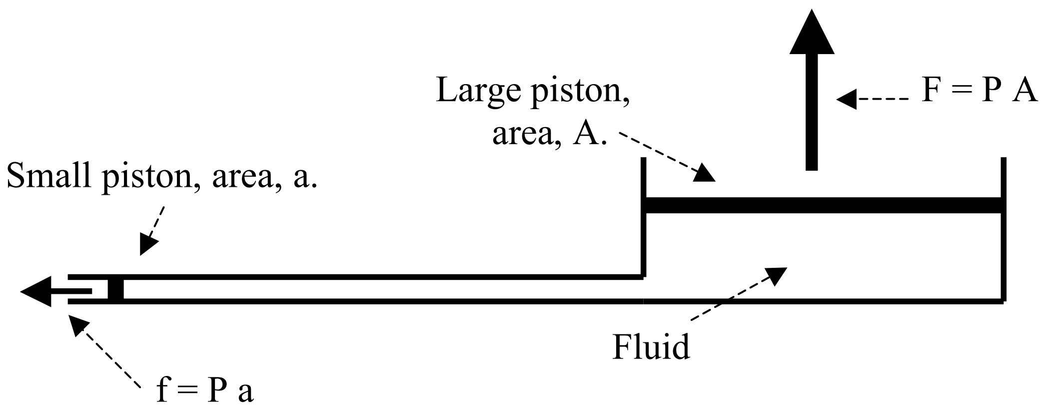


Fig. IV. 6: The hydraulic lift

So what's the catch? There has to be a catch! And there is. Yes, a human can lift a truck. But to lift a truck through a *modest* height of h, the human must push the little piston through a *long* distance of D, where $a\,D = A\,h$. To lift a 6000 lb truck by exerting a 50 lb force we need $A/a = 6000/50 = 120$. So to lift the truck by 5 ft the small piston has to be moved 120 x 5 ft = 600 ft.! That's the catch. You don't really have to push the piston across two football fields. There are clever ways to have the small piston move through the same 1 ft interval 600 times instead. Still, it's a catch.

6. Air Pressure

Air is compressible. If the temperature stays constant a fixed mass, *m*, of it satisfies **Boyle's Law**, $P\,V = (\text{const.})\,m$, to a good approximation. Therefore, on a quiet day (not even a breeze!), with no temperature variation with altitude, the weight of air within a thin horizontal layer of given thickness and area (and therefore given volume) is determined by the pressure. But the pressure must drop as we rise so that the weight of the air in the layer can be supported by the pressure difference from the bottom to the top of the layer over the area of the layer. These two considerations lead to the result that the *ratio* of the pressure, P(h), at a height, h, from the ground, to the ground level pressure, P_0 , satisfies, for *any* integer, N,

$$P(h) / P_0 = [\, P(h/N) / P_0 \,]^N \,.$$

In words, the *ratio* for height, h, is the Nth *power* of the *ratio* for the height, h/N (**Fig. IV. 7**). For example, the ratio of pressure at 10,000 ft to ground pressure would be the square of the ratio of pressure at 5,000 ft to ground pressure

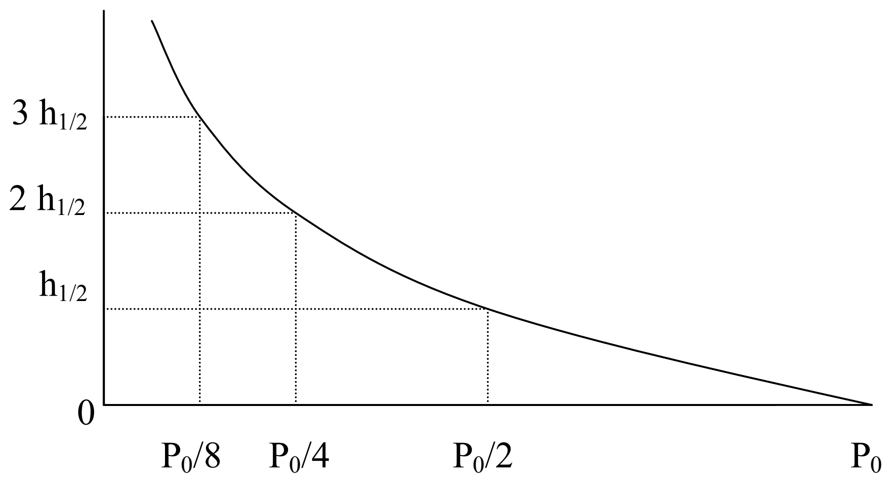


Fig. IV. 7: Exponential drop in air pressure with height

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The mathematicians call this kind of dependence **exponential** . The pressure of the air drops off exponentially with height. For comparison with the simpler water pressure variation with depth, we note that the latter can be expressed by,

$$P(D) - P_0 = N[\, P(D/N) - P_0 \,] \,,$$

where *ratio* gets replaced with *difference* and *power* gets replaced with *multiple*.

Unfortunately, our whole analysis depended on the assumption of fixed temperature – and the actual temperature ain't fixed! It first tends to drop as we rise, unless we encounter a temperature inversion. But then, sufficiently high up, there is a 'permanent' temperature inversion, etc. And we've said nothing about winds and storms. So our analysis was very simplistic. But at least it gives us a rough feel for the way in which air pressure drops with altitude.

And that way, roughly exponential, has dramatically different consequences from the case with water. Consider buoyancy. Archimedes principle works here as well. The buoyant force in air on an object equals the weight of the air displaced by the object. But at higher altitudes an object of fixed volume displaces less and less weight of air. So the buoyant force decreases with altitude and a hot air balloon of given weight and volume, for example, can not float above a maximum altitude. Weather balloons manage to reach very high altitudes only by expanding greatly as they rise. But even they, once they reach their maximum volume, can only go so high. Even airplanes, which fly not by buoyancy, but by the pressure difference their motion generates between the bottom and top surfaces of their wings, can generate the requisite pressure difference at a given velocity only if the ambient air density is high enough. But Boyle's law has the density drop as the pressure does. So a given airplane has a maximum altitude at which it can fly.

7. How do airplanes fly?

How do airplanes fly anyhow? How do they generate that pressure difference between the bottom and top of their wings? The details are very complicated but the single most important factor is as follows.

First of all the engines of the plane, whether jet or propeller, exert a strong rearward force on air initially in front of the engines. The reaction force of the air on the plane pushes the plane forward. This pulls the wings through the air and divides the air into passing over or under the wing. The wing may have some inclination to the horizontal and the top surface of the wing has a more convex curve to it than the bottom, which is more nearly flat. The consequence of this is that the air passing under the wing follows a more straight line path than the air passing over the wing. Consequently, the latter follows a more accelerated trajectory than the former before they merge again behind the wing. The acceleration is produced by a dramatic pressure drop from the *front* of the wing to the *top* of the wing. The pressure rises back to normal again where the air streams merge somewhere behind the wing. That pressure drop on top of the wing compared to the pressure below the wing is the major contributor to lifting the plane (**Fig. IV. 8**).

This argument sometimes seems counterintuitive since we expect the convex upper surface to squeeze the air as the wing moves forward and why doesn't

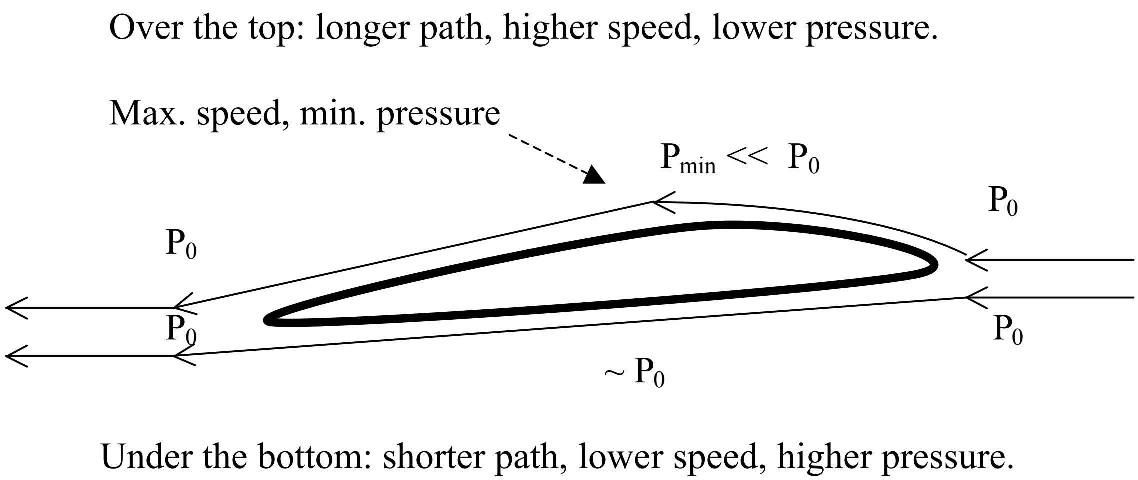


Fig. IV. 8: Lift-generating airflow past airplane wing

that *increase* the pressure on top? Well, the wing *is* squeezing the air. But unlike fluid pressure which acts equally in all directions, the airfoil squeeze is essentially vertical. The air 'escapes' the squeeze by moving horizontally, some random, pressure generating, molecular motion getting converted to organized horizontal 'escape' motion in the process.

The effectiveness of this phenomena drops as the air density decreases. Consequently, very high flying planes need very high speed and/or large wing area to produce enough lift. In the most extreme instance, atmospheric reentry of a returning Space Shuttle, there is, initially, too little air to produce lift in the described manner. In that case the Shuttle maintains a large angle of wing inclination to flight path to produce lift by simply deflecting air molecules. Pilot astronauts claim that even when close to the ground, flying the Shuttle is like flying a brick!

8. Sailing

Sailing with the wind is *not* like flying. It's like a leaf being blown across the yard. Sailing *against* the wind *is* like flying (**Fig. 9**)! Even though the sail is not rigid like a wing (some wings are actually rather flexible), setting the sail at the correct angle to the wind makes it act very much like a wing. As soon as the wind is moving faster over one side of the sail than the other, the pressure on that 'faster' side is lower than on the opposite side. That's why the sail bows out to a taut convex shape on that side. Once that happens the

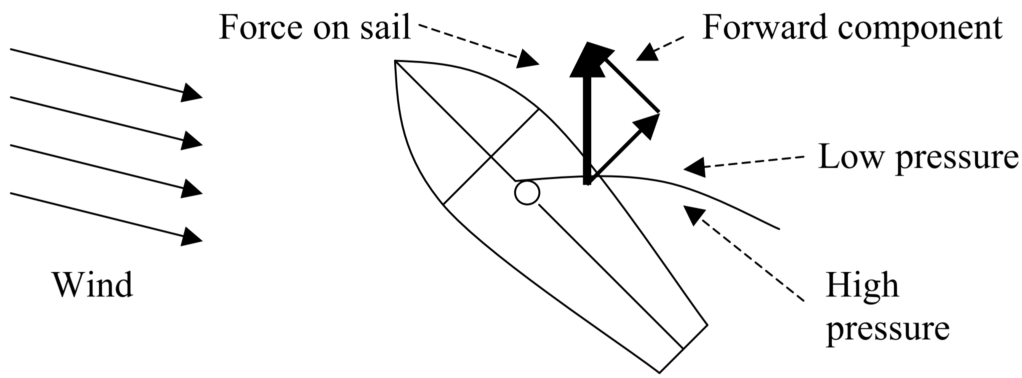


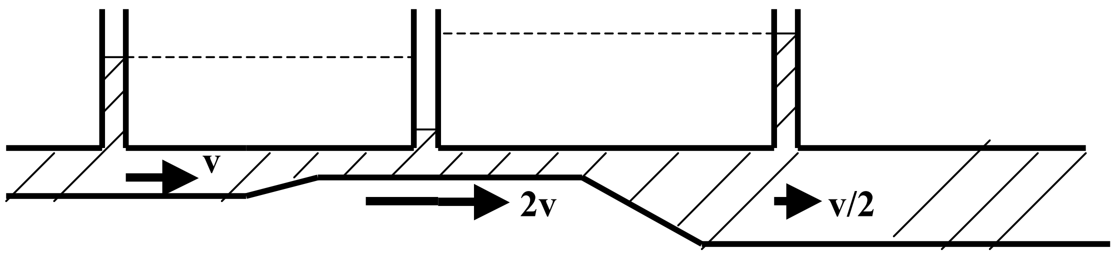
Fig. IV. 9: Sailing against the wind

wind really moves much faster over the convex side than over the concave side, where a pocket of air can almost lollygag, and the pressure difference becomes quite large. The boat is pulled perpendicular to the sail in the convex direction. So long as that pulling force has a component in the boats' forward direction, the keel and rudder of the boat can counteract the sideways component and the boat goes forward.

I've made it sound quite simple. But in shifting strong winds sailing against the wind is an art requiring alertness and quick strength! I don't sail.

This somewhat counterintuitive drop in pressure where the air moves more rapidly is also displayed by water. Indeed, by any fluid. The quantitative expression for the pressure dependence on flow speed is called **Bernoulli's Principle**. With water it can be nicely demonstrated by passing water through a horizontal tube of variable width with open vertical shafts mounted along the tube to display pressure variation (**Fig. IV. 10**). Where the width is narrow the water must flow faster and the requisite acceleration of the water as it enters the narrower channel must be provided by higher pressure behind the water than in front of it. When the channel widens the water needs to slow down and that slowing is provided by rising pressure in front of the water.

Pressure indicated by water height in vertical shafts



Water velocity indicated by velocity arrows

Fig. IV. 10: Pressure variation with flow speed in a 'horizontal' water channel

This may still seem counterintuitive because of our own 'pressure' rising when we're forced to slow down due to a traffic bottleneck from a lane closing. The point is that the water molecules aren't concerned about avoiding collisions and change their velocities primarily through collisions.

An important instance of this effect occurs when boats and large ships move parallel and too close to one another. The water passing between them must move faster (because of squeezing) than the water passing around the outer hulls. This lowers the water pressure between the boats and can result in their being pushed together into a collision.

9. Pressure differences and acceleration

Einstein was the first to realize that in sufficiently small regions of space and time, where 'sufficiency' is determined by circumstances, the effects of gravity are *always* equivalent to the effects of some kind of acceleration. He used this insight to help himself construct the General Theory of Relativity. But even in our prosaic study of the forces of everyday life there are interesting instances of Einstein's insight.

Consider again a car braking frantically to avoid a collision. Imagine a child buckled into the front passenger seat holding a helium filled balloon by a string and imagine an array of toys on the back seat. As a consequence of the strong braking the childs body presses forward against the seat belt and the toys on the back seat fly forward towards the front. But the balloon does not come forward! It drifts *backward* and pulls on the string trying to get to the back of the car (**Fig. IV. 11**). This is due to the air in the car piling up towards the front of the car during the deceleration. The air is under higher

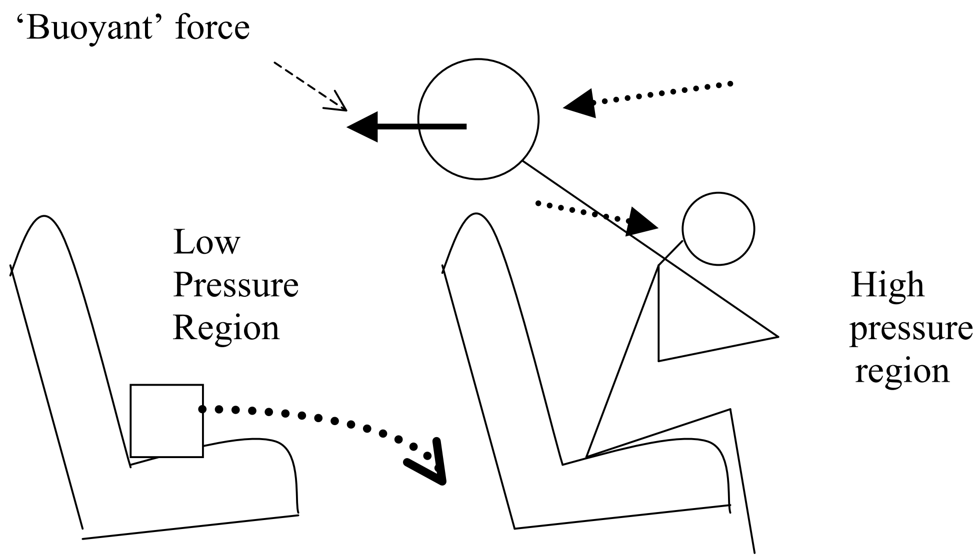


Fig. IV. 11: Inside the braking car

pressure at the front of the car than at the back and this pressure drop from front to back of the car produces a 'buoyant' force on the balloon towards the rear of the car. It is exactly as if a gravitational force directed towards the front of the car had been turned on. The balloon is pulled towards the front also but the buoyant effect of the pressed air overwhelms the 'weight' of the balloon, just as it does with ordinary gravity. This effect can easily be seen even with modest braking.

Similarly, when the car accelerates as the red light turns green, the balloon will drift towards the *front* of the car.

10. Measuring (and feeling) atmospheric pressure

Remember a ways back where I said that if we could feel the pressure of the atmosphere it would feel like the pressure at a depth of about 34 ft under water? Well, just as in the hydraulic lift, that air pressure on a ground level water surface gets transmitted throughout the body of incompressible water.

Consequently, if there was a patch of the water surface not subjected to the atmospheric pressure, we would somehow have to push down on the water under the patch with pressure equal to atmospheric to keep the water from being pushed up through the patch. We actually see this effect 'often' (those of us who still hand wash our dishes, that is) when we lift an inverted, submerged drinking glass, filled with water, from the dishwashing basin (**Fig. IV. 12**). So long as the mouth of the glass doesn't break the surface, atmospheric pressure keeps the water pushed up into the glass.

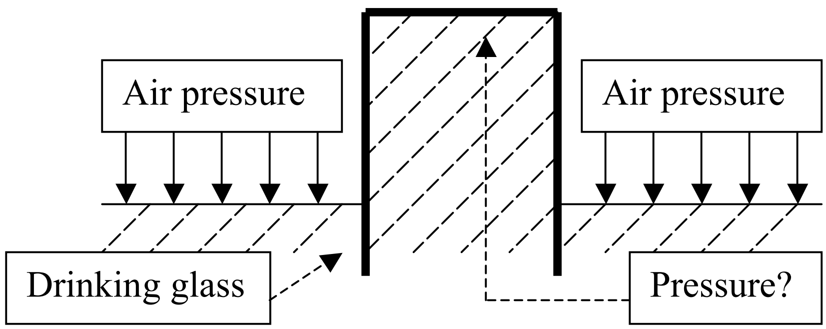


Fig. IV. 12: Inverted, full, drinking glass, partially lifted above water surface

Now some of us own taller glasses than others. But I'll bet no one has a drinking glass so tall that the water wouldn't be pushed all the way up into the glass even when the mouth of the glass was just ready to break the surface! But is there no limit to this effect?! Just how far can atmospheric pressure lift water into an otherwise empty inverted container? Well, remember that 34 ft depth? The atmosphere is pushing on the outside water surface with a pressure equal to the pressure that a column of water 34 ft high could generate by its' weight alone. So the atmospheric pressure, pushing up, as it effectively is inside the glass, can support a column of water 34 ft high.

So if you go out and buy a 35 ft tall drinking glass and do this little demonstration in a very large dishwashing basin, you will find that just before the mouth of the glass breaks the water surface there is 34 ft of water in the inverted glass and, on top of that, 1 ft of (to a good approximation) empty space! Yep, empty space - - vacuum - - nothingness, a concept that scared the ancients to death! Why empty space? How do we know?

Well, if there was some gas in that space it would exert some pressure on the water surface at the top of the column. But that pressure would add to the weight of the column and the atmospheric pressure couldn't support 34 ft of water *and* that extra pressure. The column would have to be less than 34 ft high. Of course, there might be just a *little* gas on top of the column exerting so little pressure that we wouldn't notice the discrepancy from 34 ft. And in fact, there is. It's water vapor, water molecules that have broken away from the liquid below. It exerts a small (under 'normal' conditions) pressure, called the vapor pressure of water (about 2% of a standard atmosphere). So we don't have a perfect vacuum, only a partial vacuum.

When we do this realistically with ordinary full inverted containers we notice that as they come further out of the water they seem to get heavier. It feels as though we are lifting the contained water. But we are not. The atmosphere is lifting the water. While the force we have to balance is *numerically equal* to the weight of the water coming up with the container, *what we are balancing is the atmosphere pushing down on the bottom of the inverted container against diminishing pressure at the top of the contained water as we lift the container higher and higher*. We are feeling more and more of the force of the atmosphere on the container and if we could continue to 34 ft we would finally feel the full force of the atmosphere! If our 35 ft long container had a base area of 10 in^2 , roughly the base of a yogurt tub, we would have to hold the container up with at least a force of 147 lbs!

But, you say, good skeptics that you are, has this ever actually been done?! I mean, a 35 ft long container! Well, historically the first indicators occurred with efforts to pump water out of deep mines during the middle ages. One could not pump the water more than 34 ft up in one stage. The physics was not well understood then but the serious technological problem motivated later study with the coming of the renaissance and the scientific revolution. But while water is always the most important liquid for humans, the effect we're looking for can be seen more easily with more dense liquids, like liquid mercury. Being 13.6 times more dense than water, the atmosphere can only support a (34 ft / 13.6) = 2.5 ft column of liquid mercury.

So if we fill a 3 ft long narrow tube with mercury and then hold the mouth of the tube closed as we invert it and place it below the surface of mercury in a dish of the stuff, when we open the mouth the contained mercury only flows out until a column 2.5 ft high is left in the tube. And now the vacuum left on top of the column is much better than with water because the vapor pressure of mercury is so much lower.

This arrangement is the basis of the barometer for measuring variations in atmospheric pressure (**Fig. IV. 13**). For on low pressure days (or hours) the atmosphere will only support less than 2.5 ft of mercury and in high pressure circumstances, more. By measuring the height of the column we measure the (local) atmospheric pressure.

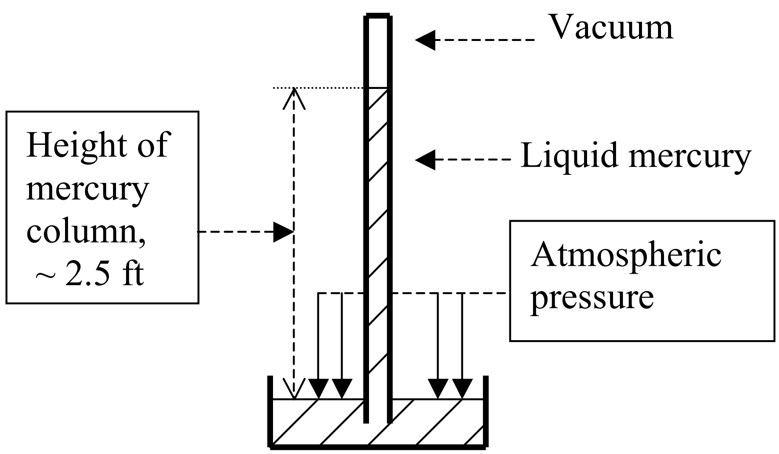


Fig. IV. 13: The basic barometer

Appendix 1: Derivation of dependence of air pressure on height

Imagine very still air divided up (in your mind) into vary many, very thin horizontal layers (**Fig. IV. A1**). Let each layer have a thickness, t. The ground has to support the weight of all the air above it. Let the pressure at the ground be P_0. Then designate the pressure at the *top* of the Nth layer, P_N.

We are going to deduce how the ratio, P_N/P_0 , for arbitrary N, depends on the ratio, P_1/P_0 . Consider an area, A, of the Nth layer. The *resultant upward force* on the air in that area of the Nth layer due to pressure variation with height is just the upwards pressure-force over A, from the top of the $N - 1$ layer, minus the downwards pressure-force over A, from the top of the Nth layer, i.e.,

$$P_{N-1}\,A - P_N\,A\ .$$

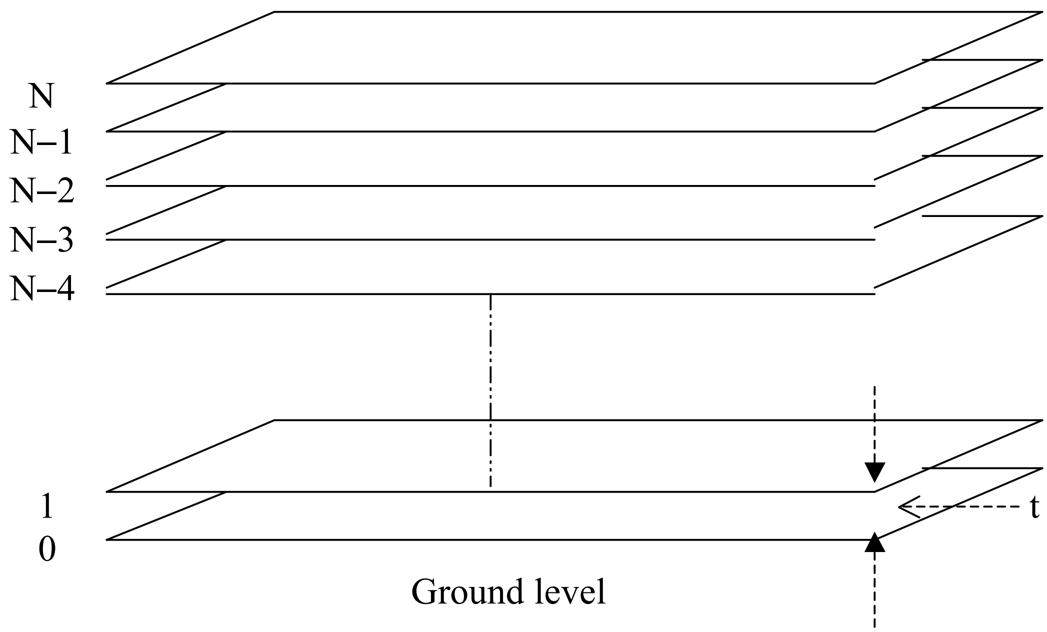


Fig. IV. A1: Surfaces of area, A, separating layers of atmosphere of thickness, t.

This force supports the weight of the air in that area of the Nth layer. The weight is $m_N\,g$, where m_N is the mass of the included air and g is the acceleration of gravity. So we have,

$$P_{N-1}\,A - P_N\,A = m_N\,g\ .$$

But the air satisfies Boyle's Law (at a fixed temperature) according to which, in the portion of the layer with area, A, and thickness, t (and, therefore, volume, A t),

$$P_{N-1}\,A\,t = (\text{const.})\,m_N\quad .$$

This equation is approximate because it treats the pressure at the bottom of the layer as if it were the pressure throughout the layer. The thinner the layer, the better the approximation.

Now we will divide this second equation into the first, left side into left side and right side into right side. The area A cancels out on the left side and the mass m_N cancels out on the right side leaving

$$(P_{N-1} - P_N) / P_{N-1}t = g / (\text{const.}) ,$$

or, multiplying both sides by the layer thickness, t,

$$1 - (P_N / P_{N-1}) = g\ t / (\text{const.}) ,$$

or, $$1 - (g\ t / (\text{const.})) = (P_N / P_{N-1}).$$

But the left hand side of this last equation doesn't depend on N. So we can say that, for any N,

$$P_N / P_{N-1} = P_1 / P_0 ,$$

the *ratio* of the pressure at the top of adjacent layers is the same at every layer (If you refer back to our discussion of pressure change with depth in incompressible water you will see that there, a similar analysis yields the result that the *difference* of pressure at the top of adjacent layers is the same at every layer).

But then we have,

$$P_N / P_0 = (P_N / P_{N-1})(P_{N-1} / P_{N-2})(P_{N-2} / P_{N-3}) \text{------} (P_1 / P_0) = (P_1 / P_0)^N,$$

the ratio of the pressure at the top of the Nth layer to the pressure at ground level equals the Nth power of the ratio of the pressure at the top of the first layer to the pressure at ground level. As we climb through each layer the pressure decreases by the same *factor*.

As mentioned above, this is what the mathematicians call an *exponential* drop in pressure with altitude. It has the interesting feature that, even though our derivation is more reliable and precise the thinner the individual layers, the final result holds regardless of how thick the layers may be.

If the distance from the ground to the top of the Nth layer is h, then the distance from ground to the top of the first layer is h/N and we have,

$$P(h) / P_0 = P_N / P_0 = (P_1 / P_0)^N = [\, P(h/N) / P_0 \,]^N \ ,$$

as we wanted to prove.

Appendix 2: Spinning a 'bucket' of water

Consider an old (empty) paint can that we half fill with water (we use a paint can rather than a bucket so as to avoid the complication of the sloping sides of a bucket). Now spin the paint can steadily about a vertical axis through its center (how you do this is up to you). Initially, the water will let the can spin around it (except for the thin boundary layer, right up against the side of the can, which immediately spins with the can). But, pretty quickly, the viscosity of the water will get all the water spinning at the same *angular rate* as the can. So every piece of water is moving in a circle concentric with the can, and, consequently, is accelerated towards the central axis of the can.

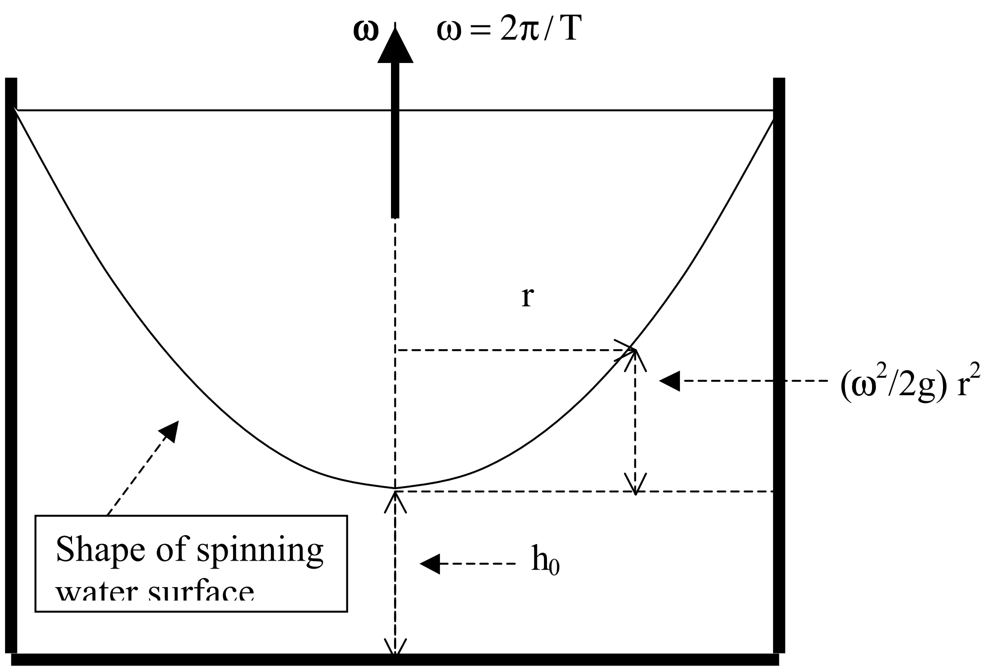


Fig. IV. A2: Parabolic surface of water spinning with paint can container

This acceleration requires a resultant force towards the central axis on the piece of water which is 'orbiting' the central axis of the can. That resultant force comes from the water pressure on the piece of water being slightly higher on the side of the piece facing away from the central axis than on the side facing the central axis. That pressure difference, in turn, comes from the water surface level rising as one goes further away from the central axis (**Fig. IV. A2**).

To see how the water surface acquires a parabolic shape consider a *very thin* cylindrical band of the spinning water between the radii, r and r' > r, from the central axis. The height of the water surface at the inner and outer radii of the cylinder will be h and h' > h, respectively.

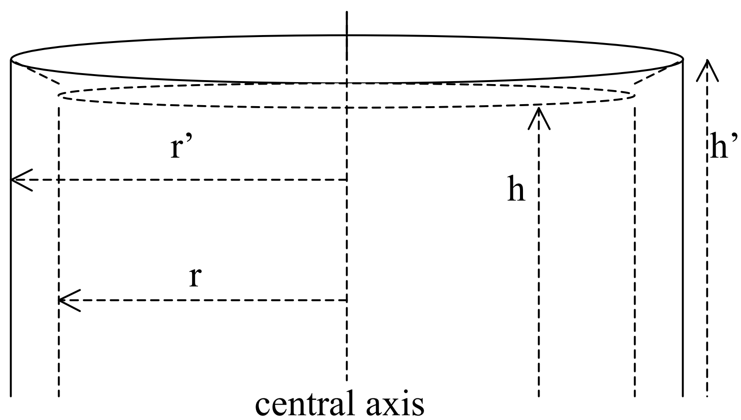


Fig. IV. A2': Thin cylindrical band of spinning water

At any vertical distance, y, from the bottom of the can and below the water surface the pressure on the inside and outside of the water cylinder is $P = \rho g\,(h - y)$ and $P' = \rho g\,(h' - y)$, respectively. So at all levels of the cylinder the pressure difference is given by,

$$\Delta P = P' - P = \rho\, g\,(h' - y) - \rho\, g\,(h - y) = \rho\, g\,(h' - h) = \rho\, g\,\Delta h.$$

This inward pushing pressure difference is the resultant *force per unit area* magnitude acting on every part of the thin cylinder of water. By our Third Rule of Forces it must equal the product of the mass per unit area, σ, of the water in the cylinder and the acceleration magnitude, a, of that mass density, i.e.,

$$\Delta P = \sigma\, a\,.$$

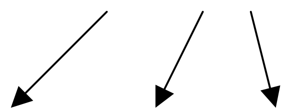
The mass per unit area, σ, is given by,

$$\sigma = \rho\ [\text{volume}] / [\text{area}] = \rho\ [(\pi r'^2 - \pi r^2)\ h] / [(2\pi r)\ h] = \rho\ \Delta(r^2) / 2r\ ,$$

where ρ is the volume mass density of water. The magnitude of the acceleration is,

$$a = \omega^2\ r\ .$$

Thus from, $\Delta P = \sigma\ a$, we get,



$$\rho\ g\ \Delta h = [\rho\ \Delta(r^2) / 2r]\ [\omega^2\ r] = \rho\ (\omega^2 / 2)\ \Delta(r^2)$$

or

$$\Delta h = (\omega^2 / 2g)\ \Delta(r^2)\ ,$$

the change in the height is proportional to the change in the square of the radius. If the height at the central axis ($r = 0$) is h_0, then the height, h, for general r is,

$$h = h_0 + (\omega^2 / 2g)\ r^2\ ,$$

the equation for a parabolic dependence of the height of the water surface on the radial distance from the center of the bucket..