The Physical Forces of Everyday Life: I.

If you're sitting down as you read this, then you can feel your body pressing against your chair. More precisely, you feel the chair pressing against your body. We feel the forces that act upon us more directly than the forces we exert on other objects. For the most part, when we think we feel ourselves exerting a force, it's because we feel some parts of our body pressing against other parts of our body in the process of bringing the exerted force to bear.

It's easier to think of *ourselves* as exerting forces than to think of pieces of *'inert' matter* as exerting forces. Forces seem to be efforts to try to change things and why would an 'inert' piece of matter try to change anything? We, on the other hand, racked by desire, passion, disgruntlement, anger and enthusiasm are always trying to change things! So perhaps we can understand that the notion of purely natural, mechanical, non-living forces was slow in coming and achieving acceptance.

1. A Brief Account of the Evolution of the Concept of Physical Force

Our conjectures are that primitive humans recognized only forces generated by beings and spirits and life forms. Even the astounding ancient Greeks, for the most part, divinized and enlivened the forces of nature. For Plato all of nature was a living organism and so the forces in nature were strivings for goals. Even the more practical minded Aristotle saw falling bodies and rising flames as objects seeking their natural place in the universe while collisions and projectiles were instances of violence done to the natural order of things. Only with Democritus and the atomists and their effort to reduce all phenomena to the motions and collisions of inert, lifeless atoms do we see some hint of the modern conception of a purely physical, non-living force. But in those days even a higher fraction than today of the educated citizenry regarded this conception as intolerably bleak and alien. It did not catch on and by the time Archimedes presented a surprisingly modern treatment of the forces of levers and buoyancy he was ahead of his time in a dying civilization. Only the later civilization of Islam would see the value of preserving his work.

Through the Middle Ages and the Renaissance a long arduous struggle with concepts of forces very gradually evolved. Slowly and erratically a branch moved in the direction of disentangling and identifying the possibility of pieces of non-living, material nature being sources of forces. If unobstructed, such forces would produce or change motion and, obstructed, could maintain stable structures. But, notwithstanding Europe's eventual recovery of Greek manuscripts, nothing as explicit as the preceding two sentences emerged from the voluminous writings of this thousand year long period!

Only in the 17th Century does the modern scientific conception of physical force begin to explicitly emerge. Some key steps in that process are as follows: The German astronomer, Johannes Kepler, anticipates Newton in conceiving the possibility of a precise quantitative and unified treatment of terrestrial and celestial attractions. He gets the details wrong, among other errors identifying the celestial forces with versions of the terrestrial magnetism discovered by Gilbert in the 16th Century. Nevertheless, his conception of physical force is a signal advance! Galileo discovers the connection of terrestrial gravity with his new and precisely defined concept of **acceleration** and identifies 'horizontal' motion, in the absence of friction or air resistance, as unaccelerated and perpetual, i.e., **inertial**. Descartes improves on Galileo in identifying the kind of motion which does not require force, inertial motion, as *unaccelerated in any fixed direction*. He also argues strongly that matter can exert forces on matter only through direct, physical contact and, as the father of modern philosophy, sharply articulates the conceptual dualism between non-living matter and living spirit. Huyghens successfully analyzes the acceleration involved in circular motion. Hooke correctly assesses the initial restorative forces of elastically distorted, solid, matter. Boyle establishes the variation of the expansive forces of gasses under compression and Pascal recognizes the transmission of forces through incompressible fluids. And then, finally, Isaac Newton improves upon and pulls these threads together and formulates the conceptual, mathematical and empirically testable scheme of physical nature in his masterpiece, "The Principia", or "The Mathematical Principles of Natural Philosophy". The central concept throughout this work is the concept of physical force.

The clarification and further development of the concepts and implications of the Principia occupied the physical scientists of the 18th Century. There were several aspects of the Principia which many found disturbing (especially the action at a distance character of Newtonian gravity which violated Descartes insistence on forces by contact only), and some problems required deep conceptual advances beyond Newton for their solution (the mechanics of continua required the concepts of torque, angular momentum, stress and strain). The leading figure, among many, in these 18th century efforts was the great mathematician and natural philosopher, Leonhard Euler.

For over two hundred years from the late 1600s the emerging system of ideas continually advanced and no successful challenge to its quantitative accuracy could be found! More than anything else, this continued success supported the confidence that humans could grasp the structure of the Universe and propelled the ever increasing advance of science! Finally, the *excessive* forms of this confidence (Newton and company had got it RIGHT and could not, fundamentally, be improved upon!) was shaken with the emergence of the limitations of this so called *Classical* scheme in the late 19th and early 20th centuries.

Nevertheless, for almost all the physical forces of everyday life, and, therefore, for this course, the pre 20th Century development of the Classical scheme is quite sufficient.

2. Our First Rule for Forces

The first rule concerning forces that we will consider asserts that

FORCES ALWAYS COME IN PAIRS,

so-called **action-reaction** pairs. For Newton this was an aspect of his *Third Law of Motion*. To get a sharper statement we need the concept of a **physical system**. A physical system is any (sufficiently) clearly delineated collection of coexisting matter and/or energy.

The qualifier '(sufficiently)' means sufficiently for the purposes at hand.

A grain of sand is a physical system. So is a pile of sand. So is a sand dune or a beach of sand. A drop of water is a physical system. So is the water in a glass, or in a swimming pool, or in a lake, or in an ocean. A bubble of air rising through water is a physical system. So is the air in an empty container, or the helium gas in a party balloon, or the swirling air of a tornado, or our whole atmosphere, or the complex of gases (and maybe liquids) comprising the planet Jupiter. A brick is a physical system. So is a house made of bricks, or a bridge made of steel, or a building of reinforced concrete, or a mountain, or the planet Mars. A single molecule or atom is a physical system. All these kinds of systems are primarily matter.

Waves traveling across the surface of water are a physical system, whether they be miniscule surface tension waves or wind driven waves or devastating tsunami's. Waves traveling through air are a physical system, whether below, within or above audible frequencies, below, within or above audible decibels. Electromagnetic waves traveling through space are a physical system, whether 60 cycle, radio, microwave, infra red, optical, ultra violet, X-ray or gamma ray frequencies, whether weak enough to be barely detectable, mild enough to tan a sunbather, strong enough to cook beef or with the intensity of a laser or an exploding supernova. A single photon is a physical system. All these kinds of systems are primarily energy.

Any coexisting combination of matter and energy systems can be a physical system.

Now, with all that under our belt, Newton's Third Law and our **First Rule** is that,

IF ANY PHYSICAL SYSTEM, A, EXERTS A FORCE ON ANY PHYSICAL SYSTEM, B, THEN (AT THE SAME TIME) THE SYSTEM, B, EXERTS AN EQUAL AND OPPOSITE FORCE ON THE SYSTEM, A.

More compactly, this rule is often expressed as: *For every action there is an equal and opposite reaction.*

3. Examples of Our First Rule

Since energy dominated systems are a bit more subtle to deal with than matter dominated systems, we will stay with matter dominated systems for awhile. If past experience is any indication, some of these examples will surprise some of you. There are twenty eight examples. The starred examples are portrayed in diagrams. The format of the examples will be:

Force of A on B (direction) – Force of B on A (direction).

(1) Sitter on chair (down) – Chair on sitter (up)

(2) Chair on floor (down) – Floor on chair (up)

(3) Earth on sitter (down) – Sitter on Earth (up)! (and, as always, equally!)

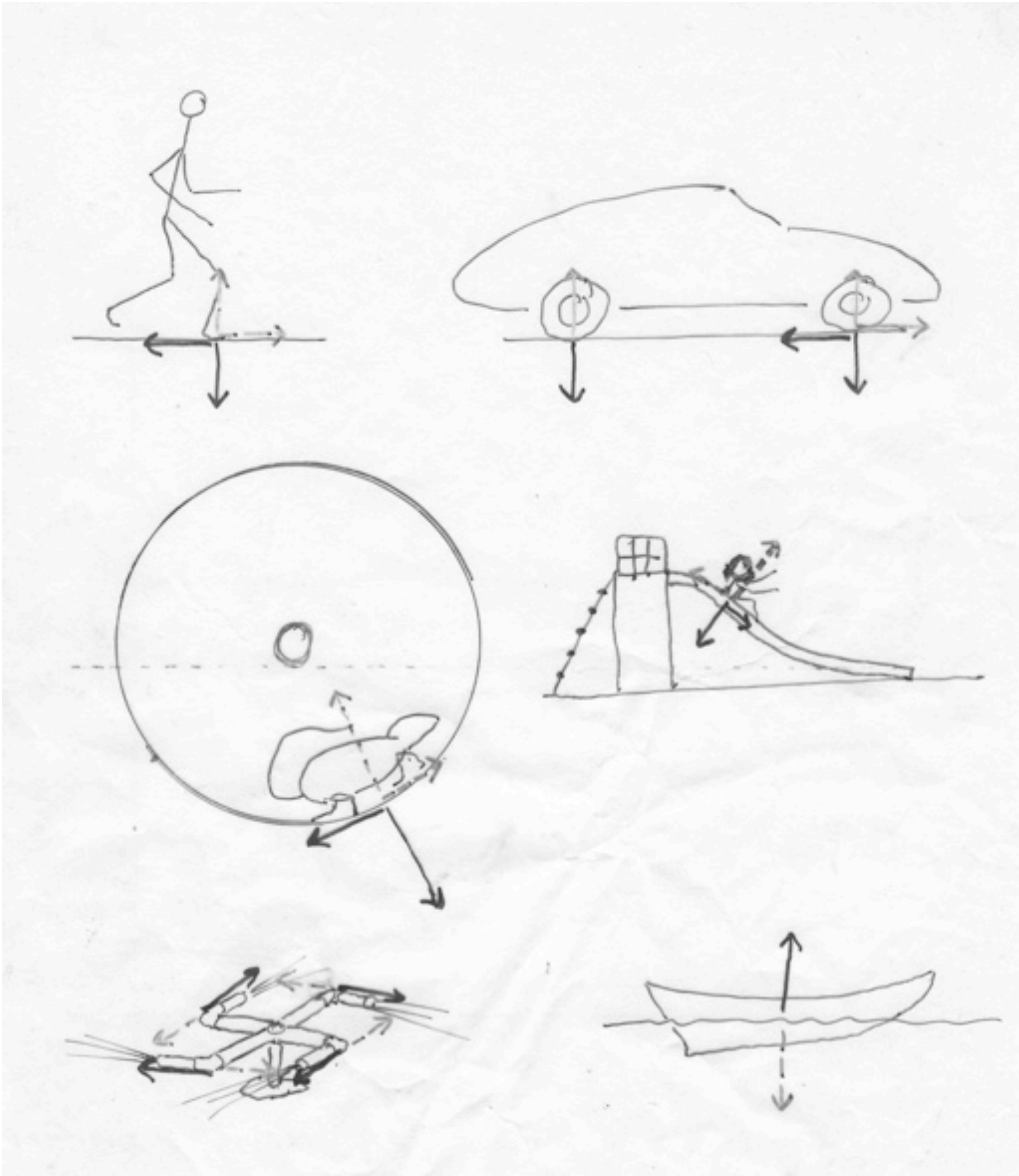


Fig. I. 1: Action – reaction force pairs. Action (bold) – reaction (light).

(4) Leaning ladder on wall (towards wall) – Wall on leaning ladder (away from wall)

(5)* Walking foot on pavement (down and rearward) – Pavement on walking foot (up and forward): Yup, the pavement is pushing you forward!

(6)* Accelerating/braking vehicle tire on road (down and rearward/forward) – Road on accelerating/braking vehicle tire (up and forward/rearward): Provided the road isn't too slippery!

(7)* Sliding child on sliding board (against and down the board) – Sliding board on sliding child (against the child and up the board)

(8)* Running mouse on turning treadmill (against and parallel to treadmill motion) – Turning treadmill on running mouse (against the mouse and 'opposed' to treadmill motion)

(9)* Water on a stationary boat (up) – boat on the water (down)

(10)* Counterclockwise spinning lawn sprinkler on spraying water (clockwise) – Spraying water on sprinkler (counterclockwise)

(11) Surrounding air on a hot air balloon (up) – Hot air balloon on the surrounding air (down)

(12)* Canoe paddle on the water (rearward) – Water on the canoe paddle (foreward)

(13) Air on a descending parachute (up) – Parachute on the air (down)

(14) One portion of tense rope on adjacent portion of tense rope (towards first portion) – Adjacent portion of tense rope on first portion of tense rope (towards adjacent portion)

(15) Air on the ground ('down') – Ground on the air ('up') : Atmospheric pressure.

(16) Exploding gas/air mix on piston (down) – Piston on exploding mix (up) : The internal combustion engine.

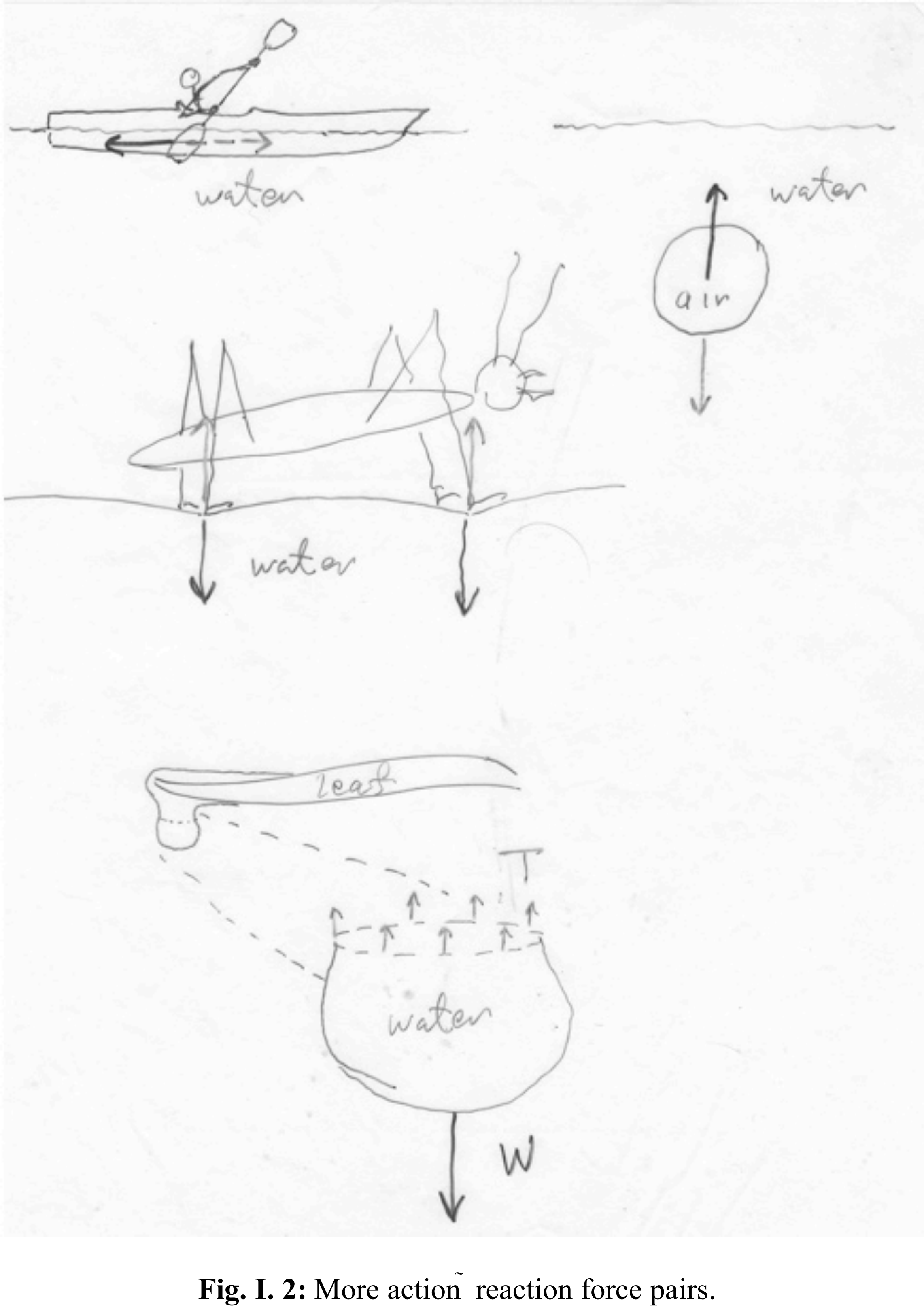


Fig. I. 2: More action˜ reaction force pairs.

(17)* Rising air bubble on surrounding water (down) – Surrounding water on rising air bubble (up)

(18)* Growing water droplet on stretching water surface (down) – Water surface on water droplet (up): Water dripping off a wet leaf.

(19)* Insect walking on water (down and rearward) – Water supporting insect (up and forward)

(20) Revolving Earth on orbiting Moon (towards Earth) – Moon on 'orbiting', revolving Earth (towards Moon) : The main source of tides.

(21) Sun on orbiting Jupiter (towards Sun) – Jupiter on 'orbiting' Sun (towards Jupiter) : How planets orbiting distant stars are discovered.

(22) Electrically charged rubber rod on paper fragment (towards rod) – Paper fragment on charged rod (towards paper)

(23)* Electrically charged clouds on charges in ground surface (up) – Ground surface charges on charged clouds (down) : The source of lightning.

(24) Magnet on a piece of iron (towards magnet) – Piece of iron on magnet (towards iron)

(25)* Stationary magnet on current carrying wire (perpendicular to current and magnetic field lines) – Current carrying wire on magnet (opposite direction): Basis for the electric motor.

(26)* Current carrying wire on parallel current carrying wire (towards first wire) – Parallel wire on first wire (towards parallel wire), i.e., mutually attractive.

(27)* Current carrying wire on anti-parallel current carrying wire (away from first wire) – Second wire on first wire (away from second wire), i.e., mutually repulsive.

(28)* Magnet on charges in wire moving perpendicular to itself and to magnetic field lines (parallel to wire) – Charges in moving wire on magnet (opposite direction) : Basis for the electric dynamo.

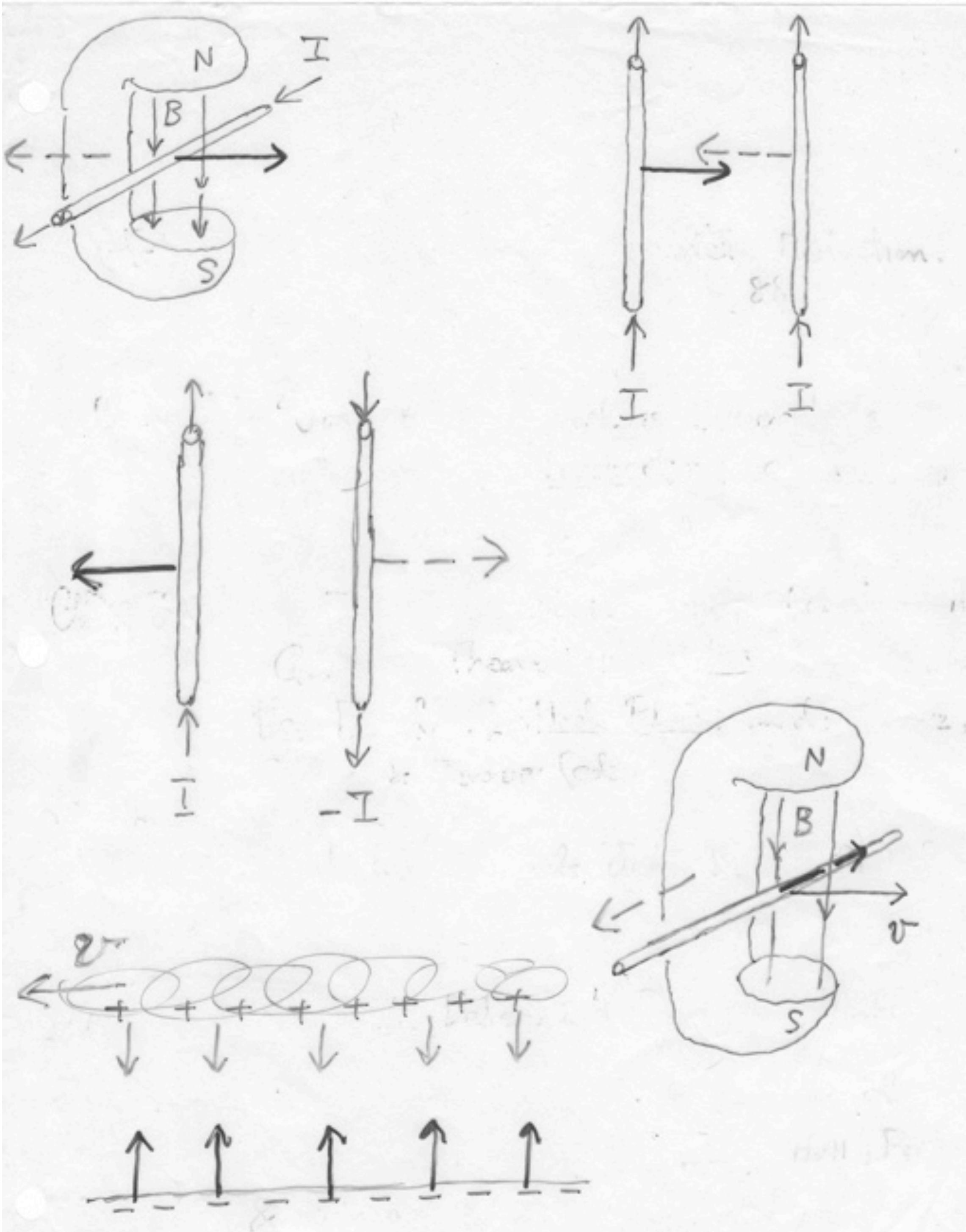


Fig. I. 3: Still more action – reaction force pairs.

Enough already!! Clearly we could go on and on. Forces *always* occur in equal and opposite pairs, each force in the pair exerted by one member of a pair of interacting physical systems and acting on the other member. Let's move on to the second rule for forces.

4. Our Second Rule for Forces

Our **Second Rule** concerns the effect of several forces being applied at the same point at the same time. The rule is,

SEVERAL FORCES APPLIED AT THE SAME POINT AT THE SAME TIME TO THE SAME SYSTEM ALWAYS HAVE THE SAME EFFECT AS IF ONLY ONE FORCE, CALLED THE **RESULTANT**, WAS APPLIED AT THAT POINT AT THAT TIME ON THAT SYSTEM.

This is terrific! Think of the simplification. No matter how many or what kind of forces are applied at a given point of a system at the same time, the effect is always the same as if only a single, equivalent force was applied! If we know how to determine this equivalent force, called the **resultant**, we're dealing with a one-force problem instead of a many-forces problem.

Notice the qualification that the forces in question must be applied to the same system. The action-reaction pairs of forces we were discussing above are frequently applied at the same point and time. But they are *always* applied to *different* systems and can never both be included in calculating the resultant force on either of *those* systems.

How are resultants determined?

Well, first we have to have a way of representing the quantitative aspects of forces. There are two quantitative aspects of forces. How strong are they? In what direction do they point?

In this regard forces are just like arrows. Of arrows we can ask; how long are they and in what direction do they point? Consequently we can represent forces by arrows. The length of the arrows will be proportional to the strength of the forces represented and the direction of the arrows will represent the direction of the forces.

Notice that other things besides forces can be represented by arrows. Changes in position for instance. The length of the arrow represents the *net* distance covered in the position change. The direction of the arrow represents the direction from the initial position to the final position. In the technical jargon, position changes are called **displacements**. We can ease our way into the concept of a resultant force by first considering how displacements combine.

Suppose someone made a sequence of several position changes, a sequence of displacements. There would be an arrow to represent each displacement in the sequence and an arrow to represent the *net* change in position resulting from the whole sequence, a *resultant* displacement arrow. That resultant arrow could be obtained from the arrows for the sequence by arranging the sequence arrows end to end and then connecting the beginning of the first arrow to the end of the last arrow. When we do this we notice that *the length and direction of the resultant arrow doesn't depend on the order in which we arrange the sequence arrows*. In other words, the resultant change in position doesn't depend on the *order* in which the position changes that comprise the sequence were made! It only depends on *what* those individual position changes were (See **Fig. I. 4**).

Well, forces combine in the same way! For a bunch of forces applied at the same point at the same time to the same system the arrow representing the resultant force is obtained by arranging the arrows representing the forces in the bunch end to end and then connecting the beginning of the first arrow to

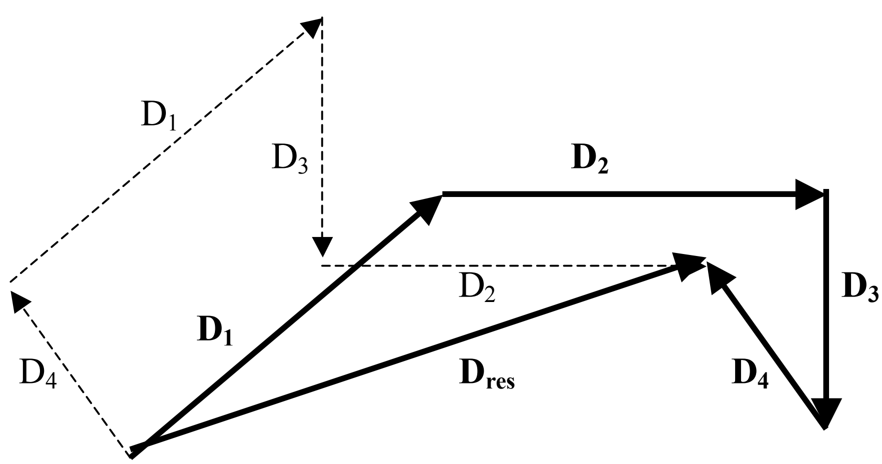


Fig. I. 4: Resultant of several displacements (or several forces). Note the independence of the resultant on the order of composition.

the end of the last arrow. And the order of the arrangement makes no difference! Furthermore, as we will later see, the resultant of several forces can be important even if they are not all applied at the same point.

5. Components

Having learned how to combine forces into resultants, we will close this first class with a discussion of how to decompose, or analyze a single force into **components**.

Since any collection of forces applied at the same point and time to a system are equivalent to applying the single resultant of that collection, it follows that any single force is, in turn, equivalent to applying a collection of forces (at the same point, time and system) provided that the collection has the single force as its resultant. This conclusion gives rise to the useful concept of a *component* of a force in a given direction.

Suppose a force is applied obliquely to an object lying on the floor. By 'obliquely' I mean the force is neither parallel to the floor nor perpendicular to the floor, but somewhere in between. If the force were perpendicular to the floor, and not so enormous as to cause the floor to collapse, the object will not move at all. If the force were parallel to the floor it would be maximally effective in moving the object. But when the force is oblique to the floor, how effective can it be in moving the object? We can answer this question by regarding the force as equivalent to the application of two forces, one perpendicular to the floor, the other parallel to the floor and such that the actual oblique force is the resultant of these two fictitious forces (**Fig. I. 5a**). It is then pretty intuitive to say that the perpendicular force makes no contribution to moving the object and the entire effect of the oblique force towards moving the object is represented by the parallel force (admittedly, the perpendicular force may undermine the effectiveness of the parallel force by pressing the object against the floor and, thereby, increasing the possible frictional resistance to motion. But only the parallel force contributes to the motion, not the perpendicular force).

These two fictitious forces, perpendicular and parallel to the floor, and having the oblique force as their resultant, are called the perpendicular and parallel components of the oblique force.

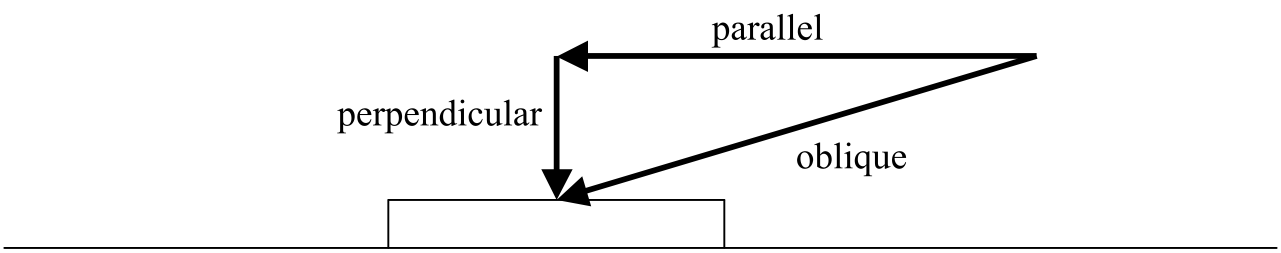


Fig. I. 5a: Components of oblique force applied to object on floor.

More generally, for any force, **F**, and any direction, D, the force, $\mathbf{F}_D$, is called the component of **F** in the direction D if, and only if (**Fig. I. 5b**),

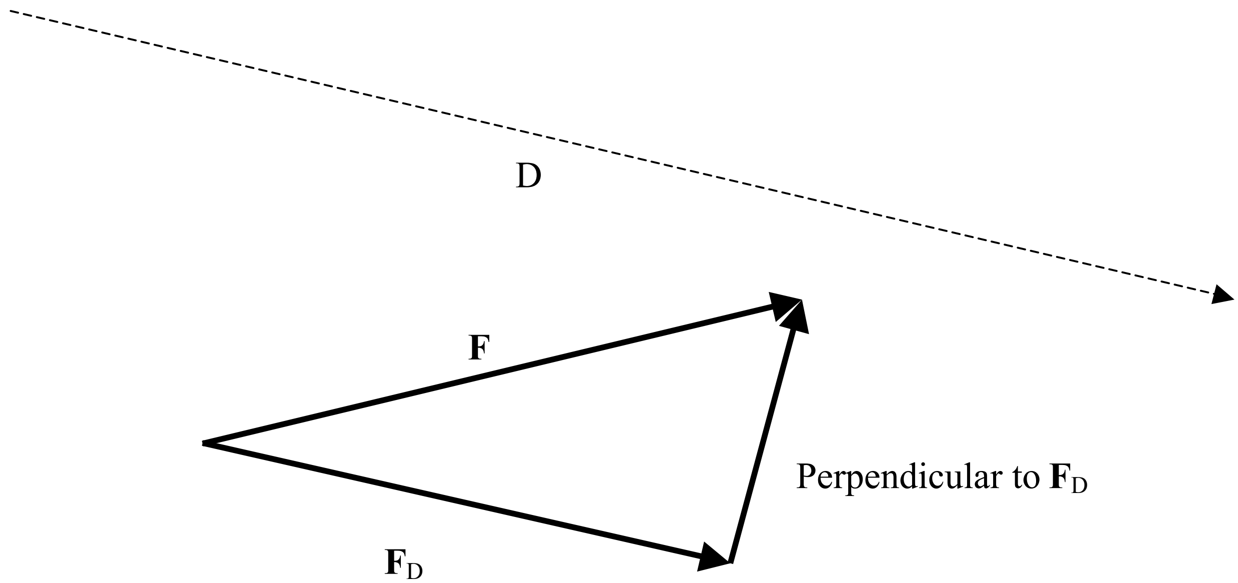
(1) $\mathbf{F}_D$ is parallel to the direction D and

(2) **F** is the resultant of $\mathbf{F}_D$ and another force perpendicular to $\mathbf{F}_D$.

A philosophical issue arises here: Since we have asserted that a collection of forces applied at one point, time and system is *completely* equivalent in their effect to applying the resultant of the collection, how can we ever know whether it is the collection or just the resultant that was actually applied? Does it make any difference? Should we regard the distinction as meaningful? In particular, was the oblique force applied to the object on the floor, or were its perpendicular and parallel components applied instead?

Being a philosophical issue, I leave you each to answer this one as your philosophical conscience and inclination to analysis dictates!

In the following classes we will discuss various kinds and examples of forces and their consequences. Our selections will comprise only a tiny fraction of the enormous variety we have to choose from. The purpose of our particular discussions is not to provide a representative survey of all the instances of forces in daily life. It is, instead, to provide a representative survey of the elementary concepts that have been found useful for developing an understanding of forces wherever and whenever we encounter them.



(notice that $\mathbf{F}_D$ may point in the *same* direction as D, as is the case here, or in the *opposite* direction, both cases being parallel to D)

Fig. I. 5b: Component, $\mathbf{F}_D$, of force, **F**, in the direction, D.

As with the concept of resultant, the concept of component is applicable to any quantity that has a magnitude and a direction and, therefore, can be represented by an arrow. Besides forces and displacements, which we've already mentioned, such quantities include velocities, accelerations, and momenta. The generic term for all such quantities and the arrows that represent them is **vectors**.

A last word about components: Consider all the directions perpendicular to a given direction, D. Any vector, **V**, or any force, **F**, will have a component in each of these directions perpendicular to D. But among all *those* components, there is one that has a special relation to the D direction component. This component is defined by the equation,

$$\mathbf{V}_{\perp D} := \mathbf{V} - \mathbf{V}_D \qquad \text{or} \qquad \mathbf{F}_{\perp D} := \mathbf{F} - \mathbf{F}_D$$

And is called *the* component perpendicular to the direction, D.

Appendix: Elements of the algebra of vectors

If two vectors of the same kind, i.e., two forces or two displacements or two velocities, etc., have the same direction but different magnitudes, then, by comparing their magnitudes quantitatively we can say that the arrows representing the vectors are numerical multiples of each other. Thus if the smaller vector is **V** and the larger has twice the magnitude of **V**, then we call the larger vector, 2**V**. Similarly for any numerical ratio between the magnitudes of vectors of the same kind having the same direction.

If two vectors of the same kind have the same magnitude but point in opposite directions and one of them is denoted by **V**, then the other can be denoted by – **V**, i.e., minus **V** or negative **V**. Accordingly, the vector, – 3/5 **V**, points in the opposite direction of **V** and has a magnitude that is 3/5 that of **V** (**Fig. I. A1**).

It follows from the independence of the order of composition of a resultant that the process of forming a resultant is, in some ways, similar to the process of adding numbers and it is advantageous to represent a resultant vector as the **addition** of the vectors that compose it. Thus if **W** is the resultant of **U** and **V**, we write, **W** = **U** + **V** = **V** + **U** (**Fig. I. A2**).

When these preceding ideas are combined we find that many of the rules for the algebra of numbers hold for vectors.

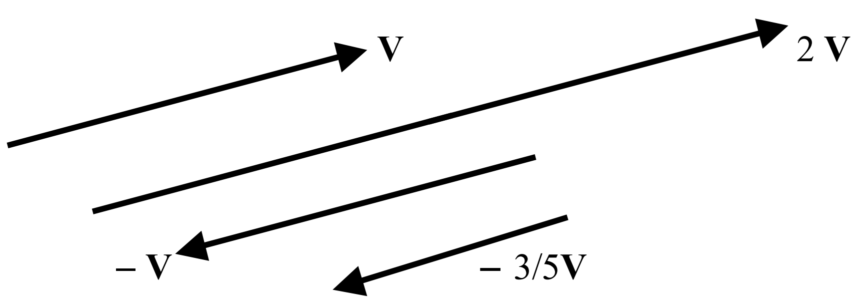


Fig. I. A1: A vector, **V**, and numerical multiples of **V**.

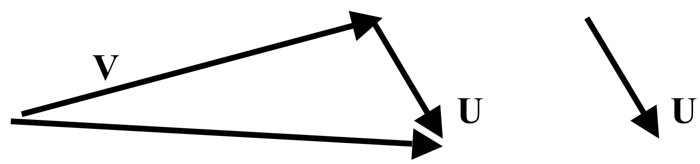


Fig. I. A2: Resultants and the addition of vectors.

(1) For any three ‘similar’ vectors (i.e., vectors of the same kind),

$$(\mathbf{U} + \mathbf{V}) + \mathbf{W} = \mathbf{U} + (\mathbf{V} + \mathbf{W}).$$

(2) For any number, λ, and two similar vectors,

$$\lambda(\mathbf{U} + \mathbf{V}) = \lambda\mathbf{U} + \lambda\mathbf{V}.$$

(3) For any two numbers, α and β, and any vector,

$$(\alpha + \beta)\mathbf{V} = \alpha\mathbf{V} + \beta\mathbf{V}, \quad \text{and} \quad (\alpha\beta)\mathbf{V} = \alpha(\beta\mathbf{V}).$$

It is also the case that any algebraic relationship between vectors holds for their components in any fixed direction. In other words if,

$$\lambda\mathbf{W} = \alpha\mathbf{U} + \beta\mathbf{V},$$

then for any direction, D, $\lambda\, \mathbf{W}_D = \alpha\, \mathbf{U}_D + \beta\, \mathbf{V}_D$.

Since all vector components in a given direction point either in that direction or opposite to it, we can introduce a so-called *algebraic component* for a direction, denoted by V_D, for a vector, $\mathbf{V}$, with component, $\mathbf{V}_D$, which is just the magnitude of $\mathbf{V}_D$, if $\mathbf{V}_D$ points in the direction, D, and is the negative of the magnitude of $\mathbf{V}_D$ if $\mathbf{V}_D$ points opposite to D. With such a definition the algebraic components of vectors, in a fixed direction, also satisfy the same algebraic relations as the vectors, i.e., referring to the previous equations,

$$\lambda W_D = \alpha U_D + \beta V_D$$

(**Fig. I. A3**).

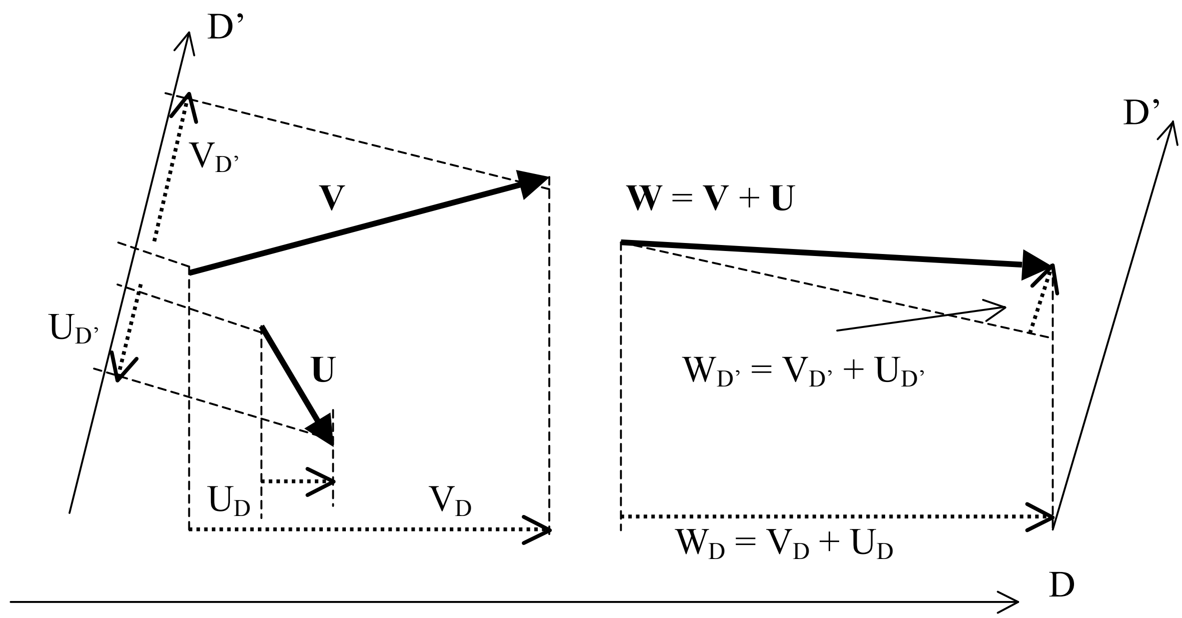


Fig. I. A3: Algebraic relations between vectors and their components in two directions. In this example all the algebraic components along directions D and D' are positive except $U_{D'}$, which is negative.