Analysis and Synthesis of Switched Systems and Its Applications

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Key Purposes of Control

- **Stability**: Tendency to return to level flight when disturbed
- **Performance**: Fast regulated response to pilot’s commands

Modern Fly-By-Wire Control Technology

- Built-in instability for maneuverability ⇒ Flight computers relieve the pilot of stabilization task.
- Aircraft motions measured and compared with pilot’s commands ⇒ Error is fed back to flight computers; performance is optimized subject to stability.
An Aircraft Control Example
Switched Control Aspect

Subsonic and Supersonic Modes

- Discontinuity in aero-dynamics between two modes
- Different control algorithms for different modes

Normal and Recovery Modes (in Supervisory Control)

- Normal mode: Normal feedback control tasks
  → If dangerous maneuvers are detected, enter recovery mode.

- Recovery mode: Flight computer blocks pilot’s commands and takes over control to bring aircraft to safe operating condition
  → If safe, return control to pilot and enter normal mode.
Mathematical Modeling of Dynamical Systems
First Principle–Based and Empirical State–Space Models

- **Newtonian Physics with Sampling Period** $\tau$:
  Define “state” by $x(t) = \begin{bmatrix} \text{“position”}(t) & \text{“velocity”}(t) \end{bmatrix}^T$
  \[
  x(t + 1) = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \tau \end{bmatrix} u(t); \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)
  \]

- **Best Fit of AR Model to Data with Sampling Period** $\tau$:
  $y(t) = a_1 y(t - 1) + a_2 y(t - 2) + b_1 u(t - 1) + b_2 u(t - 2)$
  $x(t) = \begin{bmatrix} y(t) & y(t - 1) + (b_2/a_2)u(t - 1) \end{bmatrix}^T$
  \[
  x(t + 1) = \begin{bmatrix} a_1 & a_2 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} b_1 \\ b_2/a_2 \end{bmatrix} u(t),
  \]
  $y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$
Concept of Stability
Linear Time-Invariant Systems

State-Space Representation

\[ x(t + 1) = Ax(t) + Bu(t), \]
\[ y(t) = Cx(t) \]

Equivalent Stability Conditions

- **Asymptotic Stability:**
  If \( u(t) = 0 \) for all \( t \), \( \|x(t)\| \rightarrow 0 \) for all \( x(0) \).

- \( \lim_{t \to \infty} A^t = 0 \) \( (\iff x(t) = A^tx(0) + \sum_{s=0}^{t-1} A^{t-s-1}Bu(s)) \)

- **Spectral Radius:**
  \( \rho(A) < 1 \) where
  \[ \rho(A) = \max \{ |\lambda| : \lambda \text{ is an eigenvalue of } A \} \]
  \[ = \lim_{t \to \infty} \|A^t\|^{1/t} \text{ (Any } \| \cdot \| \text{ s.t. } \|AB\| \leq \|A\|\|B\|).} \]
Outline

1 Introduction

2 Stability of Discrete Linear Inclusions
   - Definitions
   - Applications I–III

3 Analysis of Switched Linear Systems
   - Definitions
   - Applications IV and V
   - Analysis Results

4 Synthesis of Switched Linear Systems
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   - Application VI
   - Synthesis Results

5 Conclusion
**Definitions**

Discrete Linear Inclusions

- **Nondeterministic State Evolution:**
  \[ x(t + 1) \in \{ A_1 x(t), \ldots, A_N x(t) \} \]

- **Switching Sequences:** \( \theta = (\theta(0), \theta(1), \ldots) \in \{1, \ldots, N\}^\infty \)

**State-Space Representation**

\[ x(t + 1) = A_{\theta(t)} x(t), \quad \theta \in \{1, \ldots, N\}^\infty \]
Equivalent Stability Conditions

- **Asymptotic Stability:** If $u(t) = 0$ for all $t$, $\|x(t)\| \to 0$ for all $x(0)$ and for all $\theta \in \{1, \ldots, N\}^\infty$.

- $\lim_{t \to \infty} A_{\theta(t-1)} \cdots A_{\theta(0)} = \mathbf{0}$ for all $\theta \in \{1, \ldots, N\}^\infty$.

- **Joint Spectral Radius:**
  
  $\rho\{A_1, \ldots, A_N\} < 1$ where

  $\rho\{A_1, \ldots, A_N\} = \lim \sup_{k \to \infty} \max(i_1, \ldots, i_k) \|A_{i_k} \cdots A_{i_1}\|^{1/k}$.

- **Remark 1:** Any $\| \cdot \|$ s.t. $\|AB\| \leq \|A\|\|B\|$ works.

- **Remark 2:** $\rho(A_1), \ldots, \rho(A_N) < 1 \not\Rightarrow \rho\{A_1, \ldots, A_N\} < 1$

  e.g., $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ \Rightarrow \begin{align*}
  \rho(A_1) = \rho(A_2) &= 0 \\
  \rho\{A_1, A_2\} &= 1.
  \end{align*}
Application I
Stability of Piecewise Linear Systems

State-Space Representation

\[
x(t + 1) = \begin{cases} 
A_1 x(t), & x(t) \in D_1; \\
\vdots \\
A_N x(t), & x(t) \in D_N,
\end{cases}
\]

where \( \bigcup_{i=1}^{N} D_i = \mathbb{R}^n \)

- **Nondeterministic Abstraction:**

\[
x(t + 1) = A_{\theta(t)} x(t), \quad \theta \in \{1, \ldots, N\}^N
\]

- **Sufficient Condition for Stability:**

\[
\rho\{A_1, \ldots, A_N\} < 1 \iff \text{Stability under arbitrary switching} \\
\Rightarrow \text{Piecewise linear system is stable.}
\]
Magnetic Recording Channels

- High density $\Rightarrow$ High intersymbol interference
- Error probability is determined by differences between sequences.
- Small set of difference patterns dominate error probability.
- Exclusion of these difference patterns $\Rightarrow$ Performance $\uparrow$; capacity $\downarrow$

4-bit word $u$

<table>
<thead>
<tr>
<th>$u$</th>
</tr>
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<tbody>
<tr>
<td>0 1 1 1</td>
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</table>

4-bit word $v$

<table>
<thead>
<tr>
<th>$v$</th>
</tr>
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<tbody>
<tr>
<td>1 1 0 0</td>
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</table>

$u - v$

<table>
<thead>
<tr>
<th>diff. $u - v$</th>
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<tbody>
<tr>
<td>- 0 + +</td>
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</table>

$\Rightarrow$

2-Bit diff. patterns in code $\{u, v\}$: $-0$, $0+$, $++$
Application II
Difference-Constrained Codes for Magnetic Recording Channels (Cont’d)

Capacity of Constrained Codes: Illustrative Example

- Set of Disallowed Difference Patterns: \( D = \{+-, --\} \)
- Max-Length \( D \)-Constrained 2-Bit Codes:
  \[
  C_1 = \{00, 01, 11\} \quad C_2 = \{00, 10, 11\}
  \]
  \[
  \begin{align*}
  A_1 &= \begin{bmatrix} 1 & 1 \\
  0 & 1 \end{bmatrix} & A_2 &= \begin{bmatrix} 1 & 0 \\
  1 & 1 \end{bmatrix}
  \end{align*}
  \]
- Sum Norm: \( \|A\|_1 = \sum_{i,j=1}^{2} |a_{ij}| \) for \( A = [a_{11} \ a_{12} \ a_{21} \ a_{22}] \)
- Max Length of \( D \)-Constrained 2-Bit Codes:
  \[\delta_2(D) = \max_i |C_i| = \max_i \|A_i\|_1 = 3\]
Application II
Difference-Constrained Codes for Magnetic Recording Channels (Cont’d)

Capacity of Constrained Codes: Illustrative Example (Cont’d)

- **$D$-Constrained 3-Bit Codes Generated by $C_1, C_2$:**

\[
A_1A_1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \\
A_2A_1 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \\
A_1A_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \\
A_2A_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}
\]

- **Max Length of $D$-Constrained $k$-Bit Codes:**

\[
\delta_3(D) = \max_{i,j} \|A_jA_i\|_1 = 5, \\
\delta_k(D) = \max_{i_1,...,i_k} \|A_{i_k} \cdots A_{i_1}\|_1 = \delta_{k-1}(D) + \delta_{k-2}(D), \quad k \geq 4
\]

- **Capacity:**

\[
\limsup_{k \to \infty} \log_2 \delta_k(D)^{1/k} = \log_2 \rho\{A_1, A_2\}
\]
Wavelet Representation

\[ f(t) = \sum_{j,k} b_{jk} w(2^j t - k) \]

where wavelet \( w(\cdot) \) has:

- **Compact support:**
  \[ w(t) = 0 \text{ for all } t \notin [t_1, t_2] \]

- **Orthogonality:**
  \[ \int w(t)w(2^j t - k)dt = 0 \text{ for all } j, k \]

- **Continuity:**
  \[ \lim_{s \to t} w(s) = w(t) \text{ for all } t \in [t_1, t_2] \]
Application III
Continuity of Wavelet Basis Functions (Cont’d)

Construction of Wavelet: Illustrative Example

1. Pick $N = 3$ and $(c_0, c_1, c_2, c_3) = \left(\frac{3}{5}, \frac{6}{5}, \frac{2}{5}, -\frac{1}{5}\right)$ s.t.
   \[
   \sum_k c_{2k} = \sum_k c_{2k+1} = 1, \quad \sum_k c_k c_{k-2j} = 2\delta_{0j}
   \]
2. Dilation Equation: $\varphi(t) = \sum_k c_k \varphi(2t - k), \; t \in [0, N]$
3. Construction of Wavelet: $w(t) = \sum_k (-1)^k c_{N-k} \varphi(2t - k)$
   $\Rightarrow$ Compact support and orthogonality by construction

Scaling function:
$\varphi(t), \; 0 \leq t \leq 3$

Daubechies wavelet:
$w(t), \; 0 \leq t \leq 3$
Application III
Continuity of Wavelet Basis Functions (Cont’d)

Construction of Wavelet: Illustrative Example (Cont’d)

- Dilation Equation in Matrix Form:

\[ v(t) = \begin{cases} 
T_1 v(2t), & t \in [0, \frac{1}{2}]; \\
T_2 v(2t - 1), & t \in \left[\frac{1}{2}, 1\right), 
\end{cases} \]

\[ v(t) = \begin{bmatrix} \varphi(t) \\
\varphi(t+1) \\
\varphi(t+2) \end{bmatrix}, \quad T_1 = \begin{bmatrix} c_0 & 0 & 0 \\
c_2 & c_1 & c_0 \\
0 & c_3 & c_2 \end{bmatrix}, \quad T_2 = \begin{bmatrix} c_1 & c_0 & 0 \\
c_3 & c_2 & c_1 \\
0 & 0 & c_3 \end{bmatrix} \]

- Restriction of \( T_1, T_2 \) to plane \( x + y + z = 1 \):

\[ A_1 = \begin{bmatrix} 3/5 & 0 \\
-1/5 & 3/5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 6/5 & 3/5 \\
-7/5 & -4/5 \end{bmatrix} \]

- Continuity of \( w(t) \iff \rho\{A_1, A_2\} < 1 \)
  (Daubechies wavelet is indeed continuous.)
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### Definitions

#### Stability

<table>
<thead>
<tr>
<th>Switched Linear System ((\mathcal{A}, \Theta))</th>
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<tbody>
<tr>
<td>(\mathcal{A} = {A_1, \ldots, A_N}), (\Theta \subset {1, \ldots, N})∞</td>
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<tr>
<th>State-Space Representation</th>
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<tbody>
<tr>
<td>(x(t+1) = A_{\theta(t)}x(t), \quad \theta = (\theta(0), \theta(1), \ldots) \in \Theta)</td>
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<table>
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<tr>
<th>Uniform Exponential Stability</th>
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<tr>
<td>There exist (c &gt; 0) and (\lambda \in (0, 1)) s.t.</td>
</tr>
<tr>
<td>(|x(t)| \leq c\lambda^{t-t_0}|x(t_0)|)</td>
</tr>
</tbody>
</table>

for \(0 \leq t_0 \leq t\), \(x(t_0)\), and \(\theta \in \Theta\).
Definitions

$\mathcal{H}^\infty$ (or Disturbance Attenuation) Performance

**Switched Linear System ($\mathcal{G}, \Theta$)**

$$\mathcal{G} = \{(A_1, B_1, C_1, D_1), \ldots, (A_N, B_N, C_N, D_N)\}$$

**State-Space Representation**

$$x(t + 1) = A_{\theta(t)}x(t) + B_{\theta(t)}w(t),$$
$$z(t) = C_{\theta(t)}x(t) + D_{\theta(t)}w(t)$$

**Disturbance Attenuation level $\gamma$ (Under $x(0) = 0$)**

$$\sum_{0}^{\infty} \|z(s)\|^2 < \gamma^2 \sum_{0}^{\infty} \|w(s)\|^2 \quad \text{unif. for } w \in \ell^2 \text{ and } \theta \in \Theta$$
Application IV
Hybrid Automaton Model of torque generation and powertrain dynamics

- **Discrete States**: $q_1, \ldots, q_4$ — position of piston
- **Discrete Actions**: $\sigma_-, \sigma_0, \sigma_+, \sigma_{dc}$ — timing of ignition
- **Continuous State**: $x(t)$ — torque, speed, air masses
- **State Evolution**: $x(t + 1) = f(q_i, \sigma_j)$ at time $t(x(t))$
Application IV
Hybrid Automaton Model of torque generation and powertrain dynamics

- **Mode:** \( \theta(t) \in \{1, \ldots, 8\} \) (8 pairs \((q_1, \sigma_-), \ldots, (q_4, \sigma_+))\)
- **State Evolution:** \( x(t+1) = A_{\theta(t)}(x(t)) \)
- **Admissible Switching Sequences w.r.t. \((Q, q)\):**
  \[ \Theta(Q, q) = \{ (\theta(0), \theta(1), \ldots) : q_{\theta(0)} > 0, q_{\theta(t)\theta(t+1)} > 0 \} \]

Admissible transitions

\[
Q = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Possible initial modes
\( q = [1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1] \)
Application V
Distributed Network of Vehicles

- Changing neighbor sets and communication topology
  ⇒ Switched dynamics
Three-Vehicle Example

- Admissible Modes of Operation:
  - mode 1
  - mode 2
  - mode 3
  - mode 4

- Initial Mode: mode 4

- Forbidden Transitions:
  
  \[ 1 \rightarrow 2, \ 1 \rightarrow 3, \ 2 \rightarrow 1, \ 2 \rightarrow 3, \ 3 \rightarrow 1, \ 3 \rightarrow 2 \]

- Switching Path Constraint:

\[
Q = \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1
\end{bmatrix}, \quad q = [0 \ 0 \ 0 \ 1]
\]
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Classical Analysis Results
Stability of Linear Time-Invariant Systems

State-Space Representation

\[ x(t + 1) = Ax(t) \]

Equivalent Stability Conditions

- **Spectral Radius:** \( \rho(A) < 1 \)
- **Lyapunov Inequality:** There exists \( X > 0 \) s.t.

\[ A^T AX - X < 0. \]

Linear Matrix Inequality (LMI)

- Semidefinite programming (SDP): Computationally attractive
- Problem solved if formulated in terms of LMI
Classical Analysis Results

$\mathcal{H}^\infty$ Performance of Linear Time-Invariant Systems

State-Space Representation

\[
x(t + 1) = Ax(t) + Bw(t),
\]

\[
z(t) = Cx(t) + Dw(t)
\]

Kalman-Yakubovich-Popov (KYP) Lemma

LTI system is stable and its disturbance attenuation level \(< 1\) iff there exists $X > 0$ s.t.

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^T \begin{bmatrix}
X & 0 \\
0 & I
\end{bmatrix} \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} - \begin{bmatrix}
X & 0 \\
0 & I
\end{bmatrix} < 0.
\]
Theorem (Exact LMI Condition for Stability)

\((\mathcal{A}, \Theta)\) is stable iff there exist \(M\) and \(X_{i_0\ldots i_{M-1}} > 0\) s.t.

\[
A_{i_M}^T X_{i_1\ldots i_M} A_{i_M} - X_{i_0\ldots i_{M-1}} < 0
\]

for all \((i_0, \ldots, i_M)\) occurring in \(\Theta\).

Theorem (Exact LMI Condition for \(\mathcal{H}_\infty\) Performance)

\((\mathcal{G}, \Theta)\) is stable with disturbance attenuation level < 1 iff there exist \(M\) and \(X_{j_0\ldots j_{M-1}} > 0\) s.t.

\[
\begin{bmatrix}
A_{i_M} & B_{i_M} \\
C_{i_M} & D_{i_M}
\end{bmatrix}^T
\begin{bmatrix}
X_{i_1\ldots i_M} & 0 \\
0 & I
\end{bmatrix}
\begin{bmatrix}
A_{i_M} & B_{i_M} \\
C_{i_M} & D_{i_M}
\end{bmatrix}
- \begin{bmatrix}
X_{i_0\ldots i_{M-1}} & 0 \\
0 & I
\end{bmatrix} < 0
\]

for all \((i_0, \ldots, i_M)\) occurring in \(\Theta\).
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Definitions
Closed-Loop System

- **Controlled Dynamical System** (with State $x(t)$):

  \[
  \begin{align*}
  w(t) & \rightarrow \text{Controlled System} \\
  u(t) & \rightarrow y(t)
  \end{align*}
  \]

- **Dynamic Output Feedback Controller** (with State $x_K(t)$):

  \[
  \begin{align*}
  y(t) & \rightarrow \text{Feedback Controller} \\
  & \rightarrow u(t)
  \end{align*}
  \]

- **Closed-Loop System** (with State $[x(t)^T \, x_K(t)^T]^T$):

  \[
  \begin{align*}
  w(t) & \rightarrow \text{Controlled System} \\
  u(t) & \rightarrow y(t) \\
  w(t) & \rightarrow \text{Closed-Loop System} \\
  & \rightarrow z(t)
  \end{align*}
  \]
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Application VI
Networked Control Subject to Delays in Feedback Loop

- **Distributed Sensor-Actuator Network:**

- **A Sensor-Actuator Pair:**
Application VI
Networked Control Subject to Delays in Feedback Loop

- **Switched System Model:**

- **Switching Path Constraint \((Q, q)\):**

  \[\theta(t) = (\text{delay at time } t) + 1\]

  \[Q = \begin{bmatrix}
  1 & 1 & 0 & \cdots & 0 \\
  1 & 1 & 1 & \cdots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  1 & 1 & 1 & \cdots & 1 \\
  1 & 1 & 1 & \cdots & 1
\end{bmatrix}
\]

  \[q = [1 \ 1 \ 1 \ \cdots \ 1]\]
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Previous Synthesis Result

$\mathcal{H}_\infty$ Synthesis for Linear Time-Invariant Systems [Gahinet & Apkarian, '94; Packard, '94]

Controlled System

\[
x(t + 1) = Ax(t) + B_1w(t) + B_2u(t),
\]
\[
z(t) = C_1x(t) + D_{11}w(t) + D_{12}u(t),
\]
\[
y(t) = C_2x(t) + D_{21}w(t)
\]

$\mathcal{H}_\infty$ Synthesis LMI

There exists a stabilizing controller s.t. disturbance attenuation level $< 1$ iff there exist $R, S > 0$ s.t. $\begin{bmatrix} R & I \\ I & S \end{bmatrix} \geq 0$ and

\[
N^T_R \begin{bmatrix} A \\ C_1 \\ D_{11} \end{bmatrix} \begin{bmatrix} R & 0 & I \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} A \\ C_1 \\ D_{11} \end{bmatrix}^T - \begin{bmatrix} R & 0 & I \\ 0 & I & 0 \end{bmatrix} N_R < 0,
\]

\[
N^T_S \begin{bmatrix} A \\ C_1 \\ D_{11} \end{bmatrix}^T \begin{bmatrix} S & 0 & I \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} A \\ C_1 \\ D_{11} \end{bmatrix} - \begin{bmatrix} S & 0 & I \\ 0 & I & 0 \end{bmatrix} N_S < 0.
\]
Finite-Path-Dependent Feedback Control

\( H^\infty \) Synthesis for Switched Linear Systems [Lee & Dullerud, '06]

### Controlled System

\[
x(t + 1) = A_{\theta(t)}x(t) + B_{1,\theta(t)}w(t) + B_{2,\theta(t)}u(t), \\
z(t) = C_{1,\theta(t)}x(t) + D_{11,\theta(t)}w(t) + D_{12,\theta(t)}u(t), \\
y(t) = C_{2,\theta(t)}x(t) + D_{21,\theta(t)}w(t)
\]

### Finite-Path-Dependent Dynamic Output Feedback Controllers

\[
x_K(t + 1) = A_{K,\theta(t-M)\ldots\theta(t)}x_K(t) + B_{K,\theta(t-M)\ldots\theta(t)}y(t), \\
u(t) = C_{K,\theta(t-M)\ldots\theta(t)}x_K(t) + D_{K,\theta(t-M)\ldots\theta(t)}y(t)
\]

### Closed-Loop System (with \( \tilde{x}(t) = [x(t)^T \ x_K(t)^T]^T \))

\[
\tilde{x}(t + 1) = \tilde{A}_{\theta(t-M)\ldots\theta(t)}\tilde{x}(t) + \tilde{B}_{\theta(t-M)\ldots\theta(t)}w(t), \\
z(t) = \tilde{C}_{\theta(t-M)\ldots\theta(t)}\tilde{x}(t) + \tilde{D}_{\theta(t-M)\ldots\theta(t)}w(t)
\]
Future Application
Feedback Control of Epilepsy

- Temporal Lobe:

- Hippocampus:

- Electrode Positions:

- Normal EEG:

- Epilepsy EEG:
Future Application
Feedback Control of Epilepsy (Cont’d)

- **Parameter Polytope:**

- **Objective:** Minimize deviation from “normal” polytope
- **Constraints:** Closed-loop stability; regulated control effort
- **Approach:** Suffices to consider “vertices” only
  \[\Rightarrow\] Switched system
Summary
Analysis and Synthesis of Switched Linear Systems

- Stabilization, disturbance attenuation, etc.
- LMI-based exact, convex, and control-oriented results
- Offline optimal synthesis of causal finite-path-dependent controllers:

  ![Diagram]

  System parameters, Tolerance level → SDP-Based Synthesizer → $\gamma_{m-1} = \gamma_m$?

  - Yes: Path length $M$, Controller gains
  - No: $M \leftarrow M + 1$

Future Work

- More theory; applications to medicine, energy, food, etc.