Accelerated flow-acoustic boundary element solver and the noise generation of fish

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Motivation

Motivation
- Design tool for high-performance, silent devices

Goal
- Estimate the acoustic signatures due to vortex-body interactions of an individual or collective

Approach
- Develop and validate a time-domain, 2D acoustic boundary element solver
  - Essential step towards integration with fluid BEM solver

Present advancement
- First integration of a time-domain, double-layer acoustic BEM formulation accelerated by the fast multipole method
Formulation of the external 2D acoustics problem

- The acoustic field is governed by the wave equation in 2D space:

\[ \frac{\partial^2 P}{\partial t^2} - \Delta P = 0 \text{ in } V \in (0, T) \]
\[ P(x, 0) = \frac{\partial P}{\partial t}(x, 0) = 0 \text{ on } S_b \]
\[ \frac{\partial P_{\text{inc}}}{\partial n} = g_N \]

- Transform to a constant strength dipole BEM formulation

\[ -\frac{1}{2} v(x) + \int_{S_b} \frac{\partial G(x, y)}{\partial n_y} v(y) dS_b = \frac{\partial P_{\text{inc}}}{\partial n} \]

- Double-layer (dipole) enables direct application of Neumann BC
- Can combine single-/double-layer potentials in present framework
- Time domain problem recast as convolution of many frequency-domain problems
Fast multipole method

- Accelerates the boundary element solution and evaluation of the acoustic field
- The method rapidly sums potentials of the form

\[ \phi(x) = \sum G(x, y) \frac{\partial \phi(y)}{\partial n_y} + \frac{\partial G(x, y)}{\partial n_y} \phi(y) \]

by breaking them into near-field and far-field sums:

\[ \phi(x) = \sum \phi(x_{near}) + \sum \phi(x_{far}) \]

- Fast multipole method (FMM) employs a tree structure to reduce number of operations for \( N \) boundary elements from \( O(N^2) \) to \( O(N \log N) \)
Direct BEM

- Direct BEM builds full matrices to determine the influence of discrete points on each other
  - $O(N)$ operations * $N$ elements = $O(N^2)$ operations
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Fast multipole method: far field

Procedure

- Build a quadtree structure
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- Sum the potential of leaves into a multipole
Fast multipole method: far field

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- Build a quadtree structure
- Sum the potential of leaves into a multipole
- Find the influences of multipoles on one another
- Repeat up/down the tree structure
  - One bottom/top/bottom pass per time step
Fast multipole method: far field

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The multipole structure represents the *far-field* influence
Fast multipole method: near field

- Near-field calculations require the direct method to account for local influences
Acoustic BEM validation: rigid scatterer

Scatterer

\[ P_i \]

\[ P_s \]

\[ a \]

\[ x \]

\[ y \]

\[ r \]
Scatter of harmonic field by rigid cylinder

Plane wave scattering by a cylinder

\[ P_{\text{inc}}(x, t) = \exp[i(\kappa \cos(\theta) - \omega t)] \]

The scattered wave is

\[ u_s(x, t) = e^{-i\omega t} \sum_{n=0}^{\infty} i^n \left[ -\frac{J_n'(kr)}{H_n^{(1)'}(kr)} \right] \cos(n\theta) \]
Numerical convergence

\[ L_2 = \frac{\langle P_{\text{field}} - P_{\text{analytic}} \rangle}{\langle P_{\text{analytic}} \rangle} \]
Time savings of FMM

- Compare direct and FMM methods for single scatterer
  - 256 time steps per period
- Computation remains on order of seconds using FMM approach
  - Advantage for high-frequency problems requiring many elements
  - 1 to 3 orders of magnitude speed up
Scatter of soliton by multiple bodies

Multiple Scatterers
Scatter of soliton by multiple bodies

- Time-dependent scatter of single wave
- Current method manages multiple closed bodies in arbitrary configurations
- Primary and secondary scattering events handled by BEM solver
Scatter of soliton by multiple bodies
Vortex-airfoil noise generation

\[ \Gamma = \epsilon a U_\infty 4\pi \]
\[ \frac{h}{a} = 0.4 \]
\[ U_\infty = 0.5 \]
\[ \epsilon = 0.2 \]

NACA 0012 airfoil

- Difference between BEM and exact solutions attributed to airfoil thickness
- Doublet BEM enables investigation of arbitrarily thin airfoils
  - Neumann condition difficult with single-layer potential in this geometric limit

Scattered Pressure

\[ U[t]/a \]
Noise generation in swimmer’s wake

Find vortex street parameters:

\[ St = \frac{fA}{U} = 0.3 \]
\[ f = \frac{StU}{A} \]
\[ a = \frac{U}{f} \]
\[ \Gamma^* = 2\pi \tan^{-1}(\pi St) \]
Scattered Field – 2S
Scattered Field – 2P

Vorticity

Scattered acoustic field

Directivity
Scattered Field – 2S

Vorticity

Scattered acoustic field

Directivity

0°  45°  90°  135°  180°  225°  270°  315°
Scattered Field
Continuous 2S street

Vorticity

Scattered acoustic field

Directivity
Summary

Achievements and Conclusions

- A double-layer, time-dependent, 2D acoustic boundary element solver is developed and accelerated using the fast multipole method
  - Needed to rapidly predict and evaluate the noise generation from high-performance swimmers and fliers
- Validation for radiators and scatterers, involving single- and multiple-body arrangements
- Preliminary comparison of 2S/2P wake structures suggest similar noise level and dipole directivity
  - Collections of bodies in a wake modify the far-field directivity in a non-trivial way

Future work

- Incorporate fluid BEM solver to feed vorticity field into acoustic BEM
  - AIAA Aviation (Wagenhoffer, Moored, Jaworski): Integral Methods AA26, CAA V (Tuesday, 2:30pm)
- Extension of methods to handle 3D acoustics and fluid dynamics of animal and robotic collectives
BEM Flow Solver – Vortex Particle Wake

Transform Laplace’s Eqn into a Boundary Integral Equation

Enforce Kutta Condition at the trailing edge (finite velocity)

Wake terms are Gaussian vortex blobs

\[
\phi(r) = \frac{1}{2\pi} \int \int_{S_b} \phi \frac{\partial}{\partial n} \ln(|r - r'|) dS_b + \frac{1}{2\pi} \int \int_{S_w} \Delta \phi_{\text{wake}} \frac{\partial}{\partial n} \ln(|r - r'|) dS_w - \frac{1}{2\pi} \int \int_{S_{\Omega}} \frac{\partial \phi}{\partial n_r} \ln(|r - r'|) dS_b,
\]

\[
\frac{\partial \phi}{\partial n_r} = \hat{n}_r \cdot (V + (\Omega \times r) - \nabla \times \Psi).
\]
Fencing scheme

- Can add vorticity to any flow
- Fence placed around bodies to prevent penetration
- Exaggerated fence here
Wake evolution

Pure pitching foil

$$\theta_{\text{max}} = 0.25 \text{ rad}$$

$$\omega_r = \frac{\omega c}{2U_\infty} = \frac{\pi}{5}$$
Noise due to pure pitching

\[ P \]

SPL(dB) above ambient