Exact similitude laws for flat plate vibrations induced by a turbulent boundary layer

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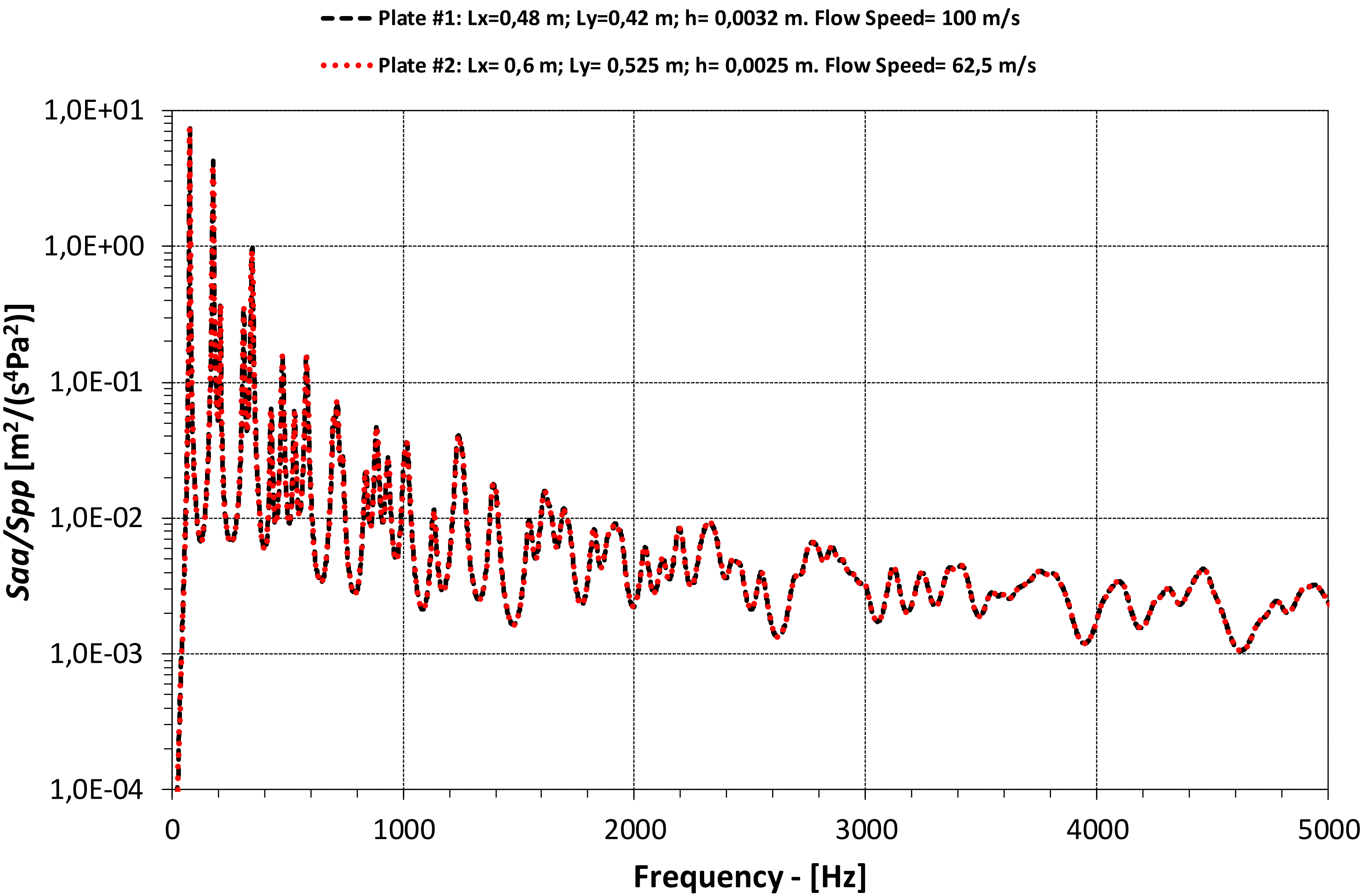

Plate #1: Lx=0,48 m; Ly=0,42 m; h= 0,0032 m. Flow Speed= 100 m/s
Plate #2: Lx= 0,6 m; Ly= 0,525 m; h= 0,0025 m. Flow Speed= 62,5 m/s
1,0E+01
1,0E+00
1,0E-01
1,0E-02
1,0E-03
1,0E-04
0
1000
2000
3000
4000
5000
Frequency - [Hz]
Saa/Spp [m2/(s4Pa2)]

Motivations

- Several research groups have addressed the problem of reducing the computational time proposing efficient numerical algorithms and/or suitable approximations for WPF load representation.
- An alternative approach can consist in measuring the structural response of plates under a TBL excitation in a wind-tunnel facility, these measurements are mandatory if the structures are complex and they cannot be easily modeled by the available numerical tools.
- Unfortunately, these experimental works need the proper wind-tunnel facility (i.e. the proper aerodynamic conditions: flow speed, boundary layer thickness, etc.) and the proper test-article sizes and structural details.

Outline

- Analytical Developments
  - ✓ Key Parameters
- Analytical Results
  - ✓ Some Approximations
- Experimental Measurements
  - ✓ Analysis of the Experimental Data
- Final Considerations

Some References

1. Ciappi E., Magionesi F., De Rosa S., Franco F., 2012. **Analysis of the scaling laws for the turbulence panel responses.** Journal of Fluids and Structures, 32, 90-103.
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4. De Rosa, S., Franco, F., Meruane, V., 2016. **Similitudes for the structural response of flexural plates.** Proc IMechE Part C: J Mechanical Engineering Science, Vol. 230(2), 174-188.
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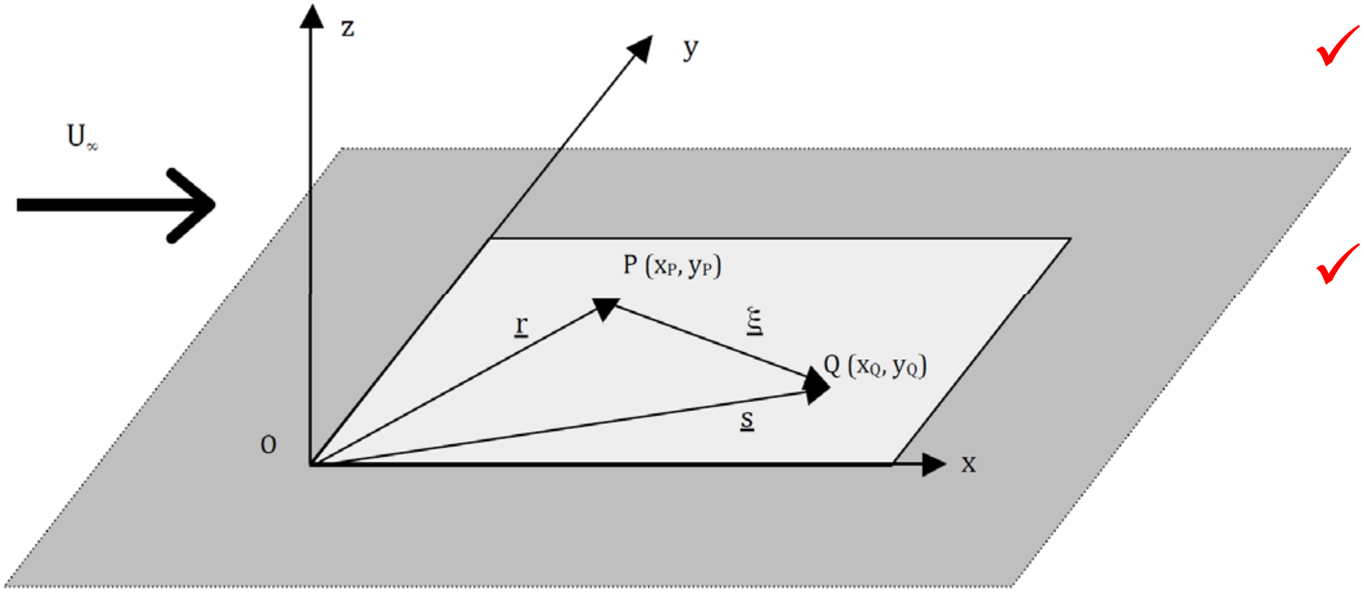
6. Xiaojian, Z., Bangcheng, A., Ziqiang, L. and Dun, L., 2016. **A scaling procedure for panel vibro-acoustic response induced by turbulent boundary layer.** Journal of Sound and Vibration, vol. 380, pp. 165–179, 2016.

The Corcos Model (1/2)



$$X_{pp}(\xi,\omega) = S_p(\omega)\Gamma(\xi,\omega)$$

$$X_{pp}(\xi_x,\xi_y,\omega) = S_p(\omega)\, e^{-\alpha_x\left|\frac{\omega\xi_x}{U_c}\right|}\, e^{-\alpha_y\left|\frac{\omega\xi_y}{U_c}\right|}\, e^{-\frac{i\omega\xi_x}{U_c}}$$

$$X_{pp}(\xi_x,\xi_y,\omega) = S_p(\omega)\, e^{-\frac{|\xi_x|}{L_x(\omega)}}\, e^{-\frac{|\xi_y|}{L_y(\omega)}}\, e^{-ik_c\xi_x}$$

- ✓ The space variables are separated.
- ✓ The phase variation is only accounted along the stream-wise direction.
- ✓ All functions have the same exponential form.
- ✓ It is independent from any couple of points and depends only on their distance.

The Corcos Model (2/2)

- It is an empirical model.
- The limitation of the Corcos model lies in its convective character; it works well in the convective domain.
- Other models have been proposed in literature overcoming some limitations of the Corcos model.
- It allows closed form expression for the response of simple structures such as simply supported flat plates.
  - ✓*It is useful for the sensitivity analyses.*
  - ✓*Its mathematical simplicity justifies the use of the Corcos model even in wide frequency ranges, as done sometimes in the scientific literature.*

The Plate – Analytical Scheme

$$X_w(x_A, y_A, x_B, y_B, \omega) = \sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{\psi_j(x_A, y_A)\ \psi_n(x_B, y_B)}{Z_j^*(\omega)\ Z_n(\omega)}\right]\cdot\left[\frac{S_{pp}(\omega)(ab)^2}{\gamma_j\gamma_n}\right]A_{Q_jQ_n}(\omega)$$

$$A_{Q_jQ_n}(\omega) = \int_0^a\int_0^a\int_0^b\int_0^b\left[\frac{X_{pp}(x, y, x', y', \omega)}{S_{pp}(\omega)\ (ab)^2}\psi_j(x, y)\ \psi_n(x', y')\right]\mathrm{d}y\mathrm{d}y'\mathrm{d}x\mathrm{d}x'$$

$$\bar{S}_{WW}(\omega) = \frac{1}{\text{Area}}\int_{\text{Area}} S_{ww}(x, y, \omega)\mathrm{d}x\mathrm{d}y$$

- ✓ ψ_n - *n*th analytical mode shape of the plate.
- ✓ η - structural damping factor.
- ✓ Z_n - plate dynamic impedance for the *n*th mode.
- ✓ γ_n - generalised mass coefficient for the plate *n*th mode.

- The symbol (^) denotes the parameter in similitude.
- Let define a set of scaling parameters, *r*, as the ratios of the similitude parameter to the original one, for example:

$$r_x = \frac{\hat{a}}{a}; \quad r_{\mathrm{y}} = \frac{\hat{b}}{b}; \quad r_{\mathrm{h}} = \frac{\hat{h}}{h}; \quad r_\eta = \frac{\hat{\eta}}{\eta}; \quad r_\omega = \frac{\hat{\omega}_i}{\omega_i}; \quad r_U = \frac{\hat{U}_C}{U_C}; \quad r_{\mathrm{p}} = \frac{\hat{S}_{pp}}{S_{pp}} \quad \ldots$$

$$r_\eta = \frac{\hat{\eta}}{\eta} = 1$$

- Assuming that the reduced frequency remains unchanged:

$$\frac{r_\omega \cdot r_x}{r_U} = \frac{r_\omega \cdot r_y}{r_U} = 1 \quad \Rightarrow \quad \hat{A}_{Q_j Q_n}(\omega) = \frac{A_{Q_j Q_n}(\omega)}{r_x \cdot r_y}$$

Similitude Laws (2/2)

- ✓ **The previous relationship on the reduced frequency can be used to scale the flow speed.**
- ✓ In addition, it limits the "complete" similitude to plates with proportionally scaled sides.

$$\hat{Z}_n = r_h \cdot r_\omega \cdot Z_n\left(\frac{\omega}{r_\omega}\right)$$

$$\hat{\gamma}_n = r_x \cdot r_y \cdot \gamma_n$$

$$r_p = \frac{\hat{S}_{pp}}{S_{pp}} = 1 \quad \Rightarrow \quad \frac{\hat{X}_w}{\hat{S}_p} = \frac{X_w}{S_p} \cdot \frac{1}{r_x r_y} \cdot \frac{1}{r_h^2 r_\omega^2}$$

- ✓ For a simply supported plate:

$$r_\omega^2 = \frac{r_h^2}{(r_x r_y)^2} \quad \Rightarrow \quad \frac{\hat{X}_w}{\hat{S}_p} = \frac{X_w}{S_p} \cdot \frac{r_x r_y}{r_h^4}$$

$$\frac{X_w}{S_p} = \left(\frac{\hat{h}}{h}\right)^4 \cdot \left(\frac{ab}{\hat{a}\hat{b}}\right) \cdot \frac{\hat{X}_w}{\hat{S}_p}$$

An additional remark

The similitude procedure assumes that the TBL excitation does not transform its characteristics between the “parent” domains.

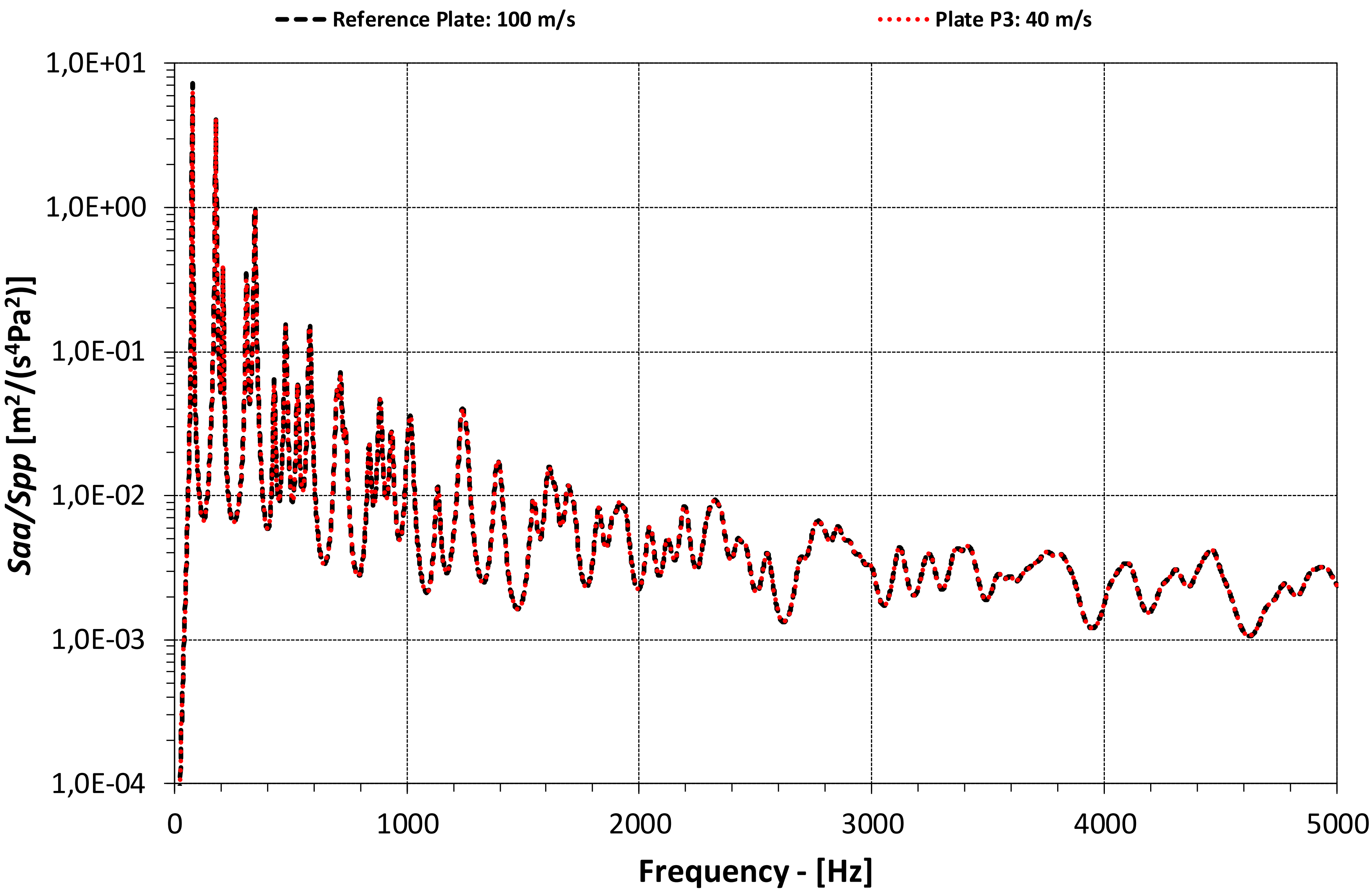
- The coefficients of the Corcos model (α_x, α_y, *convective constant*) remain unchanged.

Analytical Test-Articles

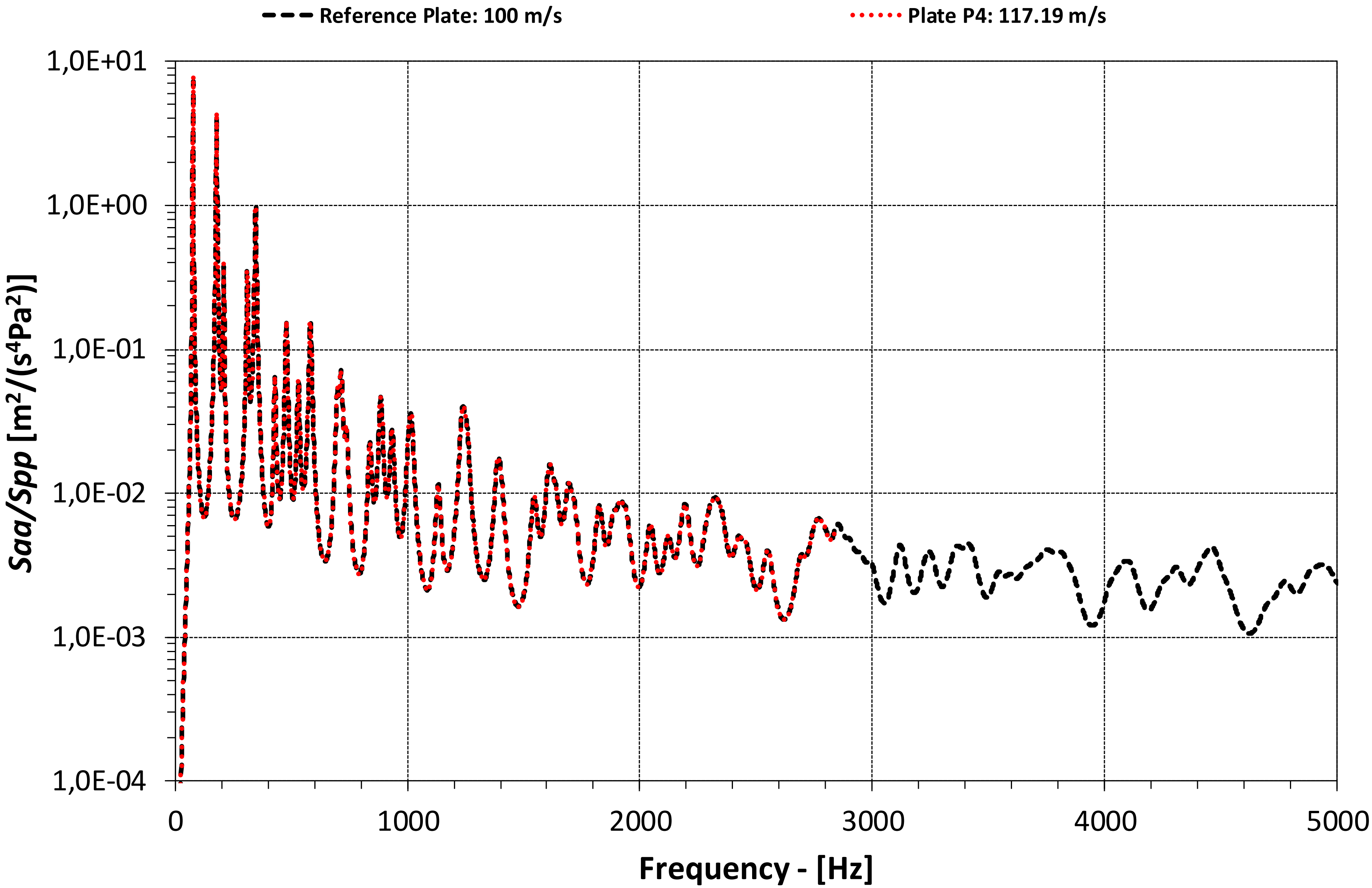
Plate	Stream-wise (a - mm)	Cross-stream (b - mm)	Thickness (h - mm)	Characteristics
P0	480	420	3.20	*Original* plate – ***Reference*** plate
P1	240	210	1.60	***Replica*** plate
P2	600	525	3.20	***Proportional Sides*** plate (the plate thickness equals the reference plate's one)
P3	600	525	1.60	***Proportional Sides*** plate (thickness is changed)
P4	320	280	2.50	***Proportional Sides*** plate (thickness is changed): the plate is scaled down

Analytical Results: $\bar{S}_{aa} / S_{pp}$

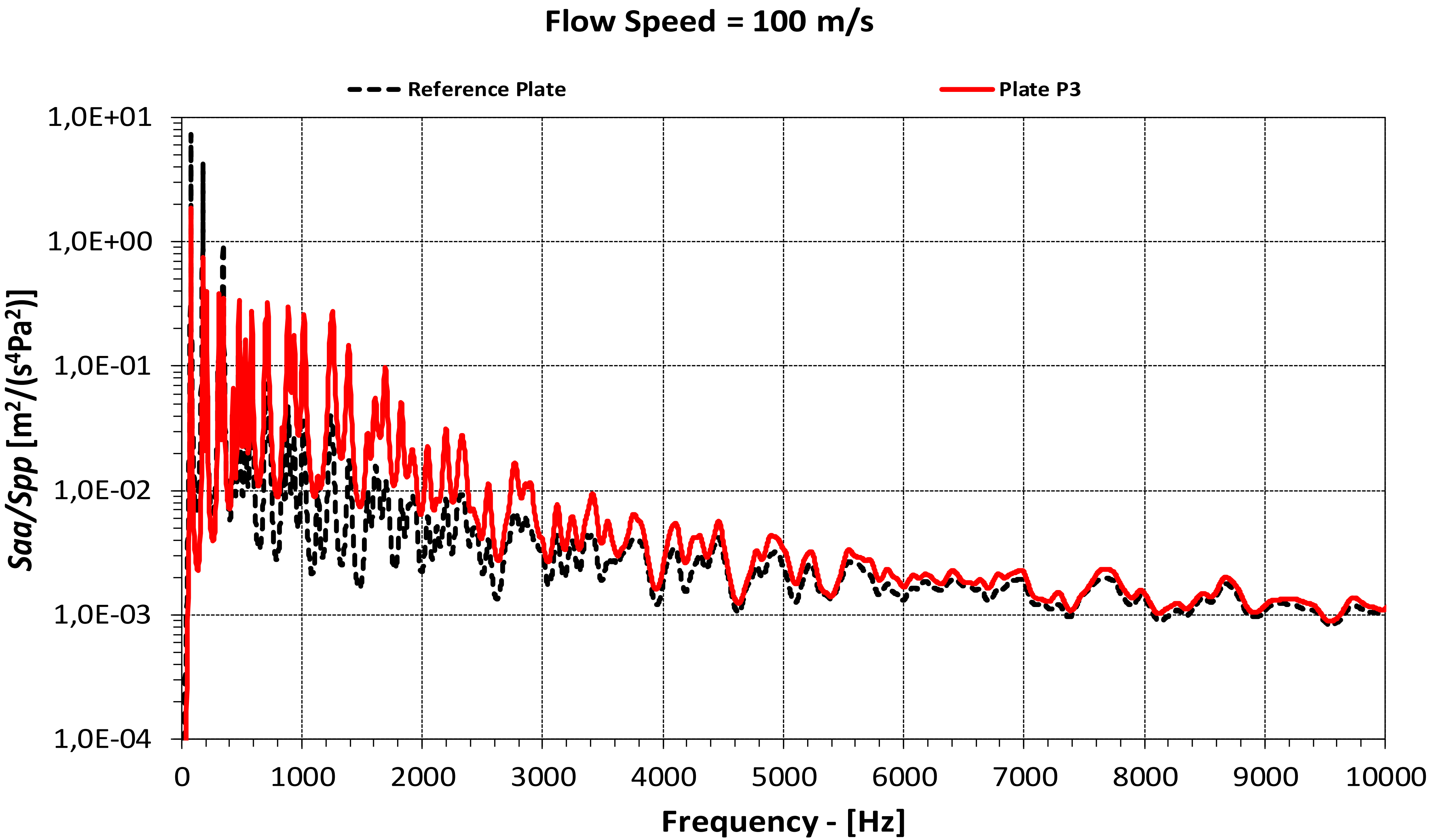
Analytical Results (1/2)



Analytical Results (2/2)

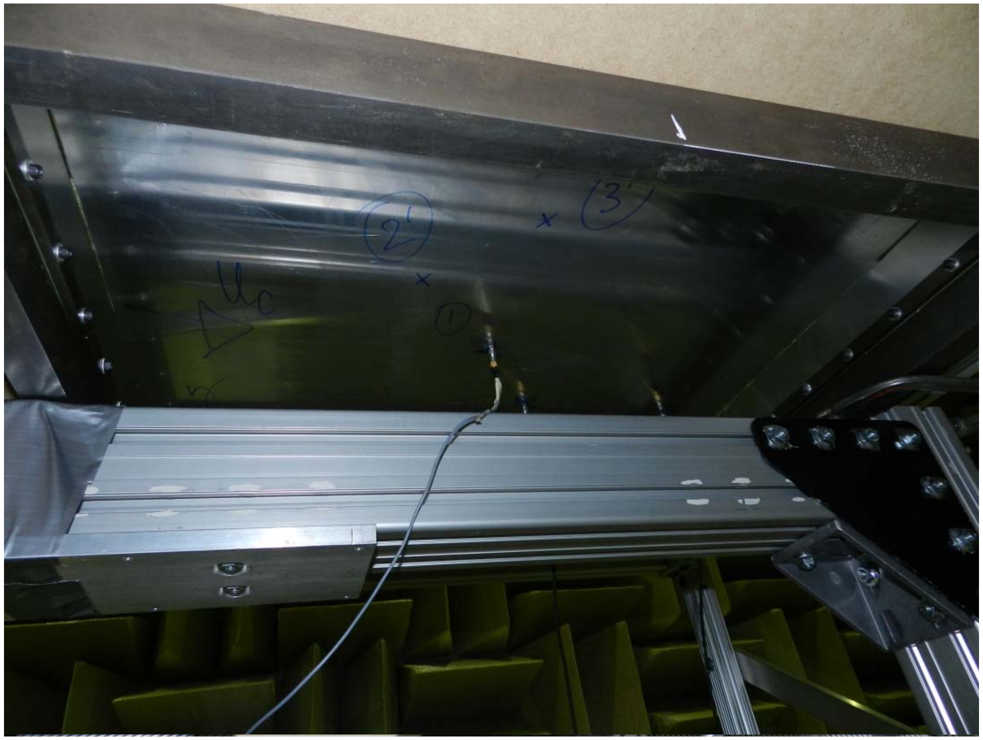


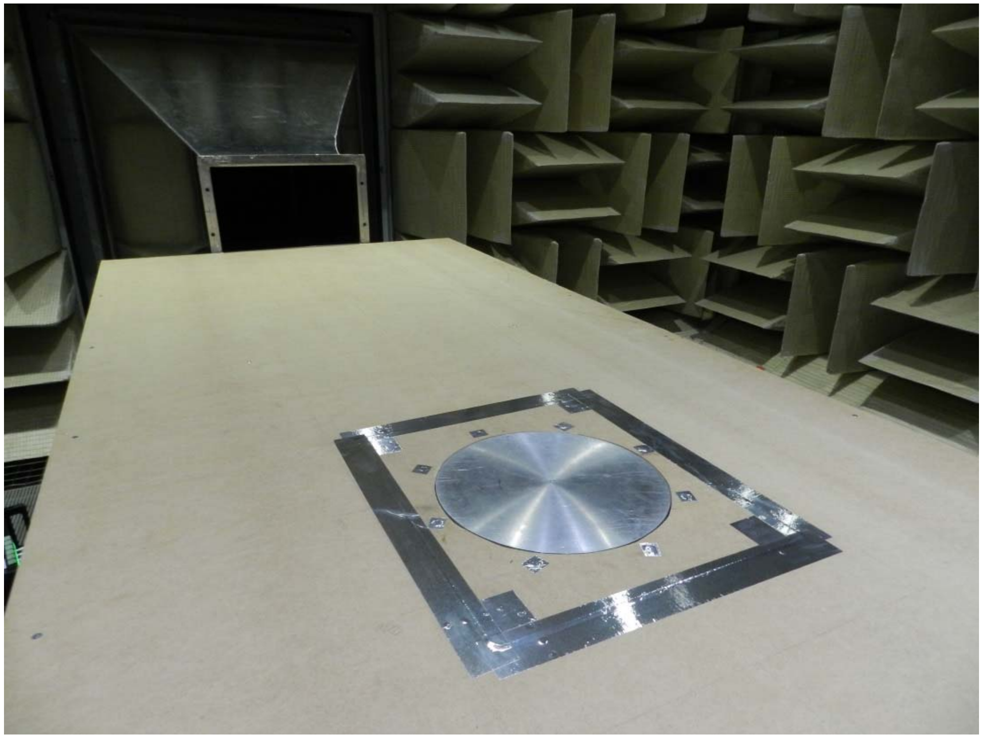
High Frequency Approximation



Experimental Facility

- Measurements held in the anechoic flow facility at Université de Sherbrooke.
- Flat wood plate placed at the convergent nozzle exit, test panels then inserted in with mechanical decoupling.
- Three low-weight accelerometers glued on the panel (added mass ≈ 10 g).
- Wall pressure statistics measured using a single surface microphone or arrays (large 2D rotative 60 microphone array or small 1D MEMS 32 microphone array).





Experimental Test-Matrix

Plates A#: aluminum alloy.

Plate S1: steel alloy.

Plate	Stream-wise	Cross-stream	Thickness	Characteristics
A1				ıum
S1				el
A2				s is ıum
A3				s is changed) Aluminum

Plate	Flow Speed #1 (m/s)	Flow Speed #2 (m/s)
A1	35	25
S1	36	25
A2	28	20
A3	21	15

All the panels have representative simply-supported boundary conditions, in agreement with theoretical scaling laws

Robin, Chazot, Boulandet, Michau, Berry and Atalla, **A plane and thin panel with accurate simply supported boundary conditions for laboratory vibroacoustic tests**. Acta acustica u. Acustica. 2016

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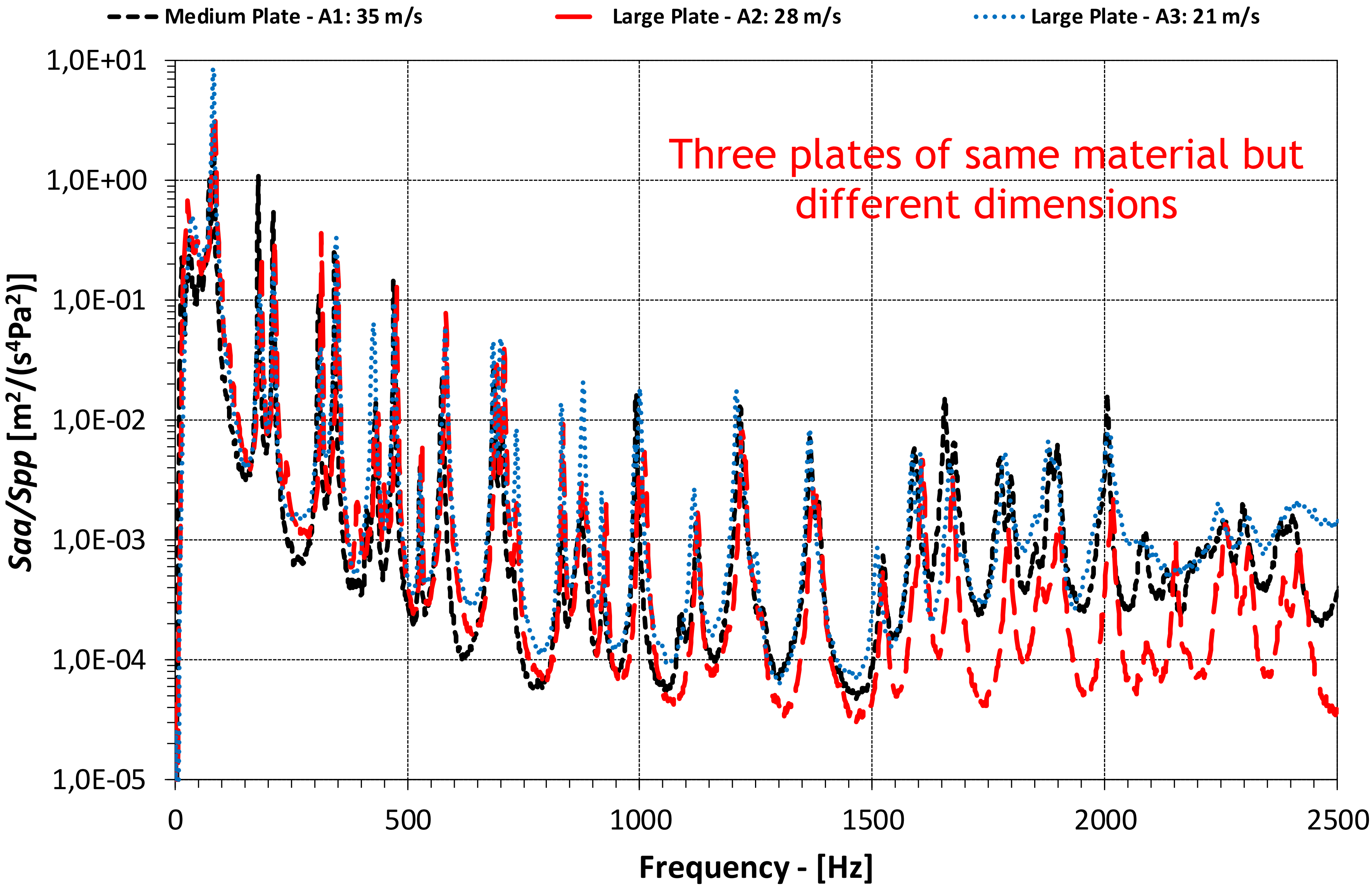
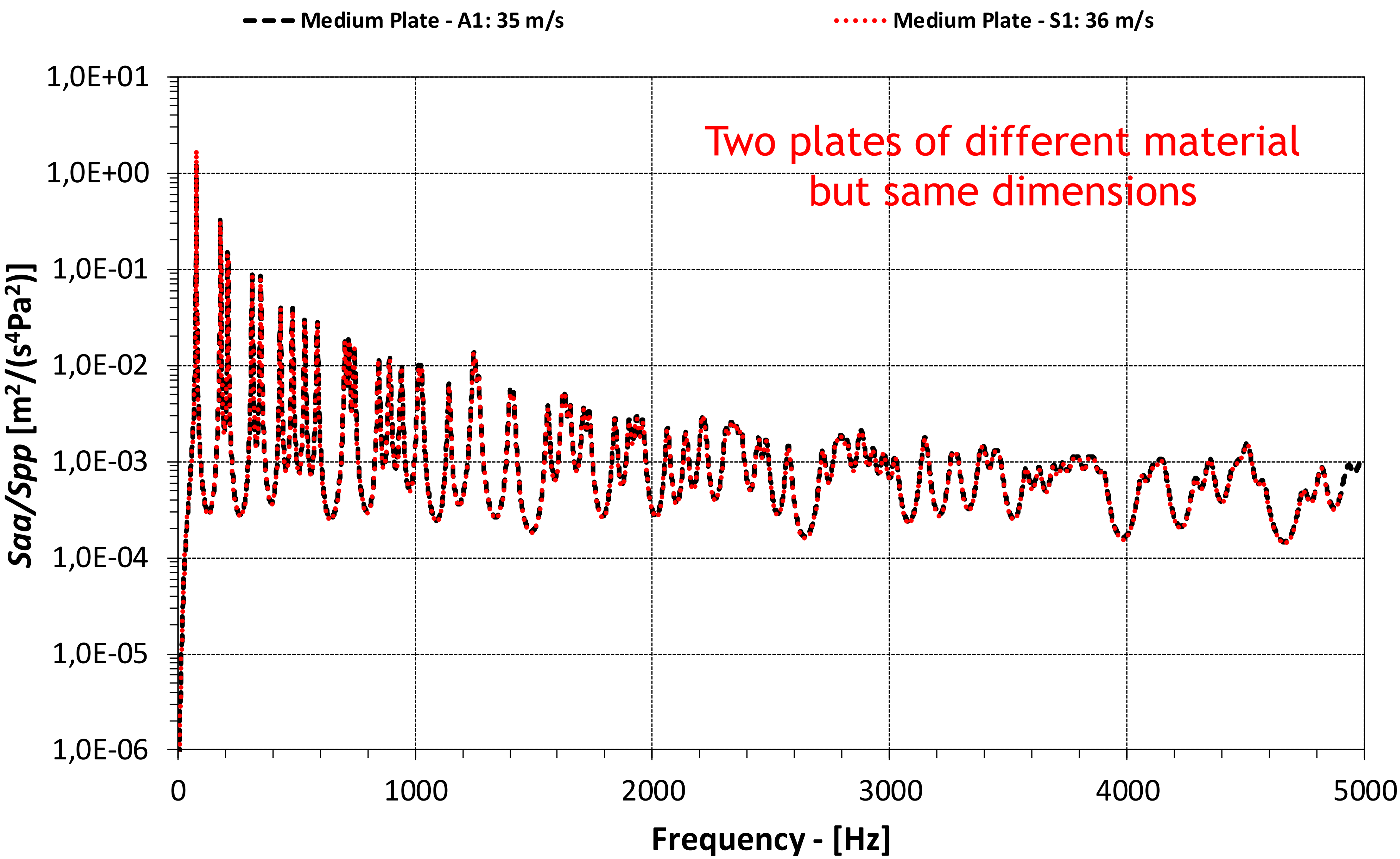


Experimental Results (2/2)

Analytical Results



Conclusions

- ✓ The analytical similitude procedure seems robust.
- ✓ This work addresses only *complete* similitudes; *distorted* similitudes are out of the scope.
- ✓ The experimental data confirms the analytical developments.
- ✓ The availability of more experimental information (for example on the TBL characteristics) could improve the quality of the results: *work in progress*.
- ✓ The role of boundary conditions and damping (for example for the add-on configurations) needs more attention.
- ✓ Different materials, composite materials … to be investigated.

Thank You !