Study of Pressure Estimation for a Human Circulatory System with a LVAD

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Abstract—Physiological control of left ventricular assist device (LVAD) can be related to the control of pressure. Based on the state space model of human circulatory system, the mean aortic pressure can be estimated. This work is to study the estimation error in the presence of variation of heart rate, systole vs. diastole ratio, left ventricular contractility, resistance of blood vessels, aortic capacitance, and so on. These variations are simulations of many scenarios that may occur during the working life of a long term LVAD. Studies demonstrated that the estimator can track the large variation of resistance, and the performance depends on the accuracy of the value for an artificial parameter and the peak to peak (p-p) value of LVAD flow rate. Simulation results show that the value of this parameter is varying in different scenarios. The result of this study can be utilized for the control of long term LVAD, or used for non-invasive evaluation of left ventricular function and properties of human circulatory system.

Keywords: Cardiovascular system, estimation, left ventricular assist device, pump head, modeling.

1. INTRODUCTION

A Rotary left ventricular assist device (LVAD) is a mechanical pump to restore the blood flow for end-stage congestive heart failure (CHF) patients due to predominant left ventricular failure. Currently, LVADs have been used successfully for short term (less than 2 years) as a bridge to heart transplant [2] with a constant pump speed. The permanent use of LVADs is expected to serve CHF patients as destination therapy or a bridge to recovery device. Scientists and researchers have been actively studying some key issues to extend the life of LVADs, such as optimizing pump design using computational fluid dynamics to increase efficiency and reduce hemolysis [20,21], suspension of LVAD impeller with magnetic forces [22-23], effect of LVAD on natural heart function and remodeling [24], and physiological control [4-19,25-26] that will be addressed in this paper.

During the long-term support of LVAD, patients may sleep, rest, or exercise slightly, like walking or climbing stairs. Besides these activities, the recipient’s left ventricle may get better or worse. In these conditions, the parameters of human circulatory system may change dramatically, so is the body need for blood. To match the body need, the pump speed of LVADs needs to be adjusted correspondingly. Therefore, one key issue to be addressed in order to achieve this long-term use of LVADs is the development of an effective and efficient (robust and adaptive) physiological control system. The objectives of physiological control can be further expressed as the requirement for mean aortic pressure [4], based on the fact that total peripheral flow (TPF) of human body and left ventricular end-diastolic pressure (LVEDP) are related to mean aortic pressure \( P_a \) [8], as \( P_a = TPR \times TPF \) and \( LVEDP \) depends on \( TPF \). Here \( TPR \) is total peripheral resistance of systemic circulation. The performance of control depends on how accurate the obtained aortic pressure information is. Since no long term pressure sensor is available yet, the pressure information has to be estimated from other signals. For a new magnetically elevated LVAD under development [27], the pump head is a long-term reliable signal that can be derived from the axial displacement of the impeller. The adaptive estimation of aortic pressure using this signal is incorporated in an adaptive control algorithm in [27]. In this paper, the accuracy of the estimation is studied in different physiological scenarios that may occur in long term LVAD support. The results of this study can be utilized by a permanent LVAD’s physiological controller, and be employed to detect the pathological states of the left ventricle and human circulatory system of a LVAD recipient.

The structure of this paper is organized as follows: the analytical results on modeling and estimation of human circulatory systems is discussed first, followed by the simulation and experiment study of estimation performance in a variety of physiological scenarios.

2. SIMPLIFIED SYSTEM MODEL

In this model, the system elements are: \( C_{LV} \), compliance of left ventricle; \( C_A \), compliance of systemic arteries; \( C_V \), compliance of systemic vein and right atrium; \( D_1 \), aortic valve; \( D_2 \), mitral valve; \( TPR \), total peripheral resistance; \( L \), blood inertia in peripheral circulation; \( R_{AV} \), resistance of aortic valve; \( R_V \), resistance of pulmonary vein and left atrium; \( \delta_m \), the pressure disturbance generated by muscle pump function in exercise; \( \delta_p \), pressure disturbance generated by pulmonary circulation; and the system signals are: \( P_{LV} \), left ventricular pressure; \( P_A \), aortic pressure; \( P_V \), central venous pressure; \( Q_{LV} \), inflow to left ventricle; \( Q_{14} \), flow rate from the vein to the left ventricle; \( Q \), LVAD flow rate; \( Q_8 \), flow rate
from the left ventricle to the aorta; $Q_A$, inflow to systemic arteries; $Q_v$, inflow to systemic vein and right atrium; TPF, total peripheral flow rate.

The LVAD is connected to left ventricle (pump inlet) and aorta (pump outlet). Therefore, from the LVAD’s point of view, only the hemodynamic response of left ventricle and aorta is pertinent. To develop an effective robust and adaptive estimation algorithm, we consider a simplified cardiovascular system model shown in Fig. 1. Pulmonary circulation is simplified as a pressure resource to help pushing blood back to the heart in dynamic exercise [3], is simplified as a pressure resource $\delta_P$, with its mean value usually less than 5 mmHg. Muscle pump function, the mechanism to help pushing blood back to the heart in dynamic exercise [3], is simplified as a pressure resource $\delta_m$, whose effect to increase central venous pressure is usually less than 5 mmHg.

For a CHF patient with LVAD, in diastole, $D_2$ is open and $D_1$ is closed. In systole, $D_2$ is closed and $D_1$ may be open depending on whether $P_{LV} > P_A$ is true or not. Taking into account the effect of $D_1$ and $D_2$, the dynamic equations of this system model [4] are

$$
\dot{x} = A^d x + B_1^d Q + B_2^d \delta_P + B_3^d \delta_m, \quad y = C x, \quad \text{in diastole} \tag{2.1}
$$

$$
\dot{x} = A^s x + B_1^s Q_s + B_2^s \delta_m + B_3^s \delta_{LV}, \quad y = C x, \quad \text{in systole} \tag{2.2}
$$

where $x = [x_1, x_2, x_3]^T$, $x_1 = P_{LV} - P_A$, $x_2 = P_v - P_A$, $x_3 = TPF$, $Q_s = Q + Q_8$, $y = \Delta P = P_A - P_v$.

$$
A^d = \begin{bmatrix}
-\frac{1}{C_v} & \frac{1}{C_v R_v} & \frac{1}{C_A} \\
-\frac{1}{C_v} & \frac{1}{C_v R_v} & \frac{1}{C_A + \frac{1}{C_v}}
\end{bmatrix},
B_1^d = \begin{bmatrix}
-\frac{1}{C_v} & -\frac{1}{C_A}
\end{bmatrix},
B_2^d = \begin{bmatrix}
\frac{1}{C_v R_v} & 0
\end{bmatrix},
B_3^d = \begin{bmatrix}
0 & 0
\end{bmatrix},
A^s = \begin{bmatrix}
0 & 0 & \frac{1}{C_A + \frac{1}{C_v}}
0 & 0 & -\frac{1}{C_v}
0 & -\frac{1}{L} & \frac{1}{L}
\end{bmatrix},
B_1^s = \begin{bmatrix}
0 & 0 & \frac{1}{C_A + \frac{1}{C_v}}
0 & 0 & -\frac{1}{C_v}
0 & -\frac{1}{L} & \frac{1}{L}
\end{bmatrix},
B_2^s = \begin{bmatrix}
0 & 0 & \frac{1}{R_v}
0 & 0 & 0
\end{bmatrix},
B_3^s = \begin{bmatrix}
0 & 0 & \frac{1}{R_v}
0 & 0 & \frac{1}{L}
\end{bmatrix},
$$

and $C = [-1, 0, 0]$, and $Q_8 = \Pi(\Delta x)$. The value of the unit step function $\Pi(\Delta x)$ is equal to 0 when $P_{LV} < P_A$ ($D_1$ is closed). This condition is true for an LVAD recipient in the majority of the heart cycle, thus $Q_8$ can be negleced [4].

In this model, $C_{LV}^d$ is a constant obtained by linearizing the exponential relation of pressure and volume in diastole, which shows strong linear relation over the working range of volume for a LVAD recipient [4]. $C_{LV}^s(t)$ is the varying capacitance of left ventricle in systole. This state space model converts the variation of left ventricular capacitance in systole to an extra input $\delta_{LV}$, whose value can be very large depending on the contractility of natural left ventricle. The output $y$ is the measured pump head of LVAD, that can be obtained from axial movement of the impeller of the magnetically levitated LVAD under development [4]. The control input $Q$, i.e. the LVAD flow rate, is assumed known accurately from the pump characteristic equation

$$
\frac{Q}{\omega} = a_2 \left( \frac{\Delta P}{\omega^2} \right)^2 + a_1 \frac{\Delta P}{\omega^2} + a_0 \tag{2.3}
$$

for the pump speed $\omega$ and some known constants $a_2$, $a_1$ and $a_0$.

The key parameter in (2.1)–(2.2) is TPR, which changes as the activity level changes. This parameter needs to be estimated for the design of a state observer for the system state $x$. The blood volume equation

$$
P_{LV} C_{LV} + P_A C_A + P_v C_v = V_s = \text{total stressed blood volume} \tag{2.4}
$$

could be used to calculate the aortic pressure $P_A$ if $x_2$ and $x_3$ were available (note that $x_1 = -y$ is measurable): $P_A = \frac{1}{C_v} \left[ \frac{1}{C_v} \Delta V + \frac{1}{C_A} V_s - \frac{1}{C_A + \frac{1}{C_v}} x_v \right] \equiv H x + N. \tag{2.5}

Stability analysis. For this system model, the system matrix switches from $A^d$ to $A^s$, when the system changes from diastole to systole. We have shown that the homogeneous part ($\dot{x} = A x$) of this system is Lyapunov stable at $x_0 = [0, 0, 0]^T$, by finding a common Lyapunov function: $V = x^T M x$ for a constant symmetric and positive definite matrix, such that $\dot{V} < 0$ for $\dot{x} = A^d x$ (diastole) and $\dot{V} \leq 0$ for $\dot{x} = A^s x$ (systole) [4] (it can be further shown to be asymptotically stable [4], and thus exponentially stable). The response of the system (2.1)–(2.2), in the steady state, is a stable oscillation.

Single-equation model. With the ensured stability, the system equation (2.1)–(2.2) can be approximated by a single-equation model

$$
\dot{x}' = A x' + B_1 Q_s + B_2' \delta_P + B_3' \delta_m, \quad y' = C x' \tag{2.6}
$$

$$
A = \begin{bmatrix}
-\frac{1}{C_v} & \frac{1}{C_v R_v} & \frac{1}{C_A} \\
-\frac{1}{C_v} & \frac{1}{C_v R_v} & \frac{1}{C_A + \frac{1}{C_v}}
\end{bmatrix},
B_1' = \begin{bmatrix}
0 & 0 & \frac{1}{C_A + \frac{1}{C_v}}
0 & 0 & -\frac{1}{C_v}
0 & -\frac{1}{L} & \frac{1}{L}
\end{bmatrix},
B_2' = \begin{bmatrix}
0 & 0 & \frac{1}{R_v}
0 & 0 & 0
\end{bmatrix},
$$

and $B_3 = B_3'$, where $R_v'$ is an equivalent venous resistance, an artificial parameter that can be calculated by measurement explained later. It is worth to point out that the disturbance caused by $\delta_{LV}$ in (2.2) does not show in (2.6).
In particular, the circulatory system (2.1)–(2.2) and the system (2.6) have the same solution at the end of systole, i.e. \( t = t_s \), supposing \( \delta_m \) remains constant in systole and diastole [4]. At \( t = t_s \), the output \( y' \) of (2.6) is equal to the output \( y \) of system (2.1)–(2.2), i.e. the measurable pump head of the LVAD: \( \Delta P = P_A - P_{LV} \), which achieve the maximum value at the same time in each heartbeat. The value of \( y' \) at other time instances is equal to \( \Delta P' \), which is obtained by passing \( (\Delta P)_{max} \), i.e. \( \Delta P|_{t=t_s} \), through a zero-hold function and a low-pass filter.

The validity of this approximation can be explained as follows. Systole and diastole are two opposing process. In diastole, state \( x_1 \) of human circulatory system tends to converge exponentially, while in systole, state \( x_1 \) tends to diverge towards the opposite direction because of the zero eigenvalues in matrix \( A' \) and the input \( \delta_{IV} \). The states of human circulatory system (2.1)–(2.2) thus exhibits periodic-like trajectory because of the alternation of systole and diastole. The bottom envelop of the periodic-like trajectory actually converge exponentially due to the proven exponential stability of the system (2.1)–(2.2) in previous paragraph, at a slower speed than the state trajectory in diastole, which is dominated solely by equation (2.1). Therefore system (2.6), which has the same structure as equation (2.1) but a different value for venous resistance, was proposed to match the bottom envelop. The equivalent venous resistance \( R'_V \) in system (2.6) is much bigger than the true venous resistance \( R_V \) in equation (2.1), and matches the slow converging speed of bottom envelop. The value of \( R'_V \) can be determined as

\[
R'_V = \frac{x_2 + \delta_P - x_1}{Q_s} = \frac{x_2 + \delta_P + \Delta P - x_1}{Q_s} \bigg|_{t=t_s}
\]

(2.7)

by the measurements in LVAD implantation surgery before switched to physiological control operation mode. (2.7) is derived from the steady state solution of (2.6) because the LVAD usually runs at constant speed during surgery. The response of human circulatory system thus follows the steady state solution of (2.6).

System (2.6) provides a linear model whose states track the bottom envelop of states of system (2.1)–(2.2). And in controlling LVAD, the mean aortic pressure, which is determined mostly by the bottom envelop of states of (2.1)–(2.2), is of more interest than the instantaneous aortic pressure determined by the instantaneous states of system (2.1)–(2.2). Based on this single-equation description of the circulatory system, a parameter estimator and a state observer can be developed. The estimated states can then be put in equation (2.5) to determine the mean aortic pressure.

**Modeling error and analysis.** With the value for \( R'_V \) in system (2.6) found accurately, the bottom envelop of states of system (2.1)–(2.2) can be approximated by the states of system (2.6). However, the assumption of this approximation, i.e. \( Q_s \) and \( \delta_m \) remain constant in systole and diastole, is only valid for non-systolic left ventricle in rest condition. For most left-sided heart failure patient, whose left ventricle may still exhibits some contractility, this assumption will be violated to some extent. With oscillating \( Q_s \), high in systole and low in diastole, the states of system (2.6) tend to be slightly larger than the states of system (2.1)–(2.2) at \( t = t_s \). Also, the perfectly matched value for \( R'_V \) may change with variation of left ventricular contractility, variation of systole vs. diastole ratio, and variation of heart rate, as will be demonstrated in Sections 4 and 5.

3. An Adaptive Estimator

In this section, we present the detailed results of our adaptive estimator whose design consists of two parts: an adaptive parameter estimation scheme to estimate the TPR and an adaptive state observer using the estimate of TPR to estimate aortic pressure.

**Parameter estimation.** From (2.6), it follows that

\[
(s^3 + a_2 s^2 + a_1 s + a_0)\gamma(t) = (b_2 s^2 + b_1 s + b_0)Q_s(s) + (b_2^p s^2 + b_1^p s + b_0^p)\delta_P(s) + (b_2^n s^2 + b_1^n s + b_0^n)\delta_m(s)
\]

(3.1)

for some parameters \( a_i \) and \( b_i \) (which depend on TPR), \( b_i^p \) and \( b_i^n \), \( i = 0, 1, 2 \). The nominal value of \( \delta_P \) for LVAD recipient can be measured in surgery, which is usually less than 5 mmHg. The variation of \( \delta_P \) in different activities is negligible. \( \delta_m \) is only effective in exercise, whose low frequency component is shown in simulation to be less than 5 mmHg [4]. A low-pass filter \( F(s) = \frac{1}{\alpha s} \) with \( \Lambda(s) = s^3 + 15 s^2 + 75 s + 125 \), can be used to remove high frequency components of \( \delta_m \). Also, low frequency component of \( \delta_m \) contributes less than 5% of \( Q_s \) and \( y' \), thus is ignored for the parameter estimation of TPR, with the use of such a filter.

Filtering both sides of (3.1) by \( F(s) \) and arranging the resulting terms, we can derive the parametric equation

\[
\theta^* \phi(s) = z(s)
\]

(3.2)

for \( \theta^* = TPR \) and some functions \( \phi(s) \) and \( z(s) \) whose denominators are \( \Lambda(s) = s^3 + 15 s^2 + 75 s + 125 \) and numerators are combinations of \( y'(s), Q_s(s) \) and \( \delta_P(s) \) (with \( \delta_m \) ignored).

In the time domain, we define the estimation error \( e(t) = \theta(t) - \theta^*(t) = z(t) \), where \( \theta(t) \) is the estimator of \( \theta^* \), and use the following adaptive law to update the estimate \( \theta(t) \):

\[
\dot{\theta}(t) = -\frac{\gamma \phi(t) \phi(t)}{1 + \phi^2(t)} \gamma > 0.
\]

(3.3)

In terms of the unknown parameter error \( \tilde{\theta}(t) = \theta(t) - \theta^* \), we can express this adaptive law as \( \dot{\theta}(t) = -\frac{\gamma \phi(t) \phi(t)}{1 + \phi^2(t)} \tilde{\theta}(t) \). In this case when \( \theta(t) \) is a scalar parameter, the error \( \tilde{\theta}(t) \) converges to zero, that is, \( \lim_{t \to \infty} \tilde{\theta}(t) = 0 \), if \( \int_0^\infty \frac{\gamma \phi(t) \phi(t)}{1 + \phi^2(t)} dt = \infty \) (it is satisfied if the signal \( \phi(t) \) does not vanish). In practice, due to modeling errors, parameter variations, and disturbances, this ideal property may only be met approximately. In our application, the adaptive law is to provide an on-line estimate of the parameter \( \theta^* \).

**State observer.** The state observer structure for the estimate \( \hat{x} \) of \( x = [x_1, x_2, x_3]^T \) in (2.1)–(2.2) is a standard one, based on the equivalent model (2.6) with all parameters but TPR known and \( \delta_m \) ignored. The adaptive estimate of TPR obtained on-line from the above parameter estimation
procedure is used, leading to an adaptive observer. The estimate of $P_A$ is given as $\hat{P_A} = H\hat{x} + N$ (see (2.5)).

4. SIMULATION STUDY

A computational model of an 11th-order human circulatory system [4] was employed to study the variation of $R'_V$ and estimation error of $TPR$ and $P_A$ in Matlab/Simulink. Different scenarios which mimic many possible variations of systemic parameters and pathological states of LVAD recipients are simulated to study the performance of adaptive estimator. These scenarios include: the variation of $TPR$, the variation of heart rate, the variation of systole vs. diastole ratio, the variation of contractility of left ventricle, and the variation of the aortic capacitance. The notation "−" and subscript "err" indicates the mean value and the estimation error of the studied variables respectively in the simulation. The estimation error is equal to the actual value of studied variable minus the estimated value of the studied variable.

The estimation error of $TPR$ is maintained less than 0.01 mmHg/(ml/sec) in this variation. The estimation error of $P_A$ is -0.39 mmHg at large $TPR$ (rest condition) and 0.26 mmHg at small $TPR$ (exercise condition). In this simulation, the LVAD flow $Q$ is set constant and the flow contribution of left ventricle is equal to 0 in rest, and less than 5% in exercise.

$R'_V$ vs. heart rate, systole/diastole ratio, and flow contribution of left ventricle. Fig. 3 illustrates the calculated $R'_V$ according to equation (2.7) using the simulation responses of human circulatory model at different heart rate and systole vs. diastole ratio. The flow contribution of natural left ventricle is maintained from 23% to 30% of the total flow $Q$, whose mean value ranges from 5.44 to 6.24 L/min. The LVAD flow rate $Q$ is set constant in each simulation. At the same heart rate, the flow contribution of natural left ventricle with larger systole vs. diastole ratio is slightly higher. As shown in Fig. 3, the value of $R'_V$ decreases when the heart rate increases, but increases with higher portion of systole, which also implies slightly higher flow contribution from the natural left ventricle.

Estimation error vs. $TPR$. To study the estimation performance when the activity level changes from rest to exercise, the effect of varying $TPR$ on estimation error is investigated. Fig. 2 illustrates the estimation of $TPR$ by adaptive estimator when $TPR$ drops approximately half. The first ten seconds is the transient period before the adaptive estimator converges.

Fig. 2. Simulation results of TPR estimation.

Fig. 3. Values of $R'_V$ with different systole vs. diastole ratios and heart rates in simulation study.

Fig. 4. Values of $R'_V$ in simulation study with different $Q_8/\bar{Q}_s$. ($Q$ is set constant in the simulation): $\bar{Q}_s$, the mean flow rate contributed by natural left ventricle; $\bar{Q}_c$, the mean total peripheral flow to the human body.

Fig. 5. Estimation errors of aortic pressure and TPR with different $Q_8/\bar{Q}_s$ in simulation study ($Q$ is set constant in the simulation): $\bar{Q}_s$, the mean flow rate contributed by natural left ventricle; $\bar{Q}_c$, the mean total peripheral flow to the human body; $P_A$ err: estimation error of mean aortic pressure; $TPR$ err: estimation error of TPR.
Fig. 4 illustrates the calculated $R'_s$ at different $\bar{Q}_s/\bar{Q}_t$. The heart rate is set at 70 bpm. The contractility of left ventricle was increased to get higher $\bar{Q}_s/\bar{Q}_t$. The value of $\bar{Q}_t$ was maintained around 5.45 L/min, with LVAD flow $Q$ set constant in each simulation. It is obvious that more flow contribution from the natural left ventricle $\bar{Q}_s/\bar{Q}_t$ implies higher value of $R'_s$.

For each simulation point in Figs 3 and 4, the LVAD flow is set constant, which may never really happen in LVAD operation. These simulations are done to study the effect of heart rate and systole vs diastole ratio on estimation error.

**Estimation error vs. $\bar{Q}_s/\bar{Q}_t$.** Figs. 5 and 6 illustrate the estimation errors of $TPR$ and $P_A$ at different $\bar{Q}_s/\bar{Q}_t$ with constant $Q$ and varying $Q$ respectively. The varying $Q$ is provided by an adaptive optimal controller [28], and exhibits oscillation because of the interaction between the controller and the heartbeat. In the simulation, left ventricular contractility was increased gradually. $\bar{Q}_s/\bar{Q}_t$ remains 0 for the first three points, while the estimation error gradually decreased. Since the fourth point, $\bar{Q}_s/\bar{Q}_t$ gradually increased. With constant $Q$ (Fig. 5), the numerical value of estimation error increases with the increase of $\bar{Q}_s/\bar{Q}_t$. However, With varying $Q$ (Fig. 6), the numerical value of estimation error decreases first, then increases with the increase of $\bar{Q}_s/\bar{Q}_t$. LVAD flow starts to turn negative in a portion of diastole when $\bar{Q}_s/\bar{Q}_t$ is larger than 16.5%.

![Fig. 6. Estimation errors of aortic pressure and TPR with different $\bar{Q}_s/\bar{Q}_t$ in simulation study ($Q$ is regulated by optimal adaptive controller): $P_{A,err}$: estimation error of mean aortic pressure; $TPR_{err}$: estimation error of TPR.](image)

Fig. 7 shows the estimation error of $TPR$ and $P_A$ with different aortic elastance with varying $Q$ regulated by the adaptive optimal controller [28]. Aortic elastance here is the reciprocal of aortic capacitance, which is a portion of the capacitance of systemic artery-$C_A$ in the state-space model (2-6). The increase of aortic elastance results in the increase of the peak to peak (p-p) value of $Q_t$ and $Q$. As shown in Fig. 7, the numerical value of $P_{A,err}$ and $TPR_{err}$ decreases with the increase of the p-p value of $Q_t$ and $Q$. And, the magnitude of $P_{A,err}$ and $TPR_{err}$ increases with increasing aortic capacitance beyond 0.5 ml/mmHg. The flow contribution of left ventricle in Fig. 7 ranges from 2.5% to 5.3%, and is higher with larger aortic elastance.

![Fig. 7. Estimation errors of aortic pressure and TPR with different aortic capacitance in simulation study ($Q$ is regulated by an optimal adaptive controller): $P_{A,err}$: estimation error of mean aortic pressure; $TPR_{err}$: estimation error of TPR.](image)

All these findings about the variation of $R'_s$ and estimation error can be explained by the effects of different factors on $(\Delta P)_{max}$. The increase of heart rate will decrease the stroke volume of left ventricle, e.g. higher left ventricular volume at the end of systole supposing nothing else changes. Therefore, $(\Delta P)_{max}$ will decrease, so is the value of $R'_s$ according to (2-7). Without the update of the value of $R'_s$, the numerical value of $P_{A,err}$ will increase. The effect of $\bar{Q}_s/\bar{Q}_t$, however, is opposite. With larger $\bar{Q}_s/\bar{Q}_t$ (e.g. stronger left ventricle), the values of $Q$ and $\bar{Q}_t$ in systole are higher, and their values in diastole are lower. As a result, the volume in left ventricle at the end of systole is smaller, so is the left ventricular
Fig. 8. Experimental results of aortic pressure estimation: $P_A$: aortic pressure; $\hat{P}_A$: estimated aortic pressure; $\text{TPR}_{\text{CAL}}$: calculated TPR according to $\frac{P_A-P_V}{T_P F}$; $\text{TPR}_{\text{EST}}$: estimated TPR by adaptive estimator; $T_P F$: total peripheral flow; $Q$: pump flow rate; $L_1$: the first element of the adaptive observer.
pressure. Therefore \((\Delta P)_{\text{max}}\) will be larger at the end of systole with the same mean value of \(Q_s\). This higher value of \((\Delta P)_{\text{max}}\) implies larger value of \(R'_V\) according to (2-7). Without the correction of \(R'_V\), the numerical value of \(\hat{P}_A\) will increase, it thus decreases the numerical value of \(P_{A,\text{err}}\). In the meanwhile, using the measured \((\Delta P)_{\text{max}}\), the states of system (2.6) underestimate the states of system (2.1)-(2.2) further with larger oscillation of \(Q_s\). As a result, the numerical value of \(P_{A,\text{err}}\) will increase. The effect of unmatched \(R'_V\) on estimation error is dominant at small oscillation of \(Q_s\), while the effect of state underestimation takes over at large oscillation of \(Q_s\).

5. EXPERIMENT STUDY

A hydraulic loop to mimic the human circulatory system was set up as an in vitro test rig for different versions of prototype LVADs [28]. This mock loop can simulate different normal or pathological states and activities of a cardiovascular system. A small centrifugal pump MY2 (Speck Pump Inc., Jacksonville, FL) was used in the place of an LVAD in the testing.

The values of \(R'_V\), \(V_s\), and \(\delta_P\) in (2.6) were derived in advance through an initial test with MY2 running at constant speed. In this preparation test, the mock circulatory loop was kept at one particular physiological state. This initial test is similar to the initial process during the surgery of LVAD implantation. During the experiment study, the values of \(R'_V\), \(V_s\), and \(\delta_P\) in the adaptive estimator were set constant. After the preparation test, the MY2 pump was controlled by a simple fixed control algorithm, which measured the aortic pressure directly using the pressure sensor and maintained the aortic pressure at roughly 95 mmHg. The element to simulate the peripheral resistance in the mock loop was adjusted to change the value for \(TPR\). The estimation performance of \(TPR\) and \(P_A\) was plotted in Fig. 8. \(L'_s\), the first element of the adaptive observer, exhibits a change at the presence of the variation of \(TPR\) showing the adaption of observer parameter.

As seen in Fig. 8, the LVAD flow \(Q\) was higher than the total peripheral flow \(TPF\). This difference is due to the flow leakage from the pump outlet to the left ventricle through the imperfect one-way valve between the left ventricle and the aorta tank. The average flow leakage is 0.62 L/min. Because of this flow leakage, the estimation of \(TPR\) was smaller than the \(TPR\). If correcting this flow leakage, the mean estimation error for \(TPR\) is 0.0162 mmHg/ml/sec. The estimated aortic pressure \(\hat{P}_A\) tracked the true aortic pressure \(P_A\) very well in the presence of \(TPR\) variations with an average estimation error of aortic pressure to be approximately 0.0121 mmHg. The value of \(P_A\) shows less oscillation than the actual \(P_A\). There is approximately one heartbeat delay in the \(TPR\) and \(P_A\) estimation because of the delay effect in the signal processing from the pump head \(\Delta P\) to \(\Delta P'\), the signal to the adaptive observer. The estimation error of the adaptive observer was further tested with different systole-diaastole ratio and heart rates, and shown in Fig. 9. In general, the estimation error of aortic pressure increases with the increase of heart rate and decreases with the increase of systole-diaastole ratio. This trend implied that the value of \(R'_V\) is smaller at higher heart rate, and larger with higher systole vs. diaastole ratio. This finding is consistent with Fig. 3, obtained from the simulation study in Section 4.

6. CONCLUSION REMARK

The design of an estimator for the mean aortic pressure of a human circulatory system is described in this paper. With the single-equation model (2.6) derived for the human circulatory system, the adaptive estimation has been applied in the estimator design. Computer simulation and mock circulatory loop test consistently show that the adaptive estimator was able to track large variation of \(TPR\). Meanwhile, the estimation performance can be influenced by other factors, such as the flow contribution ratio of the natural left ventricle, the \(p-p\) value of the LVAD flow, and the capacitance of aorta. If using a fixed value of \(R'_V\), the numerical value of estimation error was shown to increase at higher heart rate or at larger left ventricular contractility, but decrease at larger ratio of systole vs. diaastole. It is important to notice that the range of the studies in this paper is far beyond the possible physiological variations that may occur in LVAD recipient. The estimation error in normal LVAD operation, in general, is smaller than the results in Sections 4 and 5, and can be utilized for the physiological control of LVAD.

Another significant use of this work is: by comparing \(\hat{P}_A\) from adaptive observer with the mean aortic pressure that may be measured non-invasively by sphygmomanometer, the contractility of the left ventricle and capacitance of aorta may be derived, and used to evaluate the possibility of waning patient from LVAD support.

Future work includes the update of the value of \(R'_V\) in online estimation to minimize the estimation error. Also, the method to extract \(\Delta P\) from \(\Delta P\) need to be readdressed. The modeling error will be decreased and the disturbance rejection ability of an adaptive estimator will be improving using more data of \(\Delta P\) in diaastole, instead of only maximum value in each heartbeat. These two issues are currently under investigation by the authors.
REFERENCES


