Report on the Dissertation

“On the Dynamics of some Complex Fluids”

by Francesco De Anna

The thesis presented by Francesco De Anna deals with the analysis of various models arising in the theory of complex fluids. More specifically, the main part of the thesis investigates three models of nematic liquid crystal flow, the Ericksen-Leslie, the Beris-Edwards as well as the Qian-Shen model, whereas the last chapter is dealing with the Boussinesq equations with temperature dependent viscosity. All the systems are studied within the framework of the simplified systems and within the setting of homogeneous Besov spaces on the whole space $\mathbb{R}^n$ for $n = 2$ or $n \geq 2$; associated boundary value problems are not considered. Models of these types are intensively and internationally being discussed since the pioneering work of Lin and Liu [77] in 1995, who introduced a simplification of the original Ericksen-Leslie model by considering a simplified Leslie stress tensor. These type of models are nowadays called simplified Ericksen-Leslie models.

The thesis presented is an excellent thesis regarding its mathematical deepness and clearness as well as style and representation of the results are concerned. Its main results generalize in particular various previous results due to Danchin, Mucha, Paicu and Zernescu on inhomogeneous fluid flow, see e.g., [99], [29], [30], to the case of nematic liquid crystal systems associated to inhomogeneous fluids.

The thesis starts after a short summary in Chapter 1 in Chapter 2 with models describing various phases of liquid crystals rangig from nematic over smectic to cholesteric. It gives the reader a very useful and valuable introduction into these models. Chapter 3 is devoted to the modeling of nematic liquid crystals based on balance laws for momentum, mass and angular momentum, hereby following the approach developed by Ericksen and Leslie and taking into account the Oseen-Frank energy. The content of this chapter is again very valuable to the reader.
since the original articles are quite difficult to read and to understand, at least for a mathematician. In order to describe the structure of so-called defects, the Q-tensor model is a very suitable model. For this reason, the author describes in the second half of Chapter 3 also a convincing derivation of the Beris-Edwards as well as of the Qian-Shen model.

Chapter 4 summarizes the main results of this thesis, which are then presented with proofs in the following Chapters 5-9. The Chapters 5-9 represent the beautiful core of this thesis.

Chapter 5 studies in detail the simplified version of the Ericksen-Leslie system in the whole space in the case of inhomogeneous fluids, i.e. for fluids with non constant density. F. De Anna aims for the particularly interesting situation of minimal smoothness assumptions on the data allowing then in particular discontinuities along an interface. The main result of this section, Theorem 4.1.1, asserts the existence of a global weak solution to the simplified Ericksen-Leslie system under the regularity assumption (5.3) and the smallness assumption (5.4) on the data in critical Besov spaces. Uniqueness holds under additional regularity assumptions on the data, which makes a Lagrangian description possible. The proof of this first main result is fairly involved and relies on an iteration scheme inspired by the one described by Danchin and Mucha in [30], dyadic decomposition and paradifferential calculus. Two cornerstones, which will be used later on again, are of central importance, the maximal $L^p - L^q$-regularity of the heat and Stokes equation, as well as the characterization of homogeneous Besov spaces in terms of the heat semigroup.

The Beris-Edwards system is investigated in Chapters 6 and 7 in two dimensions. Consider first the corotational case. Then again, existence and uniqueness of a global, weak solution is proved for (not necessarily small) data in appropriate Sobolev spaces, hereby generalizing a former result by Paicu and Zernescu [99]. The uniqueness proof is again very involved and makes use of paradifferential calculus, double logarithmic estimates and Osgood’s lemma. The uniqueness result is then carried over to the general Beris-Edwards system in Chapter 7.

Chapter 8 analyzes the Qian-Shen model in $\mathbb{R}^n$ for $n \geq 2$. The existence of a Lyapunov function is proved first and Theorem 4.3.2 then shows the existence and uniqueness of a classical solution to this system under suitable smallness assumptions on the data.

The following Chapter 9 shows the existence of a global, weak solution to the Boussinesq system in the case of temperature dependent viscosity. The case of constant viscosity has been considered earlier by Danchin and Paicu [29]. Here only the horizontal components are ass-
umed to satisfy a smallness condition.

The thesis presented is clearly structured and well written, in particular also with precise formulation of many details. One may thus read this thesis with great pleasure. The mathematical deepness and importance of the main results could be rated without any doubts as very good.

This thesis proves F. De Anna to be an excellent analyst, who is able to develop and modify deep methods in PDE and Harmonic Analysis in order to apply them to various very difficult models in the theory of complex fluids. It is hence not surprising at all that several results of this thesis are already accepted for publication in mathematical journals or will be submitted soon.

Summing up, the thesis presented fulfills without any doubts all criteria of an excellent thesis. It is hence a great pleasure for me to recommend the Department of Mathematics at the University of Bordeaux to accept this thesis without any restrictions.

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Matthias Hieber
Professor of Mathematics