Review of Quantitative Finance and Accounting, 23: 299–312, 2004

Evaluation of Black-Scholes and GARCH Models Using Currency Call Options Data*

T. HARIKUMAR AND MARIA E. DE BOYRIE
Associate Professor, Department of Finance, MSC 3FIN, College of Business Administration & Economics, New Mexico State University, P.O. Box 30001, Las Cruces, NM 88003, USA
E-mail: sankaran@nmsu.edu
E-mail: deboyrie@nmsu.edu

SIMON J. PAK
Associate Professor, Penn State University, 30 East Swedesford Road, Malvern, PA 19355, USA
E-mail: sjp14@psu.edu

Abstract. This paper empirically examines the performance of Black-Scholes and Garch-M call option pricing models using call options data for British Pounds, Swiss Francs and Japanese Yen. The daily exchange rates exhibit an overwhelming presence of volatility clustering, suggesting that a richer model with ARCH/GARCH effects might have a better fit with actual prices. We perform dominant tests and calculate average percent mean squared errors of model prices. Our findings indicate that the Black-Scholes model outperforms the GARCH models. An implication of this result is that participants in the currency call options market do not seem to price volatility clusters in the underlying process.

Key words: GARCH, currency options, Black-Scholes

JEL Classification: G12, G13, G15

Option pricing has its origins in the seminal works of Black-Scholes (1973) and Merton (1973). Empirical testing of these models did not become possible until Feigner and Jacquillat (1979) first proposed a market for currency options. In December of 1982, American Currency Options began trading in the Philadelphia Stock Exchange (PHLX). Today, this exchange lists six dollar-based standardized currency option contracts, which settle in the actual physical currency. These are Australian dollar, British Pound, Canadian Dollar, Euro, Japanese Yen and Swiss Franc.

The Black and Scholes (1973) option-pricing model was the first to be used in pricing currency options; but, overtime and in practice, researchers have found that the prices estimated by the Black-Scholes model suffer from many biases. Duan (1995) mentions that the Black-Scholes model exhibits under pricing of out-of-the money options, under pricing of options on low volatility securities and under pricing of short-maturity options and results in a U-shaped implied volatility curve.

*We are grateful to Dr. Jin-Chuan Duan, Dr. Stewart Mayhew and the participants of the 2003 FMA International Meetings for valuable comments. We also thank Dr. Leigh Murray, New Mexico State University for statistical help and Dr. Jayashree Harikumar, Los Alamos National Laboratories, New Mexico for help with MATLAB.

In addressing these issues, some researchers have made refinements to the Black-Scholes model. Amin and Jarrow (1991) introduce a stochastic interest rate model in which the assumption of constant interest rates, both domestic and foreign, is relaxed. Hilliard, Madura and Tucker (1991) develop a currency option-pricing model under stochastic interest rates when interest rate parity holds. Their model assumes that domestic and foreign bond prices have local variances that depend only on time and not on other state variables such as the level of short-term interest rates. The authors test their option model and find that the stochastic interest rate model with domestic and foreign short term rates, driven by Arithmetic Brownian motion, exhibit greater pricing accuracy than the constant interest rate alternative. While modeling stochastic volatility, Heston and Nandi (2000) observe that it is impossible to exactly filter volatility from discrete observations of spot asset prices in a continuous time stochastic volatility model and therefore it is impossible to price an option solely on historic asset prices.[1]

Duan (1995) was the first to propose an option-pricing model based on the assumption that the exchange rate follows a GARCH process. Duan and Wei (1999) generalize the GARCH option model by allowing for stochastic volatility, unconditional leptokurtosis and a correlation between the lagged return and the conditional variance for both the exchange rate and the foreign stock price. The authors conclude that options on foreign currency can be valued by assuming a bivariate non-linear asymmetric GARCH (1, 1) model. However, they do not perform a comprehensive empirical study of foreign exchange return behavior. We draw our motivation from their study and empirically compare the performance of Black-Scholes and GARCH option pricing models in estimating call option prices on British Pounds, Swiss Francs and Japanese Yen. Our simulations indicate that the Black-Scholes model performs better than the GARCH models when pricing currency call options. We use the Kolmogorov-Smirnov test to compare error distributions between models and the average percent mean-squared error to compute the model errors.

Currency data

Daily exchange rates were obtained from the Wharton Research Database Services foreign currency (WRDS FX) database, which is based on the Federal Reserve Board's H.10 release. Exchange rates are recorded as currency units per U.S. dollar. The in-sample period is from January 5, 1987 through December 29, 1995 (2,289 observations per currency) for the British Pound, Japanese Yen and the Swiss Franc. The summary statistics for the continuously compounded spot rate returns for each of the currencies studied can be found in Table 1. The results show that the mean for each currency during the period studied is negative with a standard deviation between 0.0066 and 0.0078. This implies that during 1987 and 1995, the British Pound, the Swiss Franc and the Japanese Yen appreciated against the U.S. Dollar.

Data on call options were obtained from the Philadelphia Stock Exchange (PHLX) currency options database. This database lists every currency option trade from December 1982 to the present. Each recorded transaction has the time (up to the second) and date of the transaction, the number of contracts traded, and the underlying spot exchange rate.

Table 1. This table presents the summary statistics for the continuously compounded daily spot rate returns, $\ln(S_{t+1}/S_t)$, for British Pounds, Swiss Francs and Japanese Yen. Our data spans the period January 5, 1987 to December 29, 1995

Summary measure	British Pounds	Swiss Francs	Japanese Yen
Mean	−0.00002	−0.00015	−0.00019
Median	−0.0003	0.0000	−0.00020
Mode	0.0000	0.0000	0.0000
Standard deviation	0.0066	0.0078	0.0067
Min	−0.0289	−0.0390	−0.0339
Max	0.0330	0.0311	0.0337
Number of observations	2289	2289	2289

The daily interest rates used for risk-free rates were the British Bankers Association (BBA) settlement rates for each currency with maturities ranging from 1 month to 12 months. To perform out-of-sample tests, we use options data beginning January of 1996.

The model

The dynamics for the spot exchange rate

The valuation of currency options depends on the dynamics of the underlying spot exchange rate, the risk-free rate and the terms of the option contract. We use the Black-Scholes option-pricing model as our benchmark model. The Black-Scholes model assumes that the underlying spot exchange rate, S_t, follows a Geometric Brownian motion given as,

$$\frac{dS}{S} = \mu dt + \sigma dz \tag{1}$$

where μ denotes the instantaneous percentage rate of change in the spot rate, σ denotes the instantaneous standard deviation in the percentage rate of change in the spot rate and the term $dz \sim N(0, 1)$, follows a Weiner process.

The accuracy of the Black-Scholes option-pricing model depends critically on whether the assumptions underlying the model are faithful to the real data. There have been many extensions to the Black-Scholes model and we do not attempt to incorporate all these advances in our paper.[2] Instead, we limit our study to the stochastic process for currency exchange rates and statistically determine whether the data conforms to the diffusion process assumed in the Black-Scholes option-pricing model. One important feature of Eq. (1) is that the spot rate dynamics do not reflect any serial dependence. However, many studies examining financial time-series data have found presence of volatility clustering and consequently statistically significant serial dependence. Johnston and Scott (1999), Hsieh (1988), Kugler and Lenz (1990), McCurdy and Morgan (1988) and Taylor (1986) support an autoregressive and conditional heteroskedasticity (ARCH) and a generalized ARCH (GARCH) type process. The study by Fujihara and Park (1990) find that three out of the five currencies they study support the ARCH model. Duan (1995) and Heston and Nandi (2000) have developed a closed-form GARCH option valuation model.[3]

We conducted the Portmenteau Q-test and the Lagrange multiplier LM-test for ARCH disturbances using the discrete version of Eq. (1) given as,

$$\ln\left(\frac{S_{t+1}}{S_t}\right) = \mu + \varepsilon_{t+1} \tag{2}$$

where S_t is the exchange rate at time t, μ is the mean and $\varepsilon_t \sim N(0, \sigma_t^2)$. Note, that the empirical version in Eq. (2) indexes the volatility measure by time, thereby allowing the test to determine if volatility is constant over time. The ARCH test indicates if the volatility structure assumed in Eq. (1) needs to change.

Table 2 presents results from the ARCH test for British Pounds, Swiss Francs and Japanese Yen. The results indicate a statistically significant presence of ARCH disturbances in the daily data we used in our sample period (i.e., January 1987 to December 1995). The Q-test and the LM-test report statistical significance for each of the 12 days prior to the day of the observation. Each of the currencies we studied exhibit statistical significance at the 1% level for each of the 12 lags. Collectively, these tests suggests that the exchange rate process in Eq. (1) is not representative of the data and consequently the Black-Scholes option pricing model is likely to be somewhat inadequate.[4]

With the results from previous studies and the overwhelming evidence of volatility clusters in our sample, we use an ARCH/GARCH process given by a GARCH-M process. The process, similar to that of Duan (1995), is given as,

$$\ln\left(\frac{S_{t+1}}{S_t}\right) = \gamma(ird_{t+1}) + r_{t+1} + \lambda\sqrt{\sigma_{t+1}^2} + \varepsilon_{t+1} \tag{3}$$

where,

$\varepsilon_{t+1} \sim N(0, \sigma_{t+1}^2)$,
ird = 3-month foreign LIBOR minus 3-month domestic LIBOR.
r_{t+1} = domestic, annualized risk-free rate.

$$\sigma_t^2 = \text{ARCH0} + \sum_{j=1}^{q} \text{ARCH}(j)\varepsilon_{t-j}^2 + \sum_{i=1}^{p} \text{GARCH}(i)\sigma_{t-i}^2$$

The results are reported in Table 3.

Consistent with the previous studies, the coefficients of the GARCH (1,1) and GARCH (3, 3) processes are statistically significant at the 1% level for each currency in our study.

Table 2. This table presents the results from the ARCHTEST procedure, used to determine whether the continuously compounded return series of each of the optioned currencies contain ARCH/GARCH effects.

Panel A: Q-Test for ARCH disturbances

Lag	British Pound		Swiss Francs		Japanese Yen	
(days)	Q	p-value	Q	p-value	Q	p-value
1	41.17	(<0.0001)	41.38	(<0.0001)	18.69	(<0.0001)
2	74.41	(<0.0001)	47.44	(<0.0001)	27.70	(<0.0001)
3	116.46	(<0.0001)	58.90	(<0.0001)	33.08	(<0.0001)
4	125.27	(<0.0001)	61.76	(<0.0001)	35.73	(<0.0001)
5	164.29	(<0.0001)	75.45	(<0.0001)	55.92	(<0.0001)
6	174.47	(<0.0001)	87.71	(<0.0001)	72.89	(<0.0001)
7	184.84	(<0.0001)	113.48	(<0.0001)	76.54	(<0.0001)
8	192.15	(<0.0001)	125.35	(<0.0001)	82.58	(<0.0001)
9	213.97	(<0.0001)	127.90	(<0.0001)	89.13	(<0.0001)
10	241.80	(<0.0001)	139.06	(<0.0001)	91.91	(<0.0001)
11	277.80	(<0.0001)	158.13	(<0.0001)	106.44	(<0.0001)
12	291.40	(<0.0001)	175.28	(<0.0001)	107.94	(<0.0001)

Panel B: LM-Test for ARCH disturbances

Lag	British Pound		Swiss Francs		Japanese Yen	
(days)	LM	p-value	LM	p-value	LM	p-value
1	40.97	(<0.0001)	41.23	(<0.0001)	18.65	(<0.0001)
2	65.43	(<0.0001)	43.79	(<0.0001)	25.52	(<0.0001)
3	92.39	(<0.0001)	52.01	(<0.0001)	28.90	(<0.0001)
4	93.50	(<0.0001)	52.67	(<0.0001)	30.17	(<0.0001)
5	115.44	(<0.0001)	62.77	(<0.0001)	46.66	(<0.0001)
6	116.39	(<0.0001)	68.38	(<0.0001)	57.02	(<0.0001)
7	118.31	(<0.0001)	84.57	(<0.0001)	57.65	(<0.0001)
8	118.68	(<0.0001)	87.51	(<0.0001)	59.94	(<0.0001)
9	129.19	(<0.0001)	87.56	(<0.0001)	62.71	(<0.0001)
10	138.99	(<0.0001)	92.24	(<0.0001)	63.01	(<0.0001)
11	153.44	(<0.0001)	100.78	(<0.0001)	70.18	(<0.0001)
12	154.15	(<0.0001)	106.23	(<0.0001)	70.19	(<0.0001)

Panel A presents the Q-test with 12 days lags and p-values. Panel B presents the LM test with 12 days lags and p-values. The equation used for the SAS AUTOREG procedure is: $\ln(S_{t+1}/S_t) = \mu + \varepsilon_{t+1}$, where S_t is the spot rate at time t, μ is the intercept and $\varepsilon_t \sim N(0, \sigma_t^2)$. Sample consists of daily spot rate data from 1/5/87 to 12/29/95.

We also find that the unit risk-premium (λ) is significant at the 5% level in the case of British Pounds but not in the case of the other two currencies.

The simulation model for the spot rate dynamics

The equation given in (3) is first converted to satisfy the Local Risk Neutral Valuation Relationship (LRVNR) proposed by Duan (1995). The equivalent process under LRVNR

Table 3. This table presents the results from the GARCH-M.

	British Pounds		Swiss Francs		Japanese Yen	
	G(1,1)	G(3, 3)	G(1,1)	G(3, 3)	G(1,1)	G(3, 3)
Intercept	−0.0016*	−0.0019*	−0.0019**	−0.0018*	−0.0008	−0.0002
γ	0.0050	0.0063	2.6569**	2.7768**	4.6438*	4.6402**
ARCH0	7.70E-7*	2.68E-6*	2.26E-6*	7.55E-6*	1.90E-6*	6.98E-6*
ARCH1	0.0550*	0.0517*	0.0593*	0.0760*	0.0636*	0.0808*
ARCH2	–	0.0774*	–	0.0643*	–	0.0801*
ARCH3	–	0.0539*	–	0.0667*	–	0.0753*
GARCH1	0.9277*	−0.6308*	0.9030*	−0.6014*	0.8941*	−0.5926*
GARCH2	–	0.4619*	–	0.5882*	–	0.4215*
GARCH3	–	0.9246*	–	0.6818*	–	0.7831*
λ	0.2018**	0.2396**	0.2264	0.2121	0.0247	−0.0535
N	2289	2289	2289	2289	2289	2289

The process is given as: $\ln S_{t+1}\gamma(ird)+r_{t+1}+\lambda\sqrt{\sigma^2_{t+1}}+\varepsilon_{t+1}$, where S_t is the spot rate at time t, r is the domestic, annualized risk-free rate and $\varepsilon_{t+1} \sim N(0, \sigma^2_{t+1})$. We analyze only two lag structures namely, (1, 1) and (3, 3). The variance for a GARCH (p, q) process is given as, $\sigma_t^2 = ARCH0 + \sum_{j=1}^{q} ARCH(j)\varepsilon^2_{t-j} + \sum_{i=1}^{p} GARCH(i)\sigma^2_{t-i}$. (* and ** denote 1% and 5% significance, respectively.)

is given as,

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = ird_t + r_t - \frac{\sigma_t^2}{2} + \xi_t \tag{4}$$

where,

$$\xi_t \mid \phi_{t-1} \sim N\left(0, \sigma_t^2\right)$$

and

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i\left(\xi_{t-i} - ird_t - \lambda\sqrt{\sigma^2_{t-i}} - 0.5\sigma^2_{t-i}\right)^2 + \sum_{i=1}^{p} \beta_i \sigma^2_{t-i}$$

Using the above set of equations, we employ the Monte-Carlo simulation method to generate the various paths for the exchange rate evolution under GARCH (1, 1) and GARCH (3, 3) and compute option payoffs for each path and consequently the option prices. Since, traditional simulation does not have the martingale property needed for the risk-neutral valuation of options, we employ the empirical martingale simulation proposed by Duan and Simonato (1998) and make the adjustment to the traditional simulation (see Appendix A).

The main purpose for simulating the underlying exchange rate is to be able to generate a distribution of spot prices for the terminal date of the option. These prices along with the risk-free rate, exercise price and the time to maturity are then used in valuing the option on the current date. The simulation of the underlying process involves estimation of the conditional volatilities on a daily basis and the corresponding estimates of daily exchange rates. Initially, we create a matrix of standard normal random numbers with x rows and y columns where $rn\ (x, y)$ denotes an element. We arbitrarily fixed the number of paths $y = 1000$.[5] For each of the 1000 iterations, we make sure that each time step corresponds to one day and the number of time steps denoted by x is equal to the number of days remaining until the option expires. [6]

We construct the volatility matrix where each element is denoted as $\sigma\ (x, y)$. Initially, we set the elements of the first three rows of the matrix equal to the historical daily volatility estimated from the currency data set. For the remaining time steps, we use the procedure given in Eq. (4) above. The coefficients α_0, α_j (for $j = 1, 2, 3$), and β_i (for $i = 1, 2, 3$) are the ARCH and GARCH estimates made for each currency using Eq. (3). Note that the estimate of ξ_t in Eq. (4) is computed as $\sigma_t^2\ rn\ (x, y)$ where the value of $rn(x, y)$ is drawn from the random number matrix.

The next step in this simulation uses the information in Eq. (4) and computes the simulated exchange rates for each path. In particular,

$$s(x, y) = s(x-1, y) \exp\left\{\gamma(ird) + \frac{r_f}{365} - \frac{1}{2}\sigma^2(x, y) + \sigma(x, y) rn(x, y)\right\} \tag{5}$$

where, $s(x, y)$ denotes the simulated spot rate for the xth time step in the yth path, r_f is the domestic, annualized risk-free rate, $\sigma\ (x, y)$ is estimate of the conditional volatility and $rn(x, y)$ is the current innovation from the random number matrix generated earlier. The values of $s(x, y)$ are then subject to the adjustment proposed in Duan and Simonato (1998) for a description to yield simulated spot rates having the martingale property denoted as $sa\ (x, y)$. At this point we have three ($T \times 1000$) matrices, one each for rn, σ, and sa. We construct a new set of matrices for each observation in each currency sample.

Simulation model for pricing european options

We compare the performance of the Black-Scholes model with the GARCH (1, 1) and GARCH (3, 3) models in pricing European options on British Pounds, Swiss Francs and Japanese Yen. The inputs to the closed-form Black-Scholes model are the current spot exchange rate (S_t), the exercise price (X), the risk-free rate (r_f), the time to maturity (T) and the current volatility (σ). Instead of simulating Black-Scholes option prices, we use the closed-form solution as given below,

$$C_{\mathrm{BS}} = S_t N(d_1) - X e^{-r_f T} N(d_2) \tag{6}$$

where, C_{BS} denotes the estimated European call price using the Black-Scholes model,$N(\cdot)$ denotes the cumulative normal distribution, $d_1 = (\ln(S_t/X) + (r_f + \sigma^2/2)T)/\sigma\sqrt{T}$ and $d_2 = 1 - \sigma\sqrt{T}$.

In the case of GARCH option pricing models, we chose to estimate option prices using the empirical martingale simulation technique.[7] For the GARCH (1, 1) process, we set α_2,

α_3, β_2 and β_3 in Eq. (4) to zero. The resulting volatility matrix is used as an input to Eq. (5) to simulate a GARCH (1, 1) spot exchange rate process. To calculate the option price, we compute the average terminal payoffs across all 1000 paths and discount the average at the risk-free rate for the duration of the time to maturity. More specifically,

$$C_{G11} = \frac{1}{1000} \sum_{y=1}^{1000} e^{-r_f T} \mathrm{Max}[sa(T, y) - X, 0] \tag{7}$$

where, X, r_f and T are as defined above, $sa(T, y)$ denotes the spot exchange rate at the last time step (Terminal date), y denotes paths, C_{G11} denotes estimate of a European call options when the underlying exchange rate follows a GARCH (1, 1) process. We apply the above procedure to estimate call option values under a GARCH (3, 3) process.

Options data and the GARCH parameters

We obtained the intra-day data on European options traded in PHLX on our three currencies (i.e., British Pounds, Swiss Francs and Japanese Yen) for the calendar year 1996. The GARCH estimates were made using daily currency exchange rates from October 1987 to December 1995. In order to use current estimates for GARCH and historical volatility, we consider only the January and February 1996 options transactions. Thus, we test the parameters in an out-of-sample setting.[8] The volatility estimates used in our study is the daily estimate in Table 1 times $\sqrt{252}$. For example, the annualized volatility for British Pounds is $0.006653\sqrt{252} = 0.1048$ (0.1238 for Swiss Francs and 0.1064 for Japanese Yen). The above volatility estimate is used as an input for the Black-Scholes model in valuing the January and February 1996 transactions of options on British Pounds.[9]

Performance measures and results

We conducted the simulation for call options on the three currencies under study. Our focus is the analysis of errors, defined as the market price of the option *minus* the price estimated by a model. Thus, for each of the currencies in our sample, we have a column of errors each for the Black-Scholes, the GARCH (1, 1) and the GARCH (3, 3) models. We analyze these errors by making pair-wise comparisons of the *absolute* error distributions and by examining the average percent mean-squared error.[10] We focus on the magnitude of the errors as under or over valuation will not throw additional light on relative performance of models.

Performance measures

Kolmogorov-Smirnov test of two samples: Let $F(z)$ denote the cumulative distribution of absolute errors generated by model z, where z denotes Black-Scholes (denoted as BS), GARCH (1, 1) (denoted as G11) or GARCH (3, 3) (denoted as G33) for the three models we use. The Kolmogorov-Smirnov test of two samples tells us whether the two samples are from the same population (KS-2-tail test). The KS-1-tail test tells us which of the two distributions stochastically dominates the other. As an example, we specify the test comparing the pricing errors from Black-Scholes and GARCH (1, 1) models for options on British Pounds.

KS-2-tail:	H0: $F(\text{BS}) = F(\text{G11})$;	KS-1-tail:	H0: $F(\text{BS}) = F(\text{G11})$
	H1: $F(\text{BS}) \neq F(\text{G11})$;		H1: $F(\text{BS}) > F(\text{G11})$

In the context of our study, for example, if KS-1-tail rejects H0, it implies that the Black-Scholes model has a higher valued cumulative distribution, consequently reflecting *smaller* magnitude of absolute error relative to the G11 errors. In this case, we would then conclude that the Black-Scholes model is a superior model to the GARCH (1, 1) model. The same logic is applied to each pair-wise comparison of the option pricing models.

Average percent mean-squared errors: The average percent mean-squared errors, denoted as $AMSE(z)$, is given as

$$AMSE(z) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{C_i - C_i(z)}{C_i}\right)^2 \tag{8}$$

Table 4. British Pounds. This table presents results from the analysis of pricing errors in the *British Pound* option prices estimated using the Black-Scholes model (benchmark) and the option prices estimated using an empirical martingale simulation conditional on a GARCH (1,1) underlying spot process

Kolmogorov-Smirnov Test (KS)

Models A & B	KS-2 tail	KS-1 tail	Better model
BS & G (1, 1)	Reject H0	Reject H0	Black-Scholes
BS & G (3, 3)	Reject H0	Reject H0	Black-Scholes
G (1, 1) & G (3, 3)	Reject H1	Reject H1	Cannot determine

Average percent mean squared error (AMSE)

Models A & B	MSE [A]	MSE [B]	Better model
BS & G (1, 1)	2.1262	12.1723	Black-Scholes
BS & G (3, 3)	2.1262	13.4758	Black-Scholes
G (1, 1) & G (3, 3)	12.1723	13.4758	GARCH (1, 1)

Kolmogorov-Smirnov test (KS): Let F (M) denote the cumulative distribution of the absolute pricing errors from model M. The Kolmogorov-Smirnov 2-tail and 1-tail test is stated as:
KS-2-tail: H0: F (BS) = F (G11); H1: F (BS) ≠ F(G11).
KS-1-tail: H0: F (BS) = F (G11); H1: F (BS) > F(G11)
Average percent mean squared error (AMSE): The AMSE for model M, denoted as AMSE (M) is computed as the average of (Market Price of Option − Model M Option Price)^2/(Market Price of Option)^2.

where, z denotes BS, $G11$ or $G33$ for the three models we use, C_i denotes the market price of the option, $C_i(z)$ denotes the option price estimated by model z, and n denotes the total number of observations. The *AMSE* has the advantage that a \$1 error on a \$50 option carries less weight than a \$1 error on a \$5 option.

Results

The upper panel of Table 4 presents the results from K-S tests for *British Pounds*. Based on the KS-2 tail test, we find that the error distribution for the Black-Scholes model is different from the error distributions for the GARCH models (at the 1% significance level). The one-tailed K-S test shows that the cumulative distribution of (absolute) errors generated by the Black-Scholes model is greater than the corresponding distribution for GARCH models. This indicates that the Black-Scholes model generates smaller errors than the GARCH models. There is no discernable difference in performance between the two GARCH models.

The lower panel of Table 4 presents results from the average percent mean squared errors. These results characterize the K-S tests with a measure of central tendency. The Black-Scholes model clearly outperforms the GARCH models with lower average percent mean square errors. The errors generated by the GARCH (1, 1) model are marginally lower compared to those generated by GARCH (3, 3).

Table 5 presents the results from K-S tests and the average percent mean squared errors for *Swiss Francs*. The 1-tailed and 2-tailed KS tests strongly indicate Black-Scholes model

Table 5. Swiss Francs. This table presents results from the analysis of pricing errors in the *Swiss Francs* option prices estimated using the Black-Scholes model (benchmark) and the option prices estimated using an empirical martingale simulation conditional on a GARCH (1, 1) underlying spot process.

Kolmogorov-Smirnov test (KS)

Models A & B	KS-2 tail	KS-1 tail	Better model
BS & G (1, 1)	Reject H0	Reject H0	Black-Scholes
BS & G (3, 3)	Reject H0	Reject H0	Black-Scholes
G (1, 1) & G (3, 3)	Reject H1	Reject H1	Cannot determine

Average percent mean squared error (AMSE)

Models A & B	MSE [A]	MSE [B]	Better model
BS & G (1, 1)	0.0652	0.6859	Black-Scholes
BS & G (3, 3)	0.0652	0.6136	Black-Scholes
G (1, 1) & G (3, 3)	0.6859	0.6136	GARCH (3, 3)

Kolmogorov-Smirnov test (KS): Let F (M) denote the cumulative distribution of the absolute pricing errors from model M. The Kolmogorov-Smirnov 2-tail and 1-tail test is stated as:
KS-2-tail: H0: F (BS) = F (G11); H1: F (BS) $\neq$ F(G11).
KS-1-tail: H0: F (BS) = F (G11); H1: F (BS) > F(G11)
Average percent mean squared error (AMSE): The AMSE for model M, denoted as AMSE (M) is computed as the average of (Market Price of Option − Model M Option Price)^2/(Market Price of Option)^2

Table 6. Japanese Yen. This table presents results from the analysis of pricing errors in the *Japanese Yen* option prices estimated using the Black-Scholes model (benchmark) and the option prices estimated using an empirical martingale simulation conditional on a GARCH (1, 1) underlying spot process

Kolmogorov-Smirnov test (KS)

Models A & B	KS-2 tail	KS-1 tail	Better model
BS & G (1, 1)	Reject H0	Reject H0	Black-Scholes
BS & G (3, 3)	Reject H0	Reject H1	Cannot determine
G (1, 1) & G (3, 3)	Reject H0	Reject H1	Cannot determine

Average percent mean squared error (AMSE)

Models A & B	MSE [A]	MSE [B]	Better model
BS & G (1, 1)	2.1262	12.3364	Black-Scholes
BS & G (3, 3)	2.1262	13.1211	Black-Scholes
G (1, 1) & G (3, 3)	12.3364	13.1211	Garch (1, 1)

Kolmogorov-Smirnov test (KS): Let F (M) denote the cumulative distribution of the absolute pricing errors from model M. The Kolmogorov-Smirnov 2-tail and 1-tail test is stated as:
KS-2-tail: H0: F (BS) = F (G11); H1: F (BS) $\neq$ F(G11).
KS-1-tail: H0: F (BS) = F (G11); H1: F (BS) > F(G11)
Average percent mean squared error (AMSE): The AMSE for model M, denoted as AMSE (M) is computed as the average of (Market Price of Option − Model M Option Price)^2/(Market Price of Option)^2.

to be a better model compared to the GARCH models. This is also evident based on average percent mean squared errors. The Black-Scholes model generates fewer errors compared to the GARCH models.

Unlike the case of British pounds, GARCH (3, 3) performs better than GARCH (1, 1) on the basis of the percent average mean squared errors for Swiss Francs.

The results for *Japanese Yen* are presented in Table 6. The results from K-S tests indicate that the Black-Scholes model is superior to GARCH (1, 1). The distribution of errors for the Black-Scholes model and GARCH (3, 3) are not the same. Also, the distributions of errors between the GARCH models are not the same. However, the KS-tests cannot determine which of these stochastically dominates the other.

The lower panel of Table 6 presents results from the average percent mean squared errors. The Black-Scholes model clearly outperforms the GARCH models with lower average percent mean squared errors. The errors generated by the GARCH (1, 1) model are marginally lower compared to those generated by GARCH (3, 3).

Conclusion

Currency options are widely used by individuals and corporations facing exchange rate risk exposure. These types of investors would benefit immensely from knowing a benchmark price for a call or a put option at the time they execute their buy or sell orders. Albeit the ease in day-to-day applications, the Black-Scholes model has been shown to suffer from several biases. Ironically, these biases cause further uncertainty to a hedger in pursuit of reducing uncertainty.

Our motivation stems from Duan and Wei (1999:57) which states, "...The question as to which model can best describe the foreign exchange return behavior and hence lead to more accurate pricing can only be answered empirically. The topic is best left for future research." We focus on the underlying dynamics of the exchange rate behavior. The Black-Scholes model assumes a diffusion process with a constant volatility. This study examines the daily exchange rates from 1987 to 1995 for presence of autoregressive dependencies and finds an overwhelming presence of volatility clustering. As alternate models we examine the GARCH (1, 1) and GARCH (3, 3) specifications. The model prices were generated using the closed-form model for Black-Scholes constant volatility model and the empirical martingale simulation for the GARCH models. We find that the Black-Scholes specification of the underlying exchange rate stochastic process generates more accurate prices than the GARCH models.

Appendix A

The material in this appendix is provided for reference and is based on the paper by Duan and Simonato (1998). The GARCH process is described in Eqs. (4) and (5) and is repeated here for convenience. The volatility structure is governed by Eq. (A1) below and the simulated exchange rate path is given by Eq. (A2).

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(\xi_{t-i} - ird_t - \lambda\sqrt{\sigma_{t-i}^2} - 0.5\sigma_{t-i}^2\right)^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{A1}$$

$$s(x, y) = s(x-1, y) \exp\left\{\gamma(ird) + \frac{r_f}{365} - \frac{1}{2}\sigma^2(x, y) + \sigma(x, y) rn(x, y)\right\} \tag{A2}$$

The stochastic process $s(x, y)$ does not posses the martingale property and unless the adjustment suggested in Duan and Simonato (1998) is made, it is likely to result in erroneous option values.

Let $sa\ (x, y)$ denote the adjusted simulated spot rates, having the martingale property. The first row of the (T X 1000) matrix is assigned the initial value of the current spot rate. For the remaining rows we compute $sa\ (x, y)$ using the following equation,

$$sa(x, y) = s(x, y) * \frac{z(x, y)}{z_0(x, y)} \tag{A3}$$

where $z(x, y)$ is defined as,

$$z(x, y) = sa(x-1, y) * \frac{s(x, y)}{s(x-1, y)} \tag{A4}$$

and $z_0(x)$ is defined as,

$$z_0(x) = \frac{1}{paths} * \exp\{-rf * \Delta t * x\} * \sum_{y=1}^{paths} z(x, y) \tag{A5}$$

Notes

1. It may be for this reason that MacBeth and Mervile (1980), Emanuel and MacBeth (1982), Tucker and Peterson (1988), Bates (1996), Heston and Nandi (2000), among others, find that the Black-Scholes model does not perform as well as other models.
2. Please see the book titled 'Black-Scholes and Beyond: Option Pricing Models' by Neil A. Chriss.
3. For a thorough review of ARCH/GARCH modeling in finance see Bollerslev, Chou and Kroner (1992).
4. As previously stated, the Black-Scholes model is shown to exhibit several biases such as under pricing of out-of-the money options, under pricing of options on low volatility securities, under pricing of short-maturity options and the U-shaped implied volatility curve (see Duan, 1995).
5. We are aware that an acceptable number of iterations is in the neighborhood of 10000. Some researchers have even used over 50000. Nonetheless, due to our large sample size, we found it cumbersome to go over 1,000 iterations. The results however demonstrate that the data converged with these few number of iterations.
6. In general, the life of the option $[0, T]$ is first divided into N steps as $\{0 \equiv t_0 < t_1 < \cdots < t_N \equiv T\}$. This means that $t_i - t_{i-1} = \Delta t = T/N$. The choice of N determines the length of a time step. However, we use GARCH estimates from daily data we need to make sure that each time step is one day. This means that the number of time steps in each iteration correspond to the number of days remaining in the option. We have not accounted for weekends and holidays when simulating prices.
7. We realize Duan (1995) provides a closed-form solution for a GARCH (1,1) option pricing model and Heston and Nandi (2000) provide a closed-form solution for GARCH (p, q) option pricing model. We did not use either of these models due to the complexities in estimation. The empirical martingale method simplifies the estimation at practically no additional computing cost and yields fairly stable results.
8. Initially, we felt that intra-data options data must have input parameters estimated from corresponding intra-day currency exchange rate data. We quickly realized that we were looking at a broader econometric issue concerning GARCH estimation of irregularly-spaced data. Moreover, when we conducted the ARCH tests on the time-series of irregularly-spaced exchange rates corresponding to each option transaction, we did not find any serial dependence. Clearly, we needed to examine *all* intra-day exchange rate data and not just the ones corresponding to options transactions. We also found that it was important to conduct out-of-sample tests of the parameter estimates. Additionally, we do not study the time-series properties of option prices and only employ annualized input parameters for each option transaction.
9. We did not use the implied volatility on day $t-1$ as the volatility input for day t estimation. This is because we want to compare the constant volatility Black-Scholes model to the GARCH models.
10. For a study on loss functions in option pricing, see Christoffersen and Jacobs (2001).

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