IE 597 Nonlinear Networks and the Price of Anarchy

A. Heading as it would appear in the appropriate University Bulletin:

1. Abbreviation        IE
2. Number              597
3. Title               Nonlinear Networks and the Price of Anarchy
4. Abbreviated Title   Nonlin Nets and Anarchy
5. Credits             3
6. Description (20 words or less)  
Introduction to congestion games, including elements of non-cooperative game theory, equilibrium network flows, Braess paradox, and the price of anarchy.

7. Prerequisite(s)     
Concurrent     IE 521 or equivalent linear programming course

B. Course Outline:

1. brief outline of the course content:

   Nonlinear Networks and the Price of Anarchy is a graduate level course on congestion games and mechanism design that provides an introduction to: nonlinear network optimization and equilibrium models; non-cooperative Nash games on networks; network design, congestion pricing and Stackelberg games on networks; the Braess paradox; and the price of anarchy. The application domains emphasized are traffic networks and data networks.


2. A listing of the major topics to be covered with an approximate length of time allotted for their discussion.

   This course is designed to be taught in two 75 minute lectures each week in fall semester (15 weeks)

   I. Introduction (1 week)  
      i. What is network equilibrium?  
      ii. What is the price of anarchy? 
      iii. Importance to traffic networks   
      iv. Importance to data networks 

   II. Elements of graph theory (1 weeks) 
      i. basic definitions 
      ii. total unimodularity and integrality 
      iii. shortest path algorithms 
      iv. the network simplex 

   III. Elements of NLP (2 weeks) 
      i. geometric motivation of the Kuhn-Tucker conditions 
      ii. key results concerning convexity 
      iii. the notion of a descent algorithm
iv. network and near network programs
v. the notion of decomposition
vi. Lagrangian relaxation

IV. Nash Games and Network User Equilibria (3 weeks)
i. notion of a Nash game on a network
ii. impact of explicit path flow variables
iii. Wardrop’s first principle: user equilibrium
iv. variational inequality representation
v. sensitivity analysis of user equilibrium
vi. existence and uniqueness of user equilibria
vii. algorithms

V. The Price of Anarchy (3 weeks)
i. definition of the price of anarchy
ii. bounds on the price of anarchy
iii. other theoretical properties of anarchy

VI. Network Design (3 weeks)
i. network design as a mathematical program with equilibrium constraints
ii. numerical example of the Braess paradox
iii. solving the equilibrium network design problem
iv. gap function algorithms
v. the EDO heuristic
vi. algorithms based on sensitivity analysis

VII. Congestion Pricing and Mechanism Design (2 weeks)
i. efficient tolls
ii. bi-level tolling problems
iii. capacity auctions
iv. computation

3. a succinct stand-alone course description (400 words maximum) to be made available to students and faculty on the World Wide Web.

Nonlinear Networks and the Price of Anarchy is a graduate level course. It provides an introduction to the theory of congestion games, developed originally to describe flows on congested transport networks but recently embraced to model data networks. The course begins with an introduction to so-called system optimal network flow models that explicitly incorporate network congestion. The study of system optimal flows contains an introduction to nonlinear network optimization algorithms, including feasible direction, gradient projection, simplicial decomposition and affine scaling algorithms.

Following the consideration of system optimal flows, both atomic and non-atomic network equilibrium models in the form of non-cooperative Nash games are discussed in-depth. The price of anarchy is presented as the ratio of the cost of Nash equilibrium flows to the cost of system optimal flows within the network of interest. Various theoretical bounds on the price of anarchy are derived. Numerical experiments to determine the price of anarchy are also described.

The Braess paradox, wherein global congestion can increase when local capacity is added to a nonlinear network, is introduced and its relationship to the price of anarchy demonstrated. Discrete and continuous equilibrium network design models that eliminate
any possibility for the Braess paradox to arise are articulated. Each such design model is shown to be equivalent to a Stackelberg game, which is a type of mathematical program with equilibrium constraints (MPEC).

Mechanism design in the form of network congestion pricing to alleviate the effects of congestion is also considered and shown to have an MPEC structure as well. Algorithms for solving MPECs to ascertain efficient network topology/efficient tolling will be discussed in detail, including simulated annealing and other types of computational intelligence on the one hand; and duality, penalty, decomposition and other types of nonlinear programming algorithms on the other.

Students interested in taking this course should have completed a course in linear programming; a course in nonlinear programming is also recommended.

4. The name(s) of the faculty member(s) responsible for the development of the course.

Terry L. Friesz, Harold and Inge Marcus Chaired Professor of Industrial Engineering

C. Justification Statement:

1. Instructional, Educational, and Course Objectives:

Students will learn how to formulate descriptive models of traffic and data network flows in the presence of congestion as Nash games expressed as variational inequalities (VIs). These models will be used to derive theoretical bounds on the price of anarchy (the social costs of not achieving a truly cooperative or system optimal flow). Students will also learn how to formulate normative network design problems as Stackelberg games or so-called mathematical programs with equilibrium constraints (MPECs) to avoid the Braess paradox. Numerical techniques for solving VIs and MPECs will be discussed and illustrated.

2. Evaluation Methods:

The students’ grades will be determined by an equal weighting of the following:

- Notebook of solved problems
- Term paper/presentation
- Final exam