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When does strategic debt-service matter?

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Abstract Recent work in corporate finance has suggested that strategic debt-service by equity-holders works to lower debt values and raise yield spreads substantially. We show that this is not quite correct. With optimal cash management, defaults occassioned by deliberate underperformance (strategic defaults) and those forced by inadequate cash (liquidity defaults) work as substitutes: allowing for strategic debt-service leads to a decline in the equilibrium likelihood of liquidity defaults. In some cases, this decline is sufficiently sharp that equilibrium debt values actually *increase* and yield spreads *decline*. We provide an intuitive explanation for these results in terms of an interaction of optionalities.

Keywords Strategic debt-service · Optimal cash management · Liquidity defaults · Strategic defaults · Yield spreads

JEL Classification Numbers G13 · G33 · G35

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1 Introduction

Since its introduction by Hart and Moore (1989, 1994), the notion of *strategic debt-service* has received considerable attention in the finance literature. At its core, strategic debt-service involves a simple idea: when liquidation is costly, it may be possible for equity holders to under-perform on their debt-servicing obligations without triggering liquidation, since rejecting the offer and liquidating the firm may leave debt holders even worse off. The idea is an attractive one; it indicates that default may occur not just because the firm lacks adequate cash (liquidity defaults), but also because of opportunistic behavior by equity holders (strategic defaults).

These observations appear to imply that allowing for strategic debt-service makes debt "more" risky and should result in a lowering of debt values and widening of yield spreads. In this paper, we examine whether such an implication is, in fact, valid. More generally, we look to identify conditions under which strategic debt-service has a large impact on debt values and when it does not.

Our main results are simply stated. We find that liquidity and strategic defaults are not so much complements that reinforce each others' impacts, as *substitutes*: that is, introducing the ability to service debt strategically typically *reduces* the equilibrium likelihood of liquidity-driven defaults. The extent of reduction is greater the higher is the cost to the firm of raising new capital. For firms with a high cost of raising new capital, allowing for strategic debt-service may result in a substantial decline in liquidity-driven defaults, leading to a negligible effect on debt values and yield spreads; remarkably, in some cases, it may even lead to an *increase* in debt value and a *narrowing* of spreads. However, for firms with a low cost of raising new capital, the widening effect of strategic debt service on yield spreads can be very substantial.

The intuition behind these results may be put in terms of an "interaction of optionalities," specifically, between the option to carry cash reserves and the option to service debt strategically. When the cost of raising new funds is high, firms can avoid financial distress and costly liquidation by exercising the first option and carrying larger cash reserves within the firm. However, this involves a possible cost: if distress becomes unavoidable, debt holders have first claim on the firm's assets including these reserves. The second option, that of strategic debt-service, compensates for this partially: it ensures that in the *non-liquidation* states, "excess" reserves (those over the minimum debt-service required to avoid liquidation) accrue to equity holders rather than debt holders. Absent this option, these reserves continue to accrue to the benefit of debt holders until debt-servicing obligations are fully met.

Thus, the option to carry forward cash reserves is more valuable to equity holders when the option to service debt strategically is also present. This leads to higher cash reserves under strategic debt-service, and so to fewer liquidity defaults. In turn, this offsets at least partially the negative impact on debt values of strategic debt service. In some cases, the net result is even *higher* debt values under strategic than non-strategic debt-service.

Our results clarify and extend the recent work of Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) which have received much attention. The papers each develop cash-flow based extensions of the Merton (1974) risky-debt valuation model into which strategic debt-service considerations are introduced. Each paper reports that equilibrium yield spreads are substantially wider when debt-service is strategic than when it is non-strategic.

However, neither paper considers optimal management of the periodic cash flows from the operation of the firm; rather, both assume that all cash flows left over after debt service must be paid out as dividends to equity holders. Thus, the introduction of strategic debt-service does not affect the likelihood of liquidity-driven defaults in the models. Moreover, Mella-Barral and Perraudin (1997) assume that the firm can raise new equity *costlessly*. At the other extreme, Anderson and Sundaresan (1996) assume this process is *infinitely expensive*: no new issue of securities is permitted in their model.

By allowing for optimal cash management as well as for costs of new equity issuance (which may range from zero to infinity), our paper generalizes both sets of assumptions. We confirm the finding of Mella-Barral and Perraudin (1997) that with zero costs of new equity issuance, strategic debt-service has a substantial widening effect on equilibrium spreads. However, we find that under the conditions of the Anderson and Sundaresan (1996) paper, strategic debt-service typically has a negligible widening effect on yield spreads, and, more damagingly, may even *narrow* spreads in some cases! These conslusions remain true even if the optimal cash management of our paper is replaced with their assumption of no cash management. Thus, our findings contradict theirs. Section 3.3 explains the discrepancy.

Some general points of resemblance between our analysis and others in the literature bear mention. In a paper focussing on the optimal design of debt contracts, Bolton and Scharfstein (1996) make the point that there may be a trade-off between liquidity and strategic defaults. Our analysis, conducted in a very different setting, also points to this tension. In both cases, the riskiness of debt does not increase solely as a consequence of allowing for additional types of default.

That the interaction of optionalities could lead to apparently paradoxical conclusions has also been noted in other contexts recently. Myers and Rajan (1998) describe a "paradox of liquidity" where greater liquidity (the ability to convert assets to cash) could *reduce* a firm's debt capacity. Morellec (2001) shows that asset liquidity increases debt capacity only when bond covenants restrict disposition of assets; by contrast, greater liquidity increases credit spreads on unsecured corporate debt. Titman, Tompaidis, and Tsyplakov (2000) also find that "deep pocket" borrowers under certain conditions could face greater borrowing costs than do "credit constrained" borrowers.

Our results have implications for empirical research attempting to relate the extent of strategic debt-service to yield spreads. Our finding that strategic debt service matters more for firms with a low cost of accessing outside capital suggests that empirical studies should control in the cross-section of firms for this cost, perhaps by using the firm's credit rating. More generally, our results imply that the large number of options equity holders have in practice do not necessarily all work against the interests of the firm's creditors. Empirical work on the agency-theoretic determinants of credit spreads should thus be careful in accounting for such possibilities.

The remainder of this paper has the following structure. Section 2 presents the outline of our model and two examples that motivate the more general analysis of Section 3. These examples and the analysis in Section 3 show that strategic debt service widens spreads substantially if new equity-issuance costs are low, but that with high equity-issuance costs, the impact is small and may even be negative. Section 4 shows that our main result – that strategic debt service typically has a large effect only if equity-issuance costs are low – holds up even if our assumption of optimal cash management is replaced by a requirement of zero cash reserves (as imposed by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), and other papers in the literature). Section 5 concludes.

2 Outline of the model and motivating examples

In this section, we describe a two-date model that can be embedded into an infinite-horizon setting as we explain in section 3. We then present two examples – more accurately, two specific parametrizations of the model – that are "cooked" to provide a sharp illustration of the interaction of optionalities described in the Introduction. The first example considers a setting in which new security issuance is prohibitively expensive. It shows that cash reserves may be so much higher under strategic behavior that debt values are actually higher than under non-strategic debt service. The second example shows that if new equity-issuance is costless, then strategic debt-service can lower debt values – and so widen yield spreads – substantially.

The initial time-point in the model is denoted date 0 and the terminal point date 1. The firm has debt with a face value of $\bar{c}$ maturing at date 1. The riskless rate of interest per period is ρ.

The firm has cash and non-cash assets entering date 0 whose values are, respectively, ϕ_0 and V_0. The value of the non-cash assets represents the present value of the future cash flows that can be generated using these assets.

At date 0, the firm decides how much of the cash is to be carried forward as reserves to date 1. Any amount not carried forward is paid out as dividends to equity holders at date 0. Reserves held within the firm earn interest at the rate ρ. Thus, if δ_0 denotes date 0 dividends, the cash reserves entering date 1 are

$$\phi_1 \;=\; (1+\rho)(\phi_0 - \delta_0).$$

There are two states of the world at time 1. In state u, which occurs with probability p, the firm's non-cash assets are worth V_u. In state d, which occurs with probability $1-p$, they are worth V_d. The firm can meet its debt-service obligations at date 1 using cash reserves, raising new equity, or by liquidating the firm. If the assets are liquidated, the amount realized is the value of the assets less liquidation costs. The liquidation costs are given by $f(A)$ where A represents the total (cash and non-cash) assets of the firm.

At date 1, equity holders make a "take-it-or-leave-it" debt-service offer to the debt-holders. If this offer involves underperformance, debt holders can opt to reject the offer in which case the firm is liquidated. The proceeds from liquidation first go to satisfying debt holders' claims, and any surplus goes to equity holders.

We assume the interests of equity holders and managers are perfectly aligned and that the firm's cash reserve/dividend/debt-service policy is chosen to maxmize equity value. All agents are assumed risk-neutral (alternatively, the probabilities p and $1-p$ may be regarded as risk-neutral probabilities). In the first parametrization of this model we present below, we consider a setting where the cost of raising new funds is prohibitive. The second example considers the same parametrization but assumes new equity may be raised costlessly.

Some comments before proceeding to the examples.

Remark 1 The description above assumes that the only available cash for debt-service at time 1 is that carried over as reserves from time 0. This is easily modified to also allow an additional cash flow realization from the firm's non-cash assets. In particular, we can consider a binomial cash flow process as described in Section 3 with the firm value being the present value of these cash flows. Remark 1 in section 3 describes the slight change in the numbers that would result in the examples if we used this modification. The present description keeps the exposition simpler.

Remark 2 The "bargaining" game in our model in the event of under-performance has a simple take-it-or-leave-it offer followed by liquidation in the event of rejection. This gives equity holders all the bargaining power and ensures that strategic debt-service is feasible for equity holders if liquidation costs are sufficiently high. The specific structure of this game is, however, unnecessary. It is easily seen that the intuition for our results remains intact if this is replaced with any bargaining game that leaves equity holders with at least some bargaining power (i.e., at least some power to carry out strategic debt-service).

Remark 3 "Strategic" debt-service occurs when a firm under-performs on its debt-service obligations. For such strategic underpayment to be possible in equilibrium, it is necessary that debtholders cannot do better by liquidating the firm. This means that if the firm offers debt holders debt-service b that is less that the cash-reserves ϕ that it has, liquidating the firm must result in the debt holders getting at most b. Thus, a portion of the cash reserves themselves must get destroyed in the liquidation process. This is why our liquidation costs f apply to both cash and non-cash assets. (Of course, we could use different liquidation costs for cash and non-cash assets, but this unnecessarily complicates matters.)

Remark 4 The word "liquidation" need not be taken literally. The cost $f(A)$ could be viewed instead as a cost of transferring control of the firm to debt holders (or the value loss resulting from a debtholder-run firm); this is essentially the view that Mella-Barral and Perraudin (1997) adopt. Hart and Moore (1989) use a third interpretation where control of the firm does not change hands but the manager can divert a portion $f(A)$ of the cash flows as private benefits. Then, the "pledgable" value of the firm's assets, the maximum the debt holders can expect to recover, is $A - f(A)$.

2.1 Example 1: no new security issuance

Let the initial cash reserves of the firm be $\phi_0 = 25$ and its non-case assets be worth $V_0 = 26$. Suppose that $V_u = 32.50$, $V_d = 20.80$, and the probabilities of states u and d are, respectively, 4/9 and 5/9. Let the riskless rate of interest ρ be zero. Finally, suppose that the face value of the firm's debt $\bar{c} = 25$ and that liquidation costs are given by $f(A) = l_0 + l_1 A$, where $l_0 = 20$ and $l_1 = 0.25$. We assume throughout this example that no new securities may be issued by the firm.

We show that in this setting (a) equity holders will optimally choose to carry forward lower cash reserves under non-strategic debt-service than under strategic debt-service, and (b) that, as a consequence, debt values are *higher* under strategic than non-strategic debt service.

Consider strategic debt service first. Suppose the firm carries a cash reserve of ϕ_1 to time 1. If the state u occurs at time 1, the post-liquidation value of the firm is

$$V_u + \phi_1 - l_0 - l_1(V_u + \phi_1) = 4.375 + 0.75\,\phi_1.$$

If debt holders are offered less than this quantity, they will liquidate the firm. (Note that this reservation value is increasing in ϕ_1 – this is one of the costs of carrying cash reserves). But the amount of cash available to the firm in state u is ϕ_1. Thus, for successful stategic underperformance in the state u, we must have

$$\phi_1 \;\geq\; 4.375 + 0.75\,\phi_1,$$

which means liquidation can be avoided only if the cash reserves ϕ_1 entering time 1 are at least 17.50. Suppose that at time 0, equity holders choose to pay a dividend of 7.50 and to carry forward the minimum reserves $\phi_1 = 17.50$ required to stave off liquidation to stave off liquidation in state u in period 1. (Carrying higher reserves than this is clearly suboptimal). Then, the value of equity in state u in period 1 is the value of cash and non-cash assets less the debt service which is $32.50+17.50-17.50 = 32.50$. Moreover, if the state d now occurs, the liquidation value of the firm is

$$V_d + \phi_1 - l_0 - l_1(V_d + \phi_1) \;=\; 8.725.$$

Since the firm has enough cash to offer debt holders this quantity, liquidation will be avoided in state d too and the value of equity in state d is $20.80 + 17.50 - 8.725 = 29.575$. Thus, carrying cash reserves of 17.50 to time 1 leads to a period-0 equity and debt values of

$$\begin{aligned} E_0 &= \;7.50 + (4/9)(32.50) + (5/9)(29.675) \;=\; 38.375 \\ D_0 &= (4/9)(17.50) + (5/9)(8.725) \;=\; 12.625. \end{aligned} \tag{2.1}$$

The alternative for the firm is to carry less than 17.50 of reserves into time 1 and accept liquidation in state u. A simple computation shows that the minimum level of reserves to avoid liquidation in state d is zero. (That is, if equity holders carry forward an arbitrarily small positive level of reserves into time 1, debt holders are strictly better off accepting this amount than liquidating the firm). This alternative zero-reserve policy thus leads to time 0 values for equity and debt given by

$$\begin{aligned} E_0 &= \;25 + (4/9)(0) + (5/9)(20.80) \;=\; 36.56 \\ D_0 &= (4/9)(17.50) + (5/9)(8.725) \;=\; 1.94 \end{aligned} \tag{2.2}$$

A comparison of (2.1) and (2.2) shows that equity holders rationally prefer to carry reserves of 17.50 forward and to avoid liquidation in both states. Thus, the equilibrium equity and debt values under strategic debt service are those given by (2.1).

Now, suppose debt service is non-strategic. This implies that *strategic* default is not permitted so that all available cash must go to the debt holders until debt-servicing requirements are fully met. However, *liquidity* defaults may still occur if there is inadequate cash to meet debt-servicing obligations, and in this case debt holders choose between taking all the available cash and liquidating the firm.

If the firm takes zero (i.e., arbitrarily small positive) reserves to time 1, then debtholders will liquidate the firm in state u but there is no liquidation in state d. This leads exactly to the debt and equity values identified in (2.2) above. Once again, the alternative is to take a reserve of $\phi_1 = 17.50$ and avoid liquidation in both states (anything less will lead to liquidation in state u). In state u, the outcome is identical to that under strategic debt-service. In state d, strategic underpayment is not possible, so all the available cash of 17.50 goes to the debt holders. As a consequence, debt and equity values under this choice of reserves work out to

$$E_0 = 33.50 \qquad D_0 = 17.50 \tag{2.3}$$

Thus, with non-strategic debt service, the optimal choice of reserves for equity holders is $\phi_1 = 0$, leading to the debt and equity values (2.2). In particular, reserves are smaller, debt values are smaller, and yield spreads higher under non-strategic debt service than strategic debt service.

2.2 Example 2: costless new equity issuance

Consider now the same parametrization as above, but assume that new equity may be costlessly issued. Now the firm has no incentives to carry cash reserves forward; indeed, such reserves have no benefit but do carry a cost – in the event of liquidation, debt holders have first claim on the reserves. We will show that strategic debt service now widens debt spreads.

Consider strategic debt-service first. In the state u, the firm's non-cash assets are worth $V_u = 32.50$ as a going concern while the debt holders' reservation value – the amount they can obtain by liquidation – is 4.375. Thus, any offer from equity holders of at least 4.375 will be accepted. To make the payment, equity holders issue 4.375 of new equity and use the cash to pay the debt holders. The firm is now an all-equity firm worth 32.50, of which 4.375 represents the new equity that was issued. Thus, the value of the original equity in the state u is 28.125.

In the state d, the firm's non-cash assets are worth $V_d = 20.80$ while the debt holders' reservation value is 0 (i.e., any arbitrarily small positive offer will be accepted). Thus, equity holders capture the full value of 20.80 in state d while debt holders receive zero. This leads to date-0 values for debt and equity under strategic debt-service of

$$\begin{aligned} D_0 &= (4/9)(4.375) + (5/9)(0) \;=\; 1.944 \\ E_0 &= 25 + [(4/9)(28.125) + (5/9)(20.80)] \;=\; 49.056 \end{aligned} \tag{2.4}$$

When debt-service is non-strategic, equity-holders make the promised debt-service payment in full by raising equity (unless this makes the value of equity negative. This is not a problem in the present example since equity-issuance costs have been assumed to be zero). Thus, in state u, equity holders issue 25 worth of new equity and use the case to repay debt holders in full. The firm is now an all-equity firm worth 32.50, of which 25 belongs to the new equity. Thus, the value of the original equity in the state u is 7.50.

In the state d, the firm's non-cash assets are worth only 20.80, while the promised payment on debt is $\bar{c} = 25$. So equity holders raise 20.80 in new equity, pay the debt holders this amount, and transfers the entire firm to the new equity holders. As a consequence of these actions, the date 0 value of debt and equity under non-strategic debt service is

$$\begin{aligned} D_0 &= (4/9)(25) + (5/9)(20.80) \;=\; 22.67 \\ E_0 &= 25 + [(4/9)(7.50) + (5/9)(0)] \;=\; 28.33 \end{aligned} \tag{2.5}$$

As a comparison of (2.4) and (2.5) show, strategic debt service leads to a significant decrease in debt value and a significant increase in equity values.

3 The general model

The general multi-period model is constructed from the description in Section 2 by taking as primitive the the cash flow process generated by the firm's non-cash assets and using these cash flows to define the asset value process V_t. The model is based on Anderson and Sundaresan (1996), which is itself a discrete-time cash-flow based version of the model in Merton (1974).[1] The description below generalizes the Anderson and Sundaresan (1996) framework in treating costs of equity-issuance as a parameter and allowing for optimal cash management.

As earlier, let ρ denote the riskless interest rate per period and let $\beta = 1/(1+\rho)$. We assume the firm's cash flows follow a binomial process. If f_t denotes the cash flow in period $t (= 0, 1, 2, \ldots)$, then f_{t+1} is realized according to

$$f_{t+1} \;=\; \begin{cases} u f_t, & \text{with probability } p \\ d f_t, & \text{with probability } 1-p \end{cases}$$

where $u > 1 + \rho > d$. Given the realization f_t, let V_t denote the value of present value of all current and future cash flows from time-t onwards. It is not difficult to see that V_t and f_t are related via

$$V_t \;=\; \frac{1}{\alpha} f_t$$

where $\alpha = 1 - \beta[pu + (1-p)d]$. Thus, V_t itself follows a binomial process with

$$V_{t+1} \;=\; \begin{cases} u V_t, & \text{with probability } p \\ d V_t, & \text{with probability } 1-p \end{cases}$$

Let ϕ_t be the cash reserves of the firm entering time t. Since the firm's assets generate cash of $f_t = \alpha V_t$ in period t, the total cash available to the firm in

[1] Since our model considers optimal cash management, the choice of a discrete- (as opposed to continuous-) time model seems appropriate on at least two counts. First, firms in practice pay dividends only discretely. Second, continuous-time models would facilitate extraordinary dividends just prior to declaring bankruptcy; such dividends are illegal in most countries under the principle of *fraudulent conveyance* (e.g., Baird 1998: "Transfers made and obligations incurred with the intent to delay, hinder or defraud creditors are fraudulent and void as against creditors"). Using a discrete-time model mitigates the second feature, though equity holders in our model too may choose to exhaust cash reserves completely at any point if they wish.

period t is $\phi_t + f_t$, while the total value of the firm's assets (cash and non-cash) is $A_t = V_t + \phi_t$. All cash reserves held in the firm earn interest at the rate ρ.

The firm may also decide to raise cash via issuance of new equity in period t. If the firm issues equity worth e_t in period t, it pays a cost of me_t, where $m \in [0, 1]$ is a parameter. Thus, the net proceeds from raising equity are $(1 - m)e_t$. Costless equity-issuance is the case $m = 0$ and infinitely expensive equity is the case $m = 1$.

In this section, we consider zero-coupon debt with maturity T and face value $\bar{c}$. The next section offers some comments and an example concerning coupon debt. On date T, equity holders make debt holders a debt-service offer ξ. If $\xi = \bar{c}$, debt holders must accept and the game terminates with the firm becoming an all-equity firm.

If $\xi < \bar{c}$, debt holders can accept or reject the offer. If the offer is rejected, the firm is liquidated. Liquidation costs have the form $f(A) = l_0 + l_1 A$, where A is the firm's total assets at liquidation time.

If the offer is accepted and equity holders have enough cash to make the debt payment ξ, the payment is made and the game again terminates with the firm becoming an all-equity firm. If equity holders lack sufficient cash, they must raise the required balance through issuance of new equity; if they are unable to do so, the firm is liquidated.

Equity holders optimally choose cash reserve/equity-issuance/debt-service policies to maximize the value of equity. We begin with a summary of equilibrium behavior in this model.

3.1 Equilibrium policies under strategic behavior

Consider behavior on date T. Let ϕ_T and f_T be given. If the firm is liquidated, the liquidation cost incurred is $L_T = l_0 + l_1 A_T$. Thus, the post-liquidation firm value is $(A_T - L_T)^+$. Debt holders then receive the amount

$$D_T^L = \min\{\bar{c}, (A_T - L_T)^+\}.$$

Note that this amount is non-decreasing in ϕ_T.

A debt-service offer $\xi < \bar{c}$ will be accepted only if $\xi \geq D_T^L$. If the available cash $\phi_T + f_T$ exceeds D_T^L, it is obviously optimal for equity holders to offer D_T^L and for debt holders to accept. Equity holders then receive the continuation value

$$V_T^E = A_T - D_T^L. \tag{3.1}$$

If $f_T + \phi_T < D_T^L$, the difference $D_T^L - \phi_T - f_T$ must be raised via issuance of new equity. However, this may itself be a costly process, so it may not be profitable for equity holders to do so. Specifically, let $e_T^{\min}$ denote the minimum amount of cash that must be raised in the form of new equity if liquidation is to be avoided:

$$e_T^{\min} = \min\{e \geq 0 \mid \phi_T + f_T + e - m(e) \geq D_T^L\}. \tag{3.2}$$

Intuition suggests that equity holders should issue this new equity only if the costs $m_T = m\, e_T^{\min}$ of doing so are less than the liquidation costs L_T. It is not hard to confirm this as the following proposition shows:

Proposition 3.1 *Equilibrium on date T has the following structure:*

1. *If* $f_T + \phi_T \geq D_T^L$, *it is optimal for equity holders to offer* $\xi = D_T^L$ *and for debt holders to accept. This leads to the time* T *debt and equity values*

$$V_T^D = D_T^L \qquad V_T^E = A_T - D_T^L \tag{3.3}$$

2. *If* $f_T + \phi_T < D_T^L$ *and* $m_T < L_T$, *it is optimal for equity holders to offer* $\xi = D_T^L$ *and to raise the required cash by issuing new equity. The time* T *values of debt and equity in this case are*

$$V_T^D = D_T^L \qquad V_T^E = A_T - D_T^L - m_T \tag{3.4}$$

3. *If* $f_T + \phi_T < D_T^L$ *and* $m_T \geq L_T$, *it is optimal for equity holders to allow the firm to be liquidated, leading to the time* T *debt and equity values*

$$V_T^D = D_T^L \qquad V_T^E = A_T - L_T - D_T^L \tag{3.5}$$

Proof That (3.3) holds is obvious. To see (3.4) and (3.5), define $e_T^{\min}$ as in (3.2). Suppose the equity holders raise equity $e_T \geq e_T^{\min}$. Then, the cash available to the firm is $\phi_T + f_T + e_T - m\, e_T$. Net of debt-service D_T^L, the value of the firm's assets is $V_T + \phi_T + e_T - m\, e_T - D_T^L$. Of this quantity, e_T represents the value of the new equity, so the value of the "existing" equity is simply

$$V_T + \phi_T - m\, e_T - D_T^L. \tag{3.6}$$

It is apparent from (3.6) that *conditional* on raising enough new equity to avoid liquidation, the value of existing equity is maximized by raising the minimum required amount $e_T^{\min}$, so that existing equity has the time T value

$$V_T + \phi_T - m\, e_T^{\min} - D_T^L. \tag{3.7}$$

If equity holders do not raise new equity, liquidation results and equity value is

$$V_T + \phi_T - L_T - D_T^L. \tag{3.8}$$

A comparison of (3.7) and (3.8) establishes the result. □

Now consider date $T - 1$. Let ϕ_{T_1} denote the cash reserves entering this date and let f_{T-1} denote the date-($T - 1$) cash flow. It is obviously suboptimal for equity holders to raise new equity at this point, so the only decision facing equity holders is the size of the dividend δ_{T-1}. For any given choice of δ_{T-1}, the time T cash reserves are determined according to $\phi_T = (1 + \rho)[\phi_{T-1} + f_{T-1} - \delta_{T-1}]$. This, in turn, determines equity values at date $T - 1$ as

$$V_{T-1}^E = \delta_{T-1} + \beta\, E[V_T^E], \tag{3.9}$$

where the expectation is taken over the realization of the time T state (u with probability p, d with probability $1 - p$). Equity holders choose δ_{T-1} to maximize (3.9). The value of debt on date $T - 1$ is determined now as

$$V_{T-1}^D = \beta\, E[V_T^D]. \tag{3.10}$$

At dates $t < T - 1$, the optimal dividend takes on a simpler form. It is easily seen that one optimal policy is to pay no dividends and hold all cash flows in the firm as reserves. Intuitively, giving up a dollar of dividends at date $t < T - 1$ has no cost if the dollar (plus interest at the rate ρ) will be returned to the equity holders at date $T - 1$; and, of course, the only circumstance under which the dollar will not be returned at date $T - 1$ is if retaining it in the firm enhances equity value further by helping avoid a costly cash shortfall at date T. Thus, the zero-dividend policy either leaves investors either no worse off or strictly better off than paying a positive dividend. Under this policy, debt and equity values for $t \leq T_2$ resolve simply as

$$V_t^D = \beta E[V_{t+1}^D] \qquad V_t^E = \beta E[V_{t+1}^E]. \tag{3.11}$$

' This completes the description of the equilibrium under strategic debt-sevice.

3.2 Equilibrium policies under non-strategic debt service

Under non-strategic debt-service, equity holders pay debt holders the maximum amount they can without making the value of equity negative.[2] Thus, given ϕ_T and f_T, if $\phi_T + f_T \geq \bar{c}$, debt holders receive the full promised payment $\bar{c}$ and equity holders receive $V_T + \phi_T - \bar{c}$.

Suppose $\phi_T + f_T < \bar{c}$. If equity holders issue an amount e of equity, this costs me. The cash reserves go up by $e(1-m)$, so the maximum debt service that can be offered is $\phi_T + f_t + e(1-m)$. Suppose this offer is accepted by debtholders. Then, the value of the "old" equity in the firm is the value of the firm's assets $V_T + \phi_T$ plus the new cash raised $e(1 - m)$ less the debt service $\phi_T + f_T + e(1 - m)$ and less the amount of equity e that belongs to the "new" equity holders; that is, the value of the old equity is $V_T - f_T - e$.

Now issuance of new equity involves costs and dilution. Thus, equity holders will be willing to do this only if they anticipate the offer will be accepted by the debt holders. We also require, of course, that the value of old equity remain non-negative. Subject to these two considerations, equityholders acting non-strategically aim to raise enough equity to pay off the entire debt, or at least as much as possible without making equity values negative.

The behavior in earlier periods is similar to that under strategic debt service with obvious modifications. We will not repeat the details here.

[2] An extended comment is relevant here. With infinite equity-issuance costs, "non-strategic" debt service has a natural meaning: equity holders hand over whatever cash they have up to the face value of the debt. If there is underperformance, debt holders choose between taking this and liquidating the firm. With finite equity-issuance costs, it remains true that non-strategic debt-service requires equity holders to pay off creditors in full if they have adequate cash and to offer them all available cash if they do not. But an additional question arises here: in the event of inadequate cash to meet the full debt-service requirement, should non-strategic equity-holders offer to raise the difference through issuing new equity as long as this leaves the value of equity non-negative? We assume that yes, they do; that is, that non-strategic behavior means repaying as much of the debt as possible without negative equity values. This appears to us to be the correct way to proceed. If one makes the alternative assumption that non-strategic debt-service only relates to handling of existing cash reserves, not of equity-issuance, then equity-holders who lack adequate cash will raise just enough cash to stave off liquidation, which means debt-service will effectively be strategic in equilibrium.

3.3 Numerical analysis of equilibrium

The model described above is evidently not amenable to analytic solution but may easily be solved numerically. We present the results of our numerical analysis in this section.

Our parameterizations are similar to those chosen by Merton (1974) or Anderson and Sundaresan (1996). The initial firm value is normalized throughout to $V_0 = 1$, and the zero-coupon debt is taken to have a maturity of 10 years. The constant α relating V_t to period t cash flow f_t is taken to be 0.025. Each period of the binomial tree is taken to be of 6 months. The riskless interest rate ρ in each period is set to 0.0247 which corresponds to a continuously-compounded annual rate of 5%. The annualized volatility of the $\{V_t\}$ process is set to about $\sigma = 31.6\%$ (i.e., $\sigma^2 = 0.10$). The parameters u, d and p are chosen to match this annualized volatility subject to the constraints $u = 1/d$ and $\alpha = [1 - \beta(pu + (1-p)d)]$. Concerning the other parameters, we used the following configurations:

1. Three values are considered for the variable cost of issuing equity: $m_1 \in \{0, 0.15, 0.99\}$, with the extreme values $m_1 = 0$ and $m_1 = 0.99$ corresponding, respectively, to the case of costless and (almost) infinitely-costly equity.
2. Three values are considered for $\bar{c}$: 0.25, 0.50, 0.75. Given the initial firm value of unity, these cover the range from relatively safe to high-risk debt.
3. Two values are considered for the fixed cost of liquidation ($l_0 = 0, 0.20$), and one for the variable cost ($l_1 = 0.25$).

Equilibrium equity and debt values for these configurations are reported in Table 1. The numbers there strongly bear out the intuitive arguments made above. Equity values are always higher under strategic debt service than under non-strategic debt service indicating that the presence of this additional option cannot hurt equity holders. However, the gain is maximal when equity-issuance costs are also small. When equity-issuance costs are small, debt values are lower under strategic debt service than non-strategic debt-service and yield spreads substantially wider (in some cases, the difference exceeds 200 basis points). However, the difference becomes negligible for high equity-issuance costs and in some cases even changes sign with *lower* yield spreads under strategic debt service.

The implications of Table 1 that strategic debt-service does not widen spreads much – and could even *narrow* them – when equity-issuance costs are high is in direct contradiction to the finding of Anderson and Sundaresan (1996) that strategic debt-service widens spreads significantly in this case. Indeed, as we show in section 4, the result that strategic debt-service has a relatively minor impact at high equity-issuance costs continues to obtain when – as Anderson and Sundaresan (1996) assume – cash reserves are prohibited in the model.

The source of the discrepancy appears to be the following. In their model description, Anderson and Sundaresan (1996) explicitly state that in any period $t < T$, new equity issuance is disallowed so forced liquidation occurs if there is inadequate periodic cash flow to meet the minimum debt-service requirement (see, e.g., p.47 of their paper). They maintain this condition in describing equilibrium through periods $t < T$.

However, in an apparent violation of this condition, their analysis of equilibrium allows period T debt-service to be any amount in $[0, V_T]$ rather than any amount in $[0, f_T]$. This implies that the costs of new equity-issuance are zero *at*

Table 1 Strategic versus non-strategic debt-service

		Strategic debt service			Change under non-strategic debt-service		
$\bar{c}$	m_1	V^E	V^D	Spread	Diff. in V^E	Diff. in V^D	Diff. in Spread
$l_0 = 0.0, l_1 = 0.25$							
25	0.00	86.8	13.2	128.35	−0.57	+0.57	−42.98
	0.15	86.4	13.2	125.49	−0.28	+0.24	−17.87
	0.99	86.1	13.7	93.03	0.00	0.00	0.00
50	0.00	78.6	21.4	341.32	−2.47	+2.47	−112.05
	0.15	76.9	22.0	312.90	−0.90	+0.85	−38.99
	0.99	75.7	21.9	316.18	0.00	0.00	0.00
75	0.00	73.2	26.8	530.12	−4.04	+4.04	−146.73
	0.15	70.5	27.4	505.8	−1.73	+1.60	−59.25
	0.99	67.5	27.2	515.4	0.00	+0.00	0.00
$l_0 = 20, l_1 = 0.25$							
25	0.00	90.9	9.1	506.46	−4.62	+4.62	−421.09
	0.15	90.3	9.1	506.46	−4.22	+4.32	−398.85
	0.99	88.5	11.5	270.80	−2.48	+1.35	−113.84
50	0.00	85.0	15.0	714.65	−8.87	+8.87	−485.38
	0.15	83.8	15.5	678.48	−7.74	+7.34	−404.57
	0.99	78.2	19.0	467.02	−2.74	−1.62	+93.52
75	0.00	80.7	19.3	878.24	−11.50	+11.50	−494.85
	0.15	79.1	19.4	876.04	−10.27	+9.65	−429.49
	0.99	69.6	22.0	741.19	−2.74	−1.62	+82.86

This table presents the results of our numerical analysis for the parametrizations described in section 3.3. The third, fourth and fifth columns of the table present, respectively, equity value V^E, debt value V^D, and debt yield spreads over the riskless rate (measured in basis points) when debt-service is strategic. The sixth, seventh, and eighth columns present the changes that result in these values if debt-service is non-strategic. Thus, a negative coefficient under equity values means equity values are lower under non-strategic debt-service than under strategic debt-service

date T alone. If equity issuance is costless (even if only in period T), then, of course, strategic debt service will matter for the reasons we have outlined above. However, that if equity financing is prohibited in all periods (including T), then *strategic* underperformance will rarely even be feasible, since the periodic cash flow f_T is not likely, by itself, to exceed the minimum reservation payment that must be made to debt holders; in this case, if sizeable spreads obtain, they are likely the result of *liquidity* defaults caused by the prohibition on cash reserves imposed in their model.

4 On optimal cash management policies

The driving force behind our finding that strategic debt service may *reduce* spreads under some circumstances is the "interaction of optionalities" between cash mangement and strategic debt-service. This interaction is not possible in Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) because, as mentioned in the Introduction, neither paper allows for optimal cash management. Rather, both require that all available cash flows be paid out in each period as either debt-service or dividends. It is easy to see that under this assumption, strategic debt service *cannot* narrow spreads:

Proposition 4.1 *If no cash reserves are allowed, strategic debt-service leads to wider spreads than non-strategic service.*

Proof With zero cash reserves, debt claims must be paid out of current cash flows and equity-issuance. So, in any terminal state, the payoff to debt holders under strategic debt service is less than or equal to what they would receive under non-strategic debt service. Thus, equilibrium debt values are lower, and spreads higher, under strategic debt service. □

Combined with our earlier results, Proposition 4.1 shows that the zero-reserve requirement can exaggerate the importance of strategic debt service. Nor does the requirement of zero cash-reserves seem a mild one. A prohibition on forms holding cash reserves could overstate liquidity defaults and inflate spreads. It also goes against casual observation that firms in practice hold (even substantial) reserves of cash. This raises the question: when, if ever, is a zero cash reserve requirement optimal? Here is the answer:

Proposition 4.2 *Suppose either (a) there are no liquidation costs, or (b) there are no equity-issuance costs. Then, zero cash-reserves are fully optimal; that is, it is an optimal policy to pay out all cash flows as dividends in each period up to and including* $T-1$.

Proof First, note that it is always optimal to pay *no* dividends up to and including period $T-2$, and to simply retain all cash flows up to that point as reserves. Giving up a dollar of dividends in any period $t \le T-2$ entails no present-value loss to equity holders if the dollar (plus interest at the rate ρ) will be returned as dividends in period $T-1$. Now, the only circumstance in which equity holders will choose to *not* pay out all accumulated cash as dividends in period $T-1$ is where equity value can be enhanced by retaining some cash in the firm because it helps avoid a costly cash shortfall in period T. Thus, the zero-dividend policy either leaves investors no worse off (if all accumulated cash is paid out in period $T-1$) or strictly better off (if the retained cash helps prevent costly liquidation or raising of new equity). The claim follows.

On the other hand, it is easy to see that under either of the conditions in the statement of the Proposition, there is no gain to be made by equity holders in carrying cash from $T-1$ to T, but there is an important possible loss: if liquidation occurs at T, debt holders have first claim on the firm's assets including the cash reserves. Thus, it cannot be optimal to carry cash reserves into T, so it is optimal to pay out the entire cash reserves as dividends in period $T-1$.

From a present value standpoint, retaining all cash flows in the firm up to period $T-1$ and paying them out as dividends at that point is equivalent to paying out all cash flows as dividends as they occur. □

If equity-issuance costs and liquidation costs are non-zero (as they are in practice), it is easy to see that zero reserve policies are not generally optimal since the reserves can prevent deadweight losses from liquidation or the costs of equity issuance. (Indeed, one can even construct examples where zero reserves are *strictly* suboptimal for *any* non-zero equity-issuance cost and *any* non-zero liquidation cost. As an instance, see Example 5.4 of Acharya, Huang, Subrahmanyam, and Sundaram (2002).) In general, the higher are new equity-issuance costs, the more restrictive is the prohibition on carrying cash reserves.

Table 2 Strategic vs non-strategic debt service under zero cash reserves

$\bar{c}$	m_1	V^E	V^D	Spread (bps)	Change in V^E	Change in V^D	Change in spread
$l_0 = 0.0, l_1 = 0.25$							
25	0.00	86.2	13.2	128.35	−0.57	+0.57	−42.98
	0.15	84.8	13.2	128.35	−0.32	+0.27	−20.73
	0.99	72.7	13.2	128.35	−1.92	0.00	0.00
50	0.00	78.6	21.4	341.32	−2.47	+2.47	−112.05
	0.15	75.1	21.4	341.32	−1.25	+1.06	−49.89
	0.99	64.0	21.4	341.32	0.00	0.00	0.00
75	0.00	73.2	26.8	530.12	−4.04	+4.04	−146.73
	0.15	68.7	26.8	530.12	−2.21	+1.88	−70.86
	0.99	58.6	26.8	530.12	0.00	0.00	0.00
$l_0 = 20, l_1 = 0.25$							
25	0.00	90.9	9.1	506.46	−4.62	+4.62	−421.09
	0.15	89.5	9.1	506.46	−5.08	+4.32	−398.85
	0.99	68.0	9.1	506.46	−3.88	+0.07	−8.15
50	0.00	85.0	15.0	714.65	−8.87	+8.87	−485.38
	0.15	82.6	15.0	714.65	−8.78	+7.47	−423.22
	0.99	61.6	15.0	714.65	−2.05	+0.07	−5.06
75	0.00	80.7	19.3	878.24	−11.50	+11.50	−494.85
	0.15	77.5	19.3	878.24	−10.97	+9.33	−418.98
	0.99	57.2	19.3	878.24	−2.05	+0.07	−3.99

This table presents the results of our numerical analysis for the model of Section 4 when the firm is not allowed to carry cash reserves. The third, fourth and fifth columns of the table present, respectively, equity value V^E, debt value V^D, and debt yield spreads over the riskless rate (measured in basis points) when debt-service is strategic. The sixth, seventh, and eighth columns present the changes that result in these values if debt-service is non-strategic. Thus, a negative coefficient under equity values means equity values are lower under non-strategic debt-service than under strategic debt-service

Nonetheless, from a qualitative standpoint, our main result that strategic debt-service has a large effect only under low equity-issuance costs holds up even if reserves are prohibited. Table 2 considers the same model as Table 1 but calculates spreads under strategic and non-strategic policies *assuming a compulsory zero-reserve policy*. As the table shows, the effect of strategic debt service is large at low equity-issuance costs but negligible when these costs are high.

5 Conclusions

It has been suggested in recent work that strategic underperformance of debt-service obligations by equity holders raises yield spreads and can resolve the gap between observed yield spreads and those generated by Merton (1974)–style models. However, the models offered in support have placed somewhat strong restrictions on two other important "optionalities" available in principle to equity holders: the option to carry cash reserves as protection against costly liquidation, and the option to raise cash by issuing new equity. This makes it hard to separate the effect of strategic behavior from the impact of the other constraints.

Our paper removes these restrictions and disentangles the effect of the three factors, characterizing their individual impact as well as their interdependence. We find that while each factor could have a significant impact on equilibrium, there is a strong “interaction of optionalities” that determines when a factor is important. In particular, we show that strategic debt-service has a large impact on equilibrium when new equity-issuance costs are low, but that at high equity-issuance costs, its impact is much diminished. More strikingly (and certainly more unintuitively at first sight), we show that strategic debt-service could actually *lower* spreads compared to non-strategic debt service.

In the current paper, we have considered only zero-coupon debt, but in Acharya et al. (2002), we note that similar, and even more complex, results obtain for coupon debt. Our results thus qualify the impact previous papers have claimed for strategic debt-service. Ipso facto, they also have important implications for empirical work in this field.

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