# The Physical Forces of Everyday Life: II.

## 1. Resultants More Generally and The Center of Mass

We introduced resultants for collections of *coincident-simultaneous forces applied to a single system* because, for such collections, the resultant is equivalent to the collection in *ALL* effects. We now learn that, having done that, we get a bonus. Even if the forces are not *coincident*, i.e., even if the forces are not all applied at the same point, the resultant of a collection of forces is still *important*, not equivalent, but important. The forces in the collection still have to be applied simultaneously and applied to the same system – but they don't have to be applied at the same point – for the resultant to be important.

The useful importance of such resultants depends on the concept of the **center of mass** of a material system (**Fig. II. 1**). For our present purposes, we'll not worry about how to locate, precisely, the center of mass (CM) of a system. We'll get into that later. For now it's enough to know that *the CM is a point, with respect to which the matter of the system is evenly distributed in some sense*. This always places the CM 'inside' the system, and, for geometrically symmetrical systems the CM is often coincident with what one might call the geometrical center of the system. For example, the CM of a sphere of uniform mass density, or a cube of uniform mass density, is at the geometrical center of the sphere or cube, respectively.

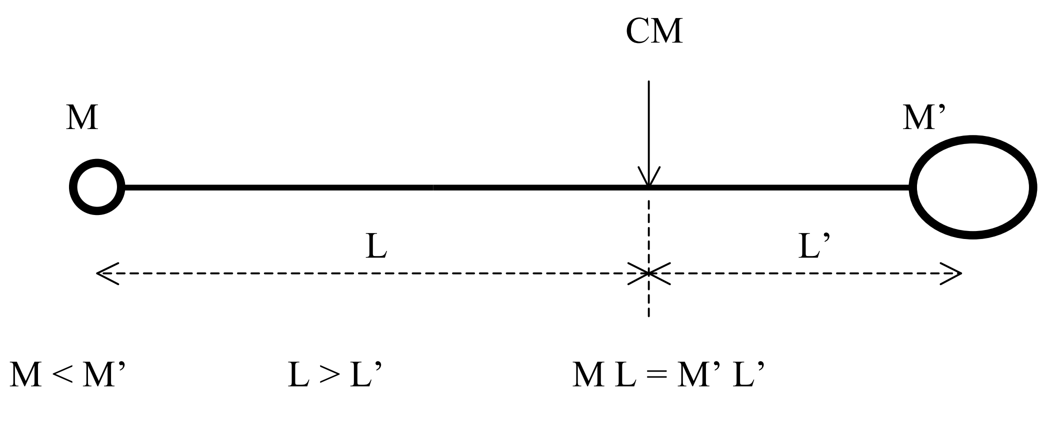



**Fig. II. 1:** Center of mass of lopsided dumbbell

If the system is a 'rigid' solid, or even a 'stiff' elastic solid, the location of the CM, relative to the rest of the system, is highly stable and inflexible. On the other hand, the CM of highly flexible solids or fluids or gasses can move around extensively, within such systems, as the distribution of the matter in such systems changes.

We're now ready to state the importance of the resultant of ALL the forces acting on a given system at the same time.

**2. Our Third Rule for Forces***

IF THE RESULTANT OF ALL THE FORCES ACTING ON A SYSTEM SIMULTANEOUSLY IS ZERO, THEN THE MOTION OF THE CM WILL NOT BE ACCELERATED. IF THE RESULTANT IS NON-ZERO, THEN THE MOTION OF THE CM WILL BE ACCELERATED IN THE DIRECTION OF THE RESULTANT AND PROPORTIONAL TO THE RESULTANT'S MAGNITUDE AND INVERSELY PROPORTIONAL TO THE SYSTEM'S MASS.

That's a bit of a mouthful! It is, in fact, a variant of **Newton's Second Law** and can be stated more compactly in the famous equation,

$$\mathbf{F} = \mathrm{M}\,\mathbf{A}.$$

Here **F** is the resultant force (to be represented by an arrow, remember), **A** is the acceleration of the CM (also representable by an arrow since it has magnitude and direction) and M **A** is the product (as in multiplication) of the mass*, M, of the system and **A**. The product (also representable by an arrow) has the same direction as **A** and a magnitude equal to the product of M and the magnitude of **A**. The historically original statement of Newton's second law did not refer to *arbitrary* systems, but to the individual minute particles of matter of which arbitrary systems are comprised. Our statement for arbitrary systems can then be derived from the original statement.

Spelling out the detailed meaning of the statements of the preceding paragraph (not to mention some detailed qualifications) would be a laborious task. Fortunately, we don't have to undertake that at this point! The initial statement of the Third Rule, given above in ALL CAPS, will suffice for us for now (but see **Figs. II. 2, 3**).

*These statements are strictly applicable only to systems of constant mass.

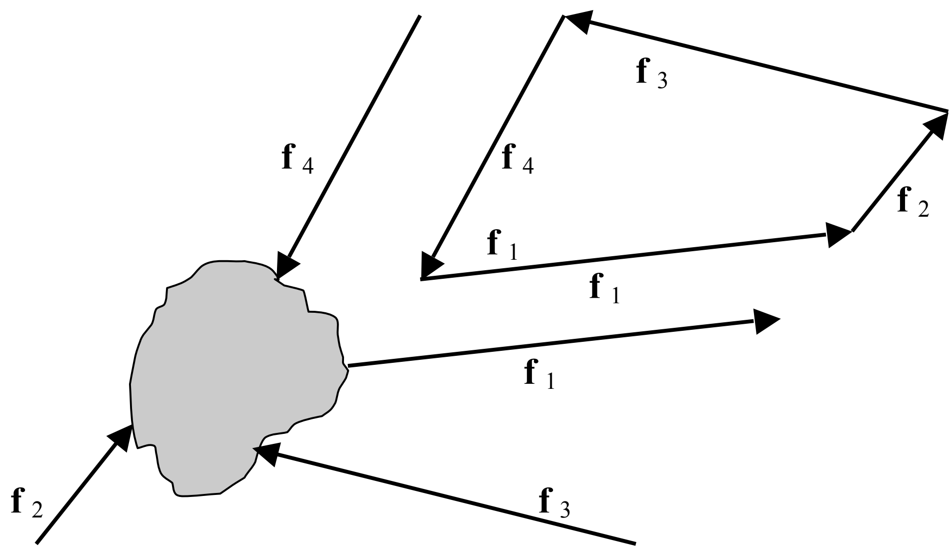
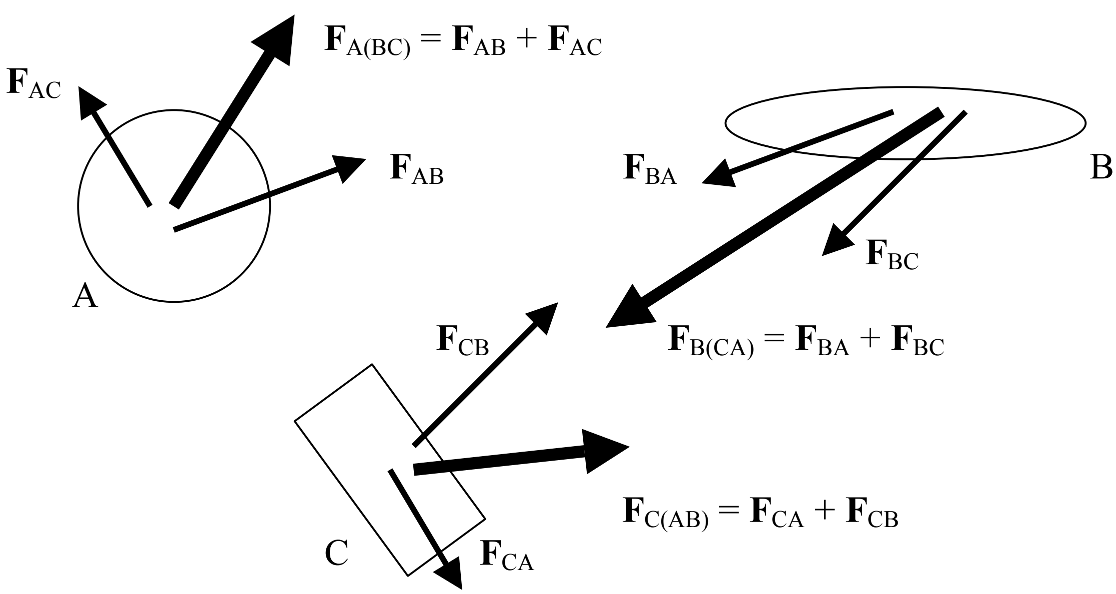



**Fig. II. 2:** The resultant of $\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{f}_3$ and $\mathbf{f}_4$ is the force, zero. If these are the only forces acting, the CM of the object on which the forces act would not be accelerated. It may be moving, but without acceleration. Note that $\mathbf{f}_1$ is pulling rather than pushing.




$$\mathbf{F}_{AB} = -\,\mathbf{F}_{BA}\,, \quad \mathbf{F}_{BC} = -\,\mathbf{F}_{CB}\,, \quad \mathbf{F}_{CA} = -\,\mathbf{F}_{AC}$$

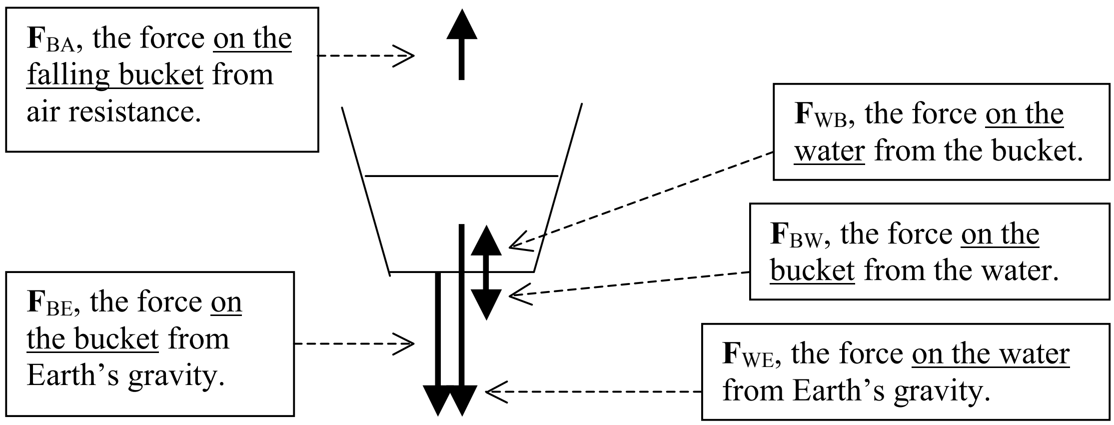
**Fig. II. 3:** Resultant forces and action-reaction pairs in a collection of three systems

There is an important useful simplification of our Third Rule that we can infer by using our First Rule.

The problem presented by our statement of the Third Rule is that the resultant force it refers to includes *all* the forces acting on the system! Now we, examining a system from the outside, so to speak, may be able to determine the forces that act on the system from outside sources. But what about *internal* forces that act on parts of the system due to other parts of the same system? These are still forces acting on the system and the Third Rule would apparently require us to include them in the assessment of the resultant force. But how can we do that?! It's usually much easier to determine the external forces acting on a system than it is to determine the internal forces. How can we know what all the detailed internal forces are?

Fortunately, the First Rule comes to our aid and tells us we can forget the problem altogether! The reason is that if any part of the system exerts a force on another part, then, according to the First Rule, that other part exerts an equal and opposite force back on the first part. But in assessing the resultant force *on the whole system*, we would have to include both of these internal forces. And being equal and opposite, they will always make mutually canceling contributions to the resultant! So the action-reaction pairs of internal forces (and they always come in such pairs) can be ignored. The Third Rule can be restated with the simplification that only the **resultant of the external forces** need be considered!

For example, consider the case of a falling bucket of water which remains upright during the fall so that no water spills out (**Fig. II. 4**). The forces on the bucket are the downward weight of the bucket, $\mathbf{F}_{BE}$ , due to the Earth's gravity, the upward resistance due to falling through the air, $\mathbf{F}_{BA}$ , which reduces the acceleration of the bucket's fall, and a downward force from the water in the bucket, $\mathbf{F}_{BW}$ , due to a transfer of part of the weight of the water to the bucket since the bucket impedes the acceleration of the falling water. The forces on the water are the downward weight of the water, $\mathbf{F}_{WE}$, due to the Earth and the upward reaction of the bucket, $\mathbf{F}_{WB}$, to the water pressing on the bucket. This upward reaction guarantees that the water does not accelerate more than the bucket during the fall. All of these forces act on the bucket-water system, but the internal action-reaction pair, $\mathbf{F}_{BW}$ and $\mathbf{F}_{WB}$, cancel each other out for the composite system and the external forces alone determine the resultant and the common bucket-water acceleration.




$\mathbf{F}_{B} = \mathbf{F}_{BE} + \mathbf{F}_{BA} + \mathbf{F}_{BW}$ $\qquad\qquad$ $\mathbf{F}_{W} = \mathbf{F}_{WE} + \mathbf{F}_{WB}$

$$\mathbf{F}_{(BW)} = \mathbf{F}_{B} + \mathbf{F}_{W} = \mathbf{F}_{BE} + \mathbf{F}_{BA} + \mathbf{F}_{BW} + \mathbf{F}_{WE} + \mathbf{F}_{WB}$$

$$= \mathbf{F}_{BE} + \mathbf{F}_{BA} \quad + \quad \mathbf{F}_{WE}\,, \quad \text{because} \quad \mathbf{F}_{BW} + \mathbf{F}_{WB} = 0.$$

**Fig. II. 4:** Forces on a falling system of water in a bucket. The internal forces cancel out (due to our First Rule).

### 3. Friction Forces

The naïve notion is that if we could only get rid of friction, we could sail through life so easily. In fact we are critically dependent upon friction for implementing most of our choices in life! Without friction we might sail through life but *we'd have almost no control over where we were sailing!*

Consider walking, for instance. Ever try walking on smooth, wet ice? On sloping, smooth, wet ice?! Efficient, relaxed walking is crucially dependent upon friction between our feet/shoes and the ground/pavement/floor/etc. We walk by propelling ourselves forward, parallel to the surface we're standing on, by pushing backwards on that surface with our feet/shoes. Normally the surface doesn't move much in response to our push – but it does push back on us (Rule One) – and we move forward.

Consider propping a ladder against a wall for climbing towards a roof or ceiling (**Fig. II. 5**). Ever try doing that when the supporting ground is covered with wet ice, or lubricating oil? The ladder will not stand (unless the legs dig into the surface). The bottom will slide away from the wall and the top will slide towards the ground. But why, you may wonder? Even with ice on it, the ground is strong enough to hold the ladder *up*! Why does it slide *away* from the wall?

Because the wall pushes it away! By virtue of leaning, the top of the ladder pushes against the wall. The wall, *in reaction*, pushes back against the ladder. If the bottom of the ladder can (due to friction) push horizontally away from the wall against the ground, the ground will react by pushing the ladder towards the wall, canceling the effect of the wall's pushing the ladder

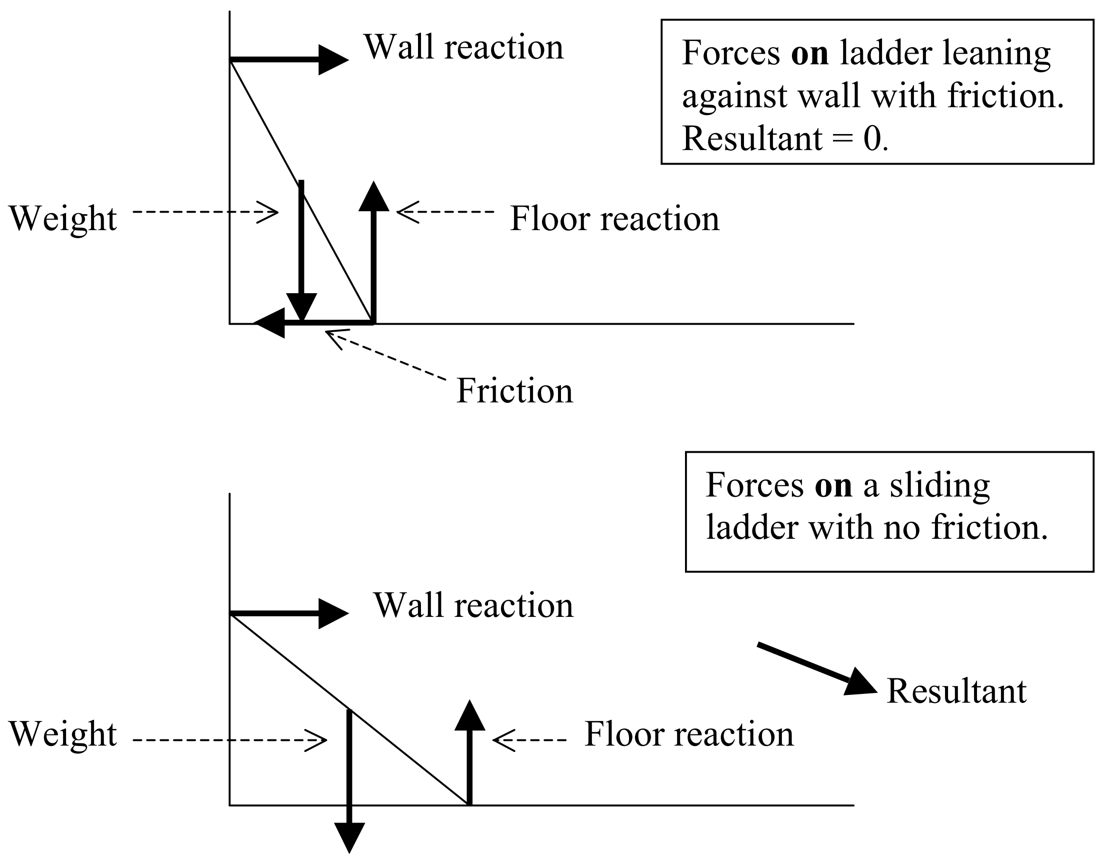



**Fig. II. 5:** Forces **on** a leaning ladder with friction (top) and **on** a sliding ladder with no friction (bottom).

away. But in the absence of friction, the ground can not react against the ladder towards the wall and the wall pushes the ladder away. Of course, the wall can push on the ladder only so long as the ladder stays in contact with the wall. But while the top stays in contact all the rest of the ladder is moving away as it slides to the ground. In fact, once the slide gets going, the horizontal motion of the ladder may well pull the ladder top away from the wall. The wall's push ceases, but the damage has been done!

As we will see later, even if friction is present for the ladder, there is usually a critical leaning angle beyond which the ladder will still slide to the ground. Qualitatively, the reason is that the wall must push hard enough to cancel the tendency of the ladder's weight to pivot the ladder counterclockwise around the floor contact. This tendency is small when the ladder is nearly vertical but it increases as the ladder deviates from the vertical. With enough deviation the compensating wall push overwhelms the static friction limit of the floor contact and the ladder slides and pivots.

**4. Static and Kinetic Friction**

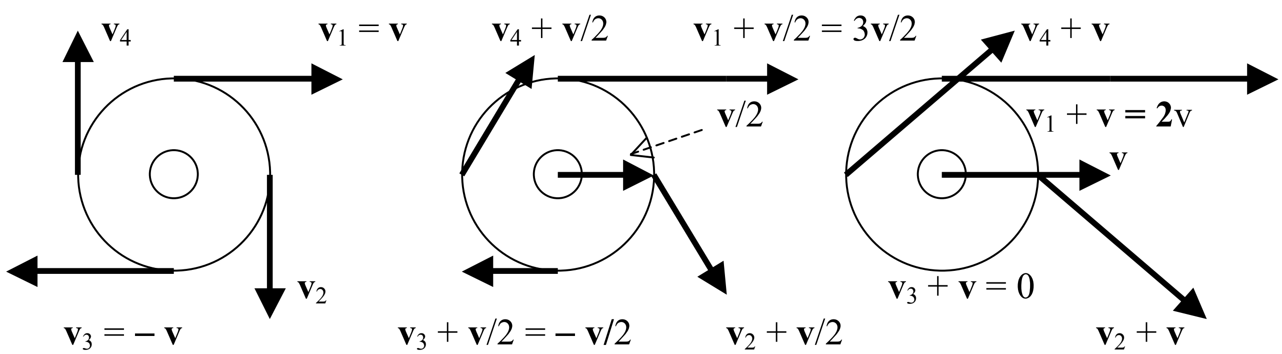
The frictional forces between two solid surfaces in contact are very different depending on whether the surfaces are sliding over each other or not. If they're not sliding but are stationary relative to each other (**static friction**), the friction force is *whatever is required* to keep them stationary, up to a *limit* which can not be exceeded. The limit depends on the forces that are pressing the surfaces together, on the materials and conditions of the surfaces, and, in some cases, on how long the surfaces have been in stationary contact.

Once the surfaces are sliding over each other (**kinetic friction**) the forces pressing the surfaces together and the material and condition of the surfaces determine the actual friction forces and these are independent of the (apparent) area of surface in contact. These 'laws' of sliding friction were first determined by Leonardo da Vinci, but his work had no influence because his notebooks were not published for hundreds of years. The 'laws' were rediscovered in the 17th century by the Frenchman, Guillaume Amontons. In the 18th century Charles Coulomb further recognized that to a good approximation kinetic friction forces are independent of how fast the surfaces are sliding.

For a given set of surface materials and conditions and given forces pressing the surfaces together, the *limiting value* for static friction is usually *greater* than the *actual value* for kinetic friction. This means that once two surfaces break free and start to slide against one another, the kinetic friction will be less than the static friction was just before the break.

The everyday context in which this static/kinetic distinction most commonly and importantly confronts us is when a vehicle has to stop as quickly as possible to avoid an accident. With the driver pressing the brake pedal, the brake shoes are pressed against the brake drums or disks of the wheels and slide against them. The kinetic friction is acting to slow the rotation of the wheels. But those forces, by themselves, would not slow down the vehicle. They are, after all, internal forces of the brake shoes on the brake drums/disks and the reaction of the drums/disks on the shoes.

What slows the vehicle is the force of the road on the tires at the 'point' of contact. This is a backward force in reaction to the slowing rotation of the tires pushing forward on the road. *That* action-reaction pair is (hopefully) an instance of static friction between the tire surface and the road. At the 'points' where the tires contact the road, if they are rolling, the tire surfaces are momentarily stationary relative to the road (**Fig. II. 6**). So the force slowing the vehicle can rise to the limit allowed by static friction but *no higher*.




| | | |
|---|---|---|
| Spinning wheel.<br>Surface speed, v.<br>Axle stationary. | Same spinning wheel.<br>Axle speed, v/2. | Same spinning wheel.<br>Axle speed, v.<br>Rolling condition. |

**Fig. II. 6:** Rim and axle velocities for spinning wheel. For each case, the rim velocities are resultants of the axle velocity and the corresponding rim velocities from the leftmost case with a stationary axle.

If the brake shoes and drums/disks slow the rotation of the wheels too rapidly (which they *are* capable of doing), the force of the road on the tires required to slow the vehicle enough to keep the tires rolling will exceed the static friction limit and the tires will break free and start to slide. Once that happens the road-tire friction is kinetic and less than the static limit. Consequently the now sliding/skidding vehicle is slowing down at a lower rate than it was before the tires broke free!

Anti-lock braking systems were designed to prevent panic stricken drivers from literally forcing their tires to break free of the road and start sliding.

For given materials/conditions of tire surface and road surface (including the slope of the road), the static friction limit places an absolute lower limit on how much distance is required to stop from a given initial vehicle speed. No matter what you do, it's *impossible* to stop more quickly! Most stops, of course, use up much more distance than the minimum possible.

**5. Lubrication**

The principal method for reducing friction where we don't want it is to lubricate the sliding surfaces. Basically this means introducing a fluid, usually a liquid, between the surfaces, so that the surfaces don't touch each other. A very thin layer of the fluid adheres to each surface and the many layers between the adhering layers slide over one another. Essentially one is replacing solid-solid sliding friction with fluid-fluid sliding friction which is much less. The technical name for fluid-fluid sliding friction is **viscosity**.

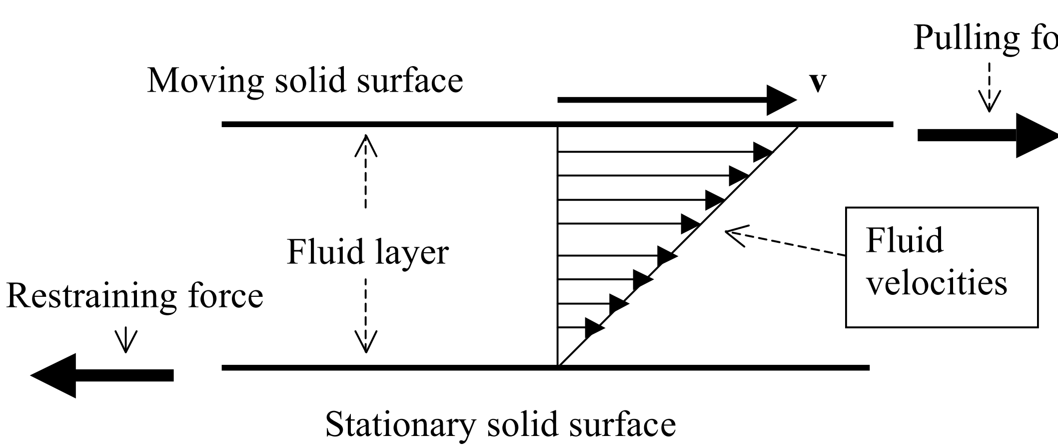



**Fig. II. 7:** Lubricating fluid between stationary and sliding solid surfaces.

Unlike solid-solid sliding friction forces, which are largely independent of the relative sliding speed, the friction forces of lubricated solid-fluid-solid sliding are dependent (essentially proportional) to the relative sliding speed when the lubrication is working 'properly' (**Fig. II. 7**). This is called **hydrodynamic lubrication** and is reasonably well understood.

Since one is trying to reduce friction, you may wonder why water, which is non-viscous, is a poor lubricant compared to oil which is more viscous. The first reason is that the forces pressing the solid surfaces together squeeze a non-viscous fluid out from between the solid surfaces much more easily than a viscous fluid. Even for viscous oil, if the sliding speed between the solid surfaces is small enough the oil film can be squeezed to such a thin layer (a few molecules thick) that proper lubrication no longer occurs and solid-solid friction, for all practical purposes, is in effect. This effect is called **boundary lubrication** and is *not yet* well understood!

The second reason why water is not a good lubricant is that friction, even fluid friction, generates heat, and water will evaporate too easily. Oil remains a good lubricant at much higher temperatures than liquid water could sustain (albeit losing some of its valued viscosity as the temperature climbs). That's why it used to be necessary to change the 'weight' of the motor oil used in vehicular engines as we passed from winter to summer and back again. The 'weight' of the oil refers to its' viscosity. More viscous oils are needed for hot summer driving and less viscous oils for cold winter driving (or, more accurately, cold winter starting). With modern additives the effective viscosity of motor oil is much less dependent on temperature than it used to be.

## 6. Viscous Damping

An important context in which viscosity is used to enhance friction rather than to reduce it is that of viscous damping. This term refers to instances in which undesirable oscillations can arise which, if not controlled and diminished, can become violent and destructive. Think of riding in a vehicle with worn out shock absorbers – on a rough road! I remember that as a common occurrence during my childhood. But even today, when smooth roads are more common, the occasional pot hole encounter would be much worse without reliable shock absorbers.

The shock absorber works in concert with a strong coiled spring. When the bump first occurs, the spring cushions the initial blow by compressing over a time interval longer than the bump itself takes. The compression is still pretty fast, and we still feel it as a bump. But it doesn't loosen our back teeth! Which it would be much more likely to do without that spring. But if the spring were the whole story, we would then be in for a wild ride of oscillations as the spring bounced back-compressed again-bounced back-compressed again, etc.

Instead the spring is connected to a sleeve and a rod which pushes a piston inside the sleeve through a viscous fluid. The fluid might be oil, or a silicon compound, or even a gas under pressure (**Fig. II. 8**). The piston has holes in it with valves that allow the piston to move in the sleeve while the fluid moves through the piston. In either direction of motion the friction force resisting the piston's motion, due to the fluid viscosity, reduces the bouncing of the spring. But usually the piston valves make the friction resistance higher when the spring is rebounding from being compressed. Effectively, the

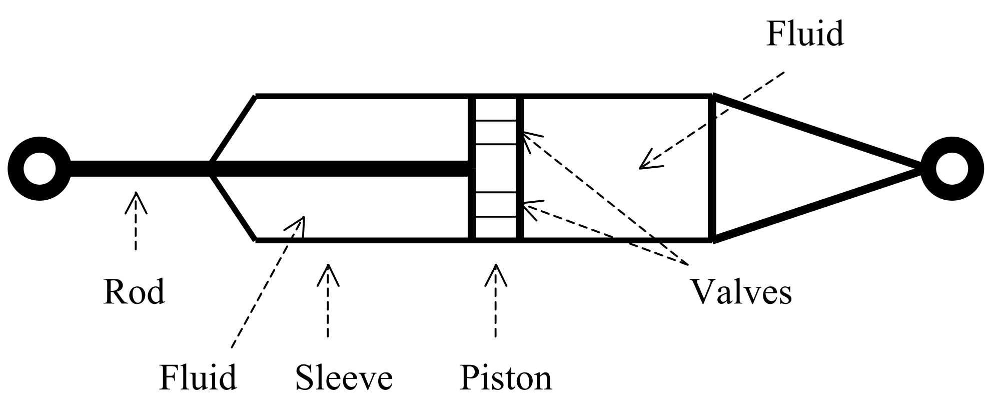



**Fig. II. 8:** Shock absorber without accompanying spring

piston sliding through the viscous fluid is sapping the spring of the energy put into it by the initial bump. The spring has lost almost all of its' energy by the time it completes its' first rebound.

A more recent application of viscous damped shock absorbers on a much larger scale is the protection of buildings from earthquake damage. Again the aim is to prevent the various parts of the building from oscillating too much and too long relative to other parts. Placing long shock absorbers, without the springs but just the viscous damped sliding piston, diagonally between floors and walls, is one of the least expensive steps toward making a building earthquake 'ready'.

**7. Friction and Heat**

Friction forces are intimately connected with the generation of heat. So far as we know, humans first learned to create fire by concentrating the heat generated by the friction in rubbing wood against wood. If laden trucks descending a long steep hill pick up too much speed, they may set their tires on fire attempting to brake to a safer speed. If racing drivers spin their tires too long or too furiously against the road surface, they can set the tires afire. (By the way, why in the world would race drivers do such a thing? Didn't we learn that static friction is greater than sliding friction? Couldn't greater acceleration be achieved by keeping the tires from spinning? Yes, but when they spin their tires, they're not trying to accelerate. They're trying to make their soft rubber tires smoother so that, when they race, their tires have better contact with the road and thereby increase the static friction limit!). We rub our hands together in the cold to make them warmer. Drilling holes in wood or metal heats the wood or metal and can produce very hot fragments of drilled out material. The temperature of water can be raised by sloshing a paddle in it for an extended time. The oil inside a shock absorber can get quite hot when driving over a bumpy road.

Why is this?! And why are friction forces generated at all?!

The details vary considerably from case to case and can be quite complicated and are still not always well understood. For instance, it used to be thought that solid-solid sliding friction was just due to the roughness of the sliding surfaces. But that is now known to be far too simplistic! A rough account that captures the general features goes like this. When substances move over one another, in contact, pieces of them at the interface, varying (depending on the circumstances) in size from merely small to almost molecular dimensions, grab hold of one another briefly and then let go. This repeated grabbing, pulling, releasing (which in some cases actually produces stick-slip, stick-slip motion rather than continuous sliding) sets up oscillations in the substances, especially near the interface. These oscillations may be large enough to be just 'visible', as they are in rubber, or they may be molecular level oscillations very similar to sound waves (but not necessarily at audible frequencies). But regardless of their character when initiated, they get distributed to the surrounding material as sound waves and then become more and more random molecular motions. The more easily sound waves can be generated within the sliding surfaces, the higher the friction force because the sound waves are consuming energy. But once the molecular oscillations become random, they are no longer sound but heat. *For that's what heat is! Random molecular motion*. **Heat energy** is the energy of that random molecular motion. **Temperature** is a measure of the average random energy per molecule. When you jerk your hand back from touching a hot stove its' because nerves in your hand have sent a signal to your brain that skin surface molecules have absorbed dangerous levels of random motional energy.

The recognition that heat was random molecular motion and carried a form of energy was established in the mid-nineteenth century and was demonstrated (among other ways) by noticing that the heat generated by the frictional process of boring cannon barrels seemed to be proportional to the work done in the boring.

When solid-solid sliding friction is at work its' converting organized energy of motion, first, into internal sound waves and thence into the random energy of heat.

**8. Elementary Quantitative Properties of Friction**

Since the mid 1980's research in the quantitative details of the physics of friction has been a large scale endeavor. This is due to the needs of modern technology and industry. Prior to that friction was not really understood very well and not many people were interested in trying to improve the situation.

One reason for the lack of interest was that for many (non-sophisticated) purposes the quantitative relationships seemed very simple.

For example, given two reasonably flat solid surfaces in stationary contact, the maximum friction force they could exert on each other before breaking free and starting to slide – what we previously called the limit of static friction – is given by the formula,

$$F_{limit} = \mu_S N,$$

where $\mu_S$ is a 'constant'* (S for static) determined by the materials and conditions of the two surfaces and N is the magnitude of the force perpendicular (normal) to the surfaces pushing them together. If the surfaces are sliding over each other then the kinetic friction force between them is given by the similar formula,

$$F = \mu_K N,$$

Where the 'constant'* $\mu_K$ (K for kinetic) is smaller than $\mu_S$ . In particular The friction force does not depend on the speed of the sliding.

In the case of viscous fluid lubrication between two sliding surfaces of area, A, separation distance, D, and relative speed, V, the friction force that needs to be overcome to keep the surfaces moving does depend on the speed of the sliding and is given by the formula,

$$F = \nu \, VA / D ,$$

where $\nu$ is a 'constant'* determined by the type and condition of the fluid used. These 'constants' are called **coefficients** of static friction, $\mu_S$ , kinetic friction, $\mu_K$ , and viscosity, $\nu$ .

The values of $\mu_S$ and $\mu_K$ can vary all over the place because of their dependence on surface 'conditions'*. Coefficients of viscosity are less volatile and some representative values are presented below.

Water: 0.01 gm/cm-sec, 'Light' oil: 0.70 gm/cm-sec,

Glycerin: 8.5 gm/cm-sec, Air: $1.8 \times 10^{-4}$ gm/cm-sec,

Water vapor: $9.6 \times 10^{-5}$ gm/cm-sec.

*All of these 'constants' depend on temperature. $\mu_S$ and $\mu_K$ can depend strongly on the preparation of the surfaces with *very* smooth, clean metal surfaces yielding very *high* values and *some* dry surfaces yielding *lower* values than with lubrication. $\mu_S$ can even depend on the *time* that the surfaces have been in contact.

You might wonder, when are water vapor or air used as lubricants? Well, when the Hover Craft lifts above the water to cross the English Channel or the French or Japanese Bullet Trains magnetically levitate above the tracks, air, with or without water vapor, is the viscous medium through which these machines move.

**Appendix: The center of mass and the concept of an average**

In this appendix we will discuss the center of mass (CM) concept as a first example of the broadly useful concept of an average.

The quantity that is being averaged in the CM case is a position, or, more precisely, a displacement, which, you may remember, is a vector represented by an arrow. The CM displacement goes from a reference point, chosen for convenience, to that point in a physical system of interest which is called the CM point in the system (**Fig. A1**).

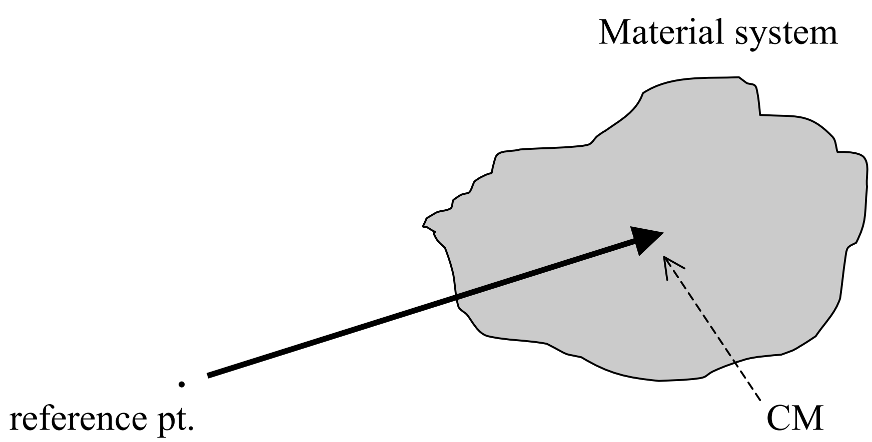



**Fig. A1:** Displacement from reference point to the CM of a material system.

The intuitive motivation for the concept is that for every material physical system there is some point in space such that the mass distribution of the system is (in some sense) balanced or symmetrical with respect to the CM point. While the CM is ‘usually’ located at a point where the mass density of the system is non-zero, for some systems it is located where there is no mass of the system, in 'empty' space, so to speak (**Fig. A2**).

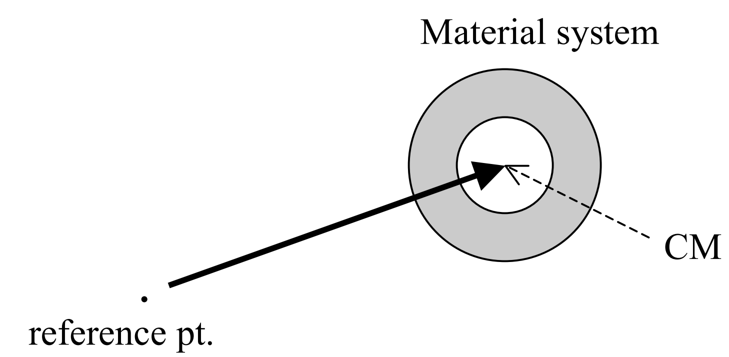



**Fig. A2:** A symmetrical material system with its' CM located in 'empty' space.

So where does the averaging come in? In determining the location with respect to which the mass distribution of the system is 'balanced' or 'symmetrical'.

Suppose the system is composed of two parts, $S = S_1 \& S_2$ . Using the same reference point for each part, denote the CM displacement vectors for each part by $\mathbf{X}_1$ and $\mathbf{X}_2$, respectively. Suppose the masses of the parts are $M_1$ and $M_2$, respectively. Then the displacement vector, $\mathbf{X}$, from the same reference point to the CM of the whole system, S, satisfies the equation,

$$(M_1 + M_2)\, \mathbf{X} = M_1\, \mathbf{X}_1 + M_2\, \mathbf{X}_2 ,$$

or

$$\mathbf{X} = (M_1\, \mathbf{X}_1 + M_2\, \mathbf{X}_2) / (M_1 + M_2) .$$

In other words the displacement vector to the CM of the whole composite system is the mass weighted average of the displacement vectors to the CMs of the partial, constituent, systems (**Fig. A3**).

The number of constituent systems can be made arbitrarily large and their sizes arbitrarily small by further subdividing the two original constituents and applying the defining relations again and again. Once the subdivided constituents are small enough, the location of their CMs are simply given by *their* locations.

Further analysis of the definition given here yields the results that the location of the CM of any system does not depend on (1) the choice of the reference point nor (2) the way in which the system is divided up into parts.

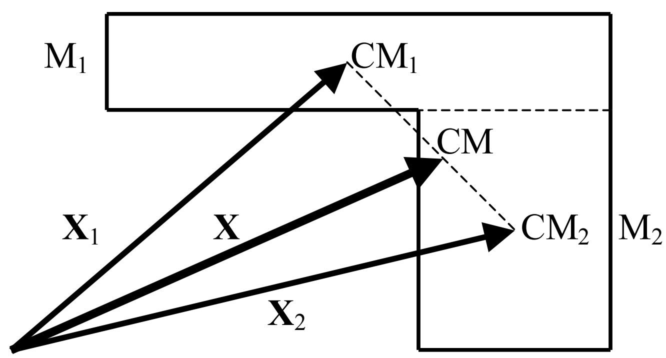



**Fig. A3:** The CM of an L shape material system (uniform mass density) as a mass weighted average of the CMs of constituent rectangles. Note that the composite CM lies on the line joining the two constituent CMs. That is a general result.

This example of an average position generalizes to a notion of any quantity, Q, averaged with respect to any additive weighting, W. In every case, if two constituent systems (1 & 2) comprise a whole system, with the W values, $W_1$, $W_2$, and $W_1 + W_2$, respectively, then the whole system Q value satisfies,

$$(W_1 + W_2)\, Q = W_1\, Q_1 + W_2\, Q_2 \,,$$

or

$$Q = (W_1\, Q_1 + W_2\, Q_2) \,/\, (W_1 + W_2) \,.$$

As before, the result for Q does not depend on how the whole system is decomposed into constituents.