Rotorcraft Acoustics and Dynamics
Group Activities

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Director, Penn State Vertical Lift Research Center

2017 CAV Workshop
Presentation Outline

• New VLRCOE Award 2016-2021
  - Sponsored by Army, Navy, NASA
  - $13.6M over 5 years (includes cost share)
  - 14 separate tasks

Project Highlights (student presentations)

• Optimization of Circular Force Generator (CFG) Placement for Cancellation of Rotorcraft Hub Loads

• Passive Tailboom Vibration Control with Fluidic Flexible Matrix Composite Tubes
New (Noise and Vibration) VLR COE Tasks for 2016

New Task 1  
*Fundamental Investigations into Future Low-Drag Single- and Co-axial Rotor Hub Systems* (Schmitz, Willits, Coder)

New Task 3  
*Nonlinear Laser Ultrasonics for Reduced Variability in Additive Manufacturing Parts* (Lissenden, Reutzel)

New Task 10  
*Fundamental Aeroacoustics of Coaxial Helicopter Rotors* (Brentner, Lee)

New Task 11  
*Enhanced Damping for High-Speed Rigid Rotors via Tailored Hybrid Nanocomposites and Flexible Fluidic Matrix Composite Blade Dampers* (Smith, Rahn, Bakis)

New Task 13  
*Experimental Validation, Noise and Dynamic Analysis and Variable Speed Attributes of High Power Density Pericyclic Transmission* (Smith, DeSmidt, Bill)

More to come in the future on all of the above.....
Spectrum of Rotorcraft Acoustics and Dynamics

- Rotor Noise
- Blade Flutter
- Airframe Vib
- Flight Control
- Blade Loads
- T & B
- Driveline Vib
- Interior Noise
- HUMS
- On-board NDE
- Anti-Icing
- Crash
- Blast
- Ballistics

- Gust Resp
- HQ
- Aeromech Stab
- T & B
- Rotor Noise

Frequency Bands:
- 0.1 Hz
- 1 Hz
- 10 Hz
- 100 Hz
- 1 kHz
- 10 kHz
- 100 kHz
- 1 MHz
- 10 MHz
Outline

• **New VLRCOE Award 2016-2021**

Project Highlights (student presentations)

• **Active Rotor Concepts for In-Plane Noise Reduction**

• **High Power Density, Low-Noise Pericyclic Transmissions**
Optimization of CFG Placement for Cancellation of Rotorcraft Hub Loads

Keerti Prakash
Dr. George A. Lesieutre

CAV Review, April 25, 2017
Motivation

- Vibration control in helicopters is important as apart from being debilitating to passengers and crew, vibration also reduces reliability and life of avionics and components.
- Circular Force Generators (CFGs) can, in principle, cancel hub loads at N/rev.
- The relationship between CFG parameters and the resulting hub loads is highly non-linear. Therefore, CFG placement and sizing is a challenge.

Objectives

- Develop a model that predicts the effect of a single and multiple CFGs on hub loads.
- Formulate and solve an optimization problem that addresses optimal placement addressing different load cases.
- Develop physical insight to guide the efficient solution of the placement problem.
**CFG Placement**

**Representative allowable volume**

**Local co-ordinate system**

- Real force (+1by)
- Spin Axis
- Imaginary force (-1bz)

**Torus specifications**
- Major Radius = 33 inches
- Minor radius = 8 inches
- Centre = (0”, 0”, +4”)

**Hub to Local Coordinate**

**Moving to actuator location**

*Figure showing representative allowable volume, local coordinate system, and torus specifications.*
Overview of the PSO algorithm

Visualization of movement of particle

Step 1 : PSO Initialization
Step 2 : Fitness Evaluation
Step 3 : Update Individual and Global Best Data
Step 4 : Update Velocity and Position of Each Particle
Step 5 : Convergence Determination

Visualization of convergence under PSO
Objectives

• Develop a model that predicts the effect of a single and multiple CFGs on hub loads.

• Formulate and solve an optimization problem that addresses optimal placement addressing different load cases.

• Develop physical insight to guide the efficient solution of the placement problem.
Model Description

The $N/\text{rev}$ hub loads are given as:

\[ \text{HL} = \{ \vec{F}_{\text{hub}}, \vec{M}_{\text{hub}} \} \]

Scaled CFG actuator forces in local coordinates can be expressed as:

\[ \vec{f}_{av} = f_{av} f_{\text{nominal}} \{0 \hat{b}_x + 1 \hat{b}_y - i \hat{b}_z\} \]

Actuator forces and moments at the hub:

\[ \vec{F}_a = [\text{ROT}_a]^T \cdot \vec{f}_{av} \quad \vec{F}_A = \sum_{a=1}^{N} \vec{F}_a \]

\[ \vec{M}_a = \vec{R}_a \times \vec{F}_a \quad \vec{M}_A = \sum_{a=1}^{N} \vec{M}_a \]

The normalized actuator-induced hub loads can be given as:

\[ \text{HL}_a = \{ \vec{F}_a, \vec{M}_a \} \quad \text{HL}_A = \{ \vec{F}_A, \vec{M}_A \} \]

Normalized actuator-induced hub loads:

\[ \text{HL}_{a-norm} = \{ \vec{F}_a / F_{\text{norm}}, \vec{M}_a / M_{\text{norm}} \} \quad \text{HL}_{A-norm} = \{ \vec{F}_A / F_{\text{norm}}, \vec{M}_A / M_{\text{norm}} \} \]

Normalized residual hub loads:

\[ \text{HL}_{\text{norm-res}} = \{ \text{HL}_{\text{norm}} + \text{HL}_{A-norm} \} \]
Single Objective

- Control variables

- Cost function
  \[ J = \text{Re}\left\{ H_{\text{norm-res}} \right\} \cdot \text{Re}\left\{ H_{\text{norm-res}} \right\} + \text{Im}\left\{ H_{\text{norm-res}} \right\} \cdot \text{Im}\left\{ H_{\text{norm-res}} \right\} \]

- Position penalty
  \[ J_{\text{augmented}} = J + p \]

Multi Objective

- Control variables

- Cost function
  \[ \{ J = (w) \cdot J_{\text{load1}} + (1 - w) \cdot J_{\text{load2}} \} \]

- Position penalty
  \[ J_{\text{augmented}} = J + p \]
Objectives

• Develop a model that predicts the effect of a single and multiple CFGs on hub loads.

• Formulate and solve an optimization problem that addresses optimal placement addressing different load cases.

• Develop physical insight to guide the efficient solution of the placement problem.
Single objective results

Representative load case for the following plots:

\[ HL_1 = \{1000, 500, 2000, 20000, 30000, 20000\} \]

- The hub force in blue is cancelled with two actuators in green, which agrees with \( J=0 \).
- The imaginary actuator forces in red cancel themselves.

Forces from hub load and actuators

Real moment from hub load and actuators

Imaginary moment from hub load and actuators
Single objective results

Representative load case for the following plots:

\[ HL_2 = \{1000, 500i, 2000, 20000, 30000i, 20000\} \]

- The imaginary actuator forces in red cancel themselves.
- The actuators are in opposite direction $\$ \text{the imaginary force produced by both the actuators are in line and pointed towards the hub.}$
Single objective results with imaginary loads

Representative load case for the following plots.

$HL_2 = \{1000, 500i, 2000, 20000, 30000i, 20000\}$

- Different hub loads represent different flight conditions physically.
- The imaginary loads are cancelled by sum of actuator forces.

**Forces from hub load and actuators**

**Real moment from hub load and actuators**

**Imaginary moment from hub load and actuators**
Multi objective results

Forces from hub load (case 1) and actuators

Moments from hub load (case 1) and actuators

J > 0

Real Imaginary

Real Imaginary

Real Imaginary
**Summary of cost functions for different cases**

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<th>J = 0 for 2 actuators</th>
<th>J &gt; 0 for 2 actuators</th>
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Robustness of system in case of actuator failure

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Conclusions

- 2 CFGs were found to be sufficient to cancel hub loads for a single (given) load.
- 14 design parameters combined (3 positions, 3 orientations and 1 force for each of the 2 actuators) completely mitigates 12 load variables (6 real forces and moments and 6 imaginary).

\[ HL = \{ \vec{F}_{hub}, \vec{M}_{hub} \} \]

\[ 2 \times \]
Design parameters
Load variables

- For the multiple load case, the use of 3 actuators can reduce the value of the fitness function to the order of \(10^{-2}\) (from 2.0).
- 24 design variables in this situation (3 positions, 3 orientations and 2 forces for each of the 3 actuators) mitigate 24 load variables (12 for each of the 2 load cases).

\[ 3 \times \]
Design parameters
Load variables

- Robustness analysis can help distinguish between different sets of non-unique solutions
Questions?
Passive Tailboom Vibration Control with Fluidic Flexible Matrix Composite Tubes

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Tailboom vibrations can be excited by turbulent flow interacting with helicopter tail surfaces. [de Waard & Trouve, 1999]

- Typical low frequency “tail wag” mode shape
- Increased discomfort
- Structural fatigue
- Lightly damped tailboom = slowly decaying transient vibration
Current passive treatments for the tail wag problem are often not weight-efficient.

Hub Fairings

Vibration Absorbers

Eurocopter tailboom absorber [US Patent 6,286,782, 2001]
Fluidic Flexible Matrix Composite (F²MC) tubes are more efficient force generators and fluid pumps than pistons.

Fiber Reinforcement Angle, $\alpha$

- **Compress axially:** increased volume, negative pressure
- **Stretch axially:** decreased volume, positive pressure
Attached F$^2$MC tubes are strained by tailboom bending, pumping a fluid mass back and forth inside a circuit.

As the tailboom bends to the right,

- Tube A elongates, pumping fluid out
- Tube B shortens, receiving pumped fluid
Attached F$^2$MC tubes are strained by tailboom bending, pumping a fluid mass back and forth inside a circuit.

As the tailboom bends to the **left**, Tube A shortens, receiving pumped fluid.

Tube B elongates, pumping fluid out.

Fluidic circuit

Tailboom Top View

A B
A lateral F²MC vibration absorber is built and tested on the lab-scale tailboom to validate the model.

Circuit Properties

**Fluid:** Water

**Track diameter:** 7.04 mm (0.277 in.)

**Branch segments:** 21.6 cm (8.5 in.)

**Main segment:** 86.5 cm (38 in.)
The model accurately predicts the F²MC treatment’s absorber frequency and magnitude of vibration reduction.

**Bending Vibration**
- baseline tailboom (model)
- baseline tailboom (experiment)
- water/copper (model)
- water/copper (experiment)

**Torsional Vibration**
- baseline tailboom (model)
- baseline tailboom (experiment)
- water/copper (model)
- water/copper (experiment)

**Treatment weight:**
- 7.7 lb. w/ copper tubing
- 6.6 lb. w/ lightweight, plastic tubing

Tailboom weighs approximately 60 lb.
The fluidic circuit can be designed to target two vibration modes simultaneously instead of just one.
The multi-mode treatment sacrifices minimal vertical effectiveness for a large benefit in the lateral direction.

Vertical Bending (12.2 Hz mode)

- baseline tailboom
- single-mode vertical absorber
- multi-mode vert./lat. absorber

63% reduction

Lateral Bending (26.7 Hz mode)

- baseline tailboom
- single-mode vertical absorber
- multi-mode vert./lat. absorber

43% reduction
65% reduction

only 0.14 lb. additional weight penalty for targeting second mode!
In conclusion, $F^2$MC absorbers show potential as realistic treatments for both lateral & vertical tailboom vibrations.

Single-mode treatment:
80% reduction in 26.7 Hz lateral mode

Multi-mode treatment:
63% reduction in 12.2 Hz vertical mode
65% reduction in 26.7 Hz lateral mode
Designing the entire F²MC vibration treatment requires consideration of many different parameters.

**Inertance (effective absorber mass)**

\[ I = k_i(r, \rho, \mu, \omega_n) \frac{\rho l}{\pi r^2} \]

**Resistance (effective damping)**

\[ R = k_R(r, \rho, \mu, \omega_n) \frac{8 \mu l}{\pi r^4} \]
Simulated time response results predict that the 26.7 Hz vibration decays much faster with the F^2MC treatment.

*Horizontal tail has a mode around 38 Hz which is visible at the beginning of the response*
The tailboom-F$^2$MC vibration absorber system is analogous to a classical mechanical absorber.

Undamped absorber frequency: \[ \sqrt{\frac{k_c}{m_a}} = \sqrt{\frac{2}{c_4 I_{tot}}} \]
The fluidic circuit can be designed to target two vibration modes simultaneously instead of just one.

**Circuit Properties**

- **Fluid**: dense isolator fluid (specific gravity ≈ 1.88)
- **Track diameter**: 5.7 mm (0.225 in.)

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**Single-mode, vertical**
- Weight: 7.2 lb.
- 8” branch
- 72” (6’) vertical segment

**Multi-mode, vertical and lateral**
- Weight: 7.3 lb.
- 14” branch