_Electron in Electric or Magnetic Field_

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(Dated: September 30, 2008)

A point charge \( q \) placed at \( \vec{r}_0 \) will induce an electric field in the space,

\[
\vec{E}(\vec{r}) = \frac{kq}{|\vec{r} - \vec{r}_0|^3} (\vec{r} - \vec{r}_0)
\]

Eqn.(1)

\[
U(\vec{r}) = \frac{kq}{|\vec{r} - \vec{r}_0|},
\]

Eqn.(2)

where \( k = \frac{1}{4\pi\varepsilon_0} \) and \( U(\vec{r}) \) is the electric potential. From the equation, electric field satisfies superposition principle, which means, it can be calculated by adding the field produced by each point charge, so does the potential.

“ElectricField2PlotApp.java” displays the electric field lines of 2 static point charges (2D). The drawing process is actualized by several steps:

1) Place the static point charges at the space. After setting an integer \( N \) as the number of electric field lines coming out of each point charge, place \( N \) points (seeds) very close to each point charge, equally rounding which in the center. Here “very close” is set by step size parameter ‘\( dr \)’.

2) For each point, calculate the electric field magnitude and x, y components (carried by “ElectricField2.java”). The next point will be,

\[
x = x + dr \times E_x(x, y) / E(x, y) \times \text{Sign}(q)
\]

\[
y = y + dr \times E_y(x, y) / E(x, y) \times \text{Sign}(q)
\]

Here, \( \text{Sign}(q) \) is used to make sure that the iteration goes out from the point charge, \( dr \) is the step size.

3) In the calculating process, something may happen to make it NEVER stop for the loop of calculation, for example, the position point is too close to one point charge, or the electric field goes to zero. Make sufficient conditions to eliminate those situations.

4) Plot the graph.

Here show some graphs got from the program. Figure 1 shows the electric field lines of two oppositely charged points and Figure 2 is that of two same charged points. Figure 3 displays the situation of \((+q, -2q)\) and Figure 4 is that of \((+q, +2q)\).

Some defects result from the finite size of \( dr \) and the eliminations in the calculation. For example, in the middle area of Figure 1, by theory, one field line coming out of left charge
should end at the right charge, i.e., two lines should coincide with each other. However, since \( dr \) is not infinitely small in the numerical calculation, we get a small difference between them. In Figure 3, for some large field, the iteration will end therefore blank area comes at the corners of the square.

The electric field applies a force on electrons in the space,

\[
\vec{F}(\vec{r}) = e\vec{E}(\vec{r})
\]

\( e \) here is the charge of the electron. The motion of an electron in electric field displayed above is simulated in “ElectronInElectricField2App.java”. Figure 5, 6, 7 and 8 display four types of 2D movement of the electron under different conditions. The total energy of the system, equal to the kinetic energy adding with the potential energy, stays a constant in the movement, which can be derived from the fact that the Hamiltonian is time-independent. The \( z \) component of angular momentum is not a constant since the force does not go through the origin (we have two point charges here).
Figure 5: Initial conditions (I) and motion of electron in electric field:
There is only one fixed point charge at (-1, 0) to provide electric field in the space.

Figure 6: Initial conditions (II) and motion of electron in electric field:

Figure 7: Initial conditions (III) and motion of electron in electric field

Figure 8: Initial conditions (IV) and motion of electron in electric field
The magnetic field induced by a static line current is defined by Biot-Savart Law, saying,

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{|\vec{r}|^3}$$  \hspace{1cm} \text{Eqn. (4)}

The total \(\vec{B}\) is the integral of the current, calculated by summation of all small \(d\vec{l}\)'s in the program “MagneticFiled2.java”. (I failed on drawing magnetic field lines in the same way as electric field lines, since magnetic field does not have a source while as field does.)

Electrons in the magnetic field are applied a force on if it is not at rest,

$$\vec{F} = e\vec{v} \times \vec{B}$$  \hspace{1cm} \text{Eqn. (5)}

According to the equation, the force is perpendicular to both the velocity and the field hence will not change the kinetic energy of the electron.

“ElectronInMagneticField2PlotApp.java” gives the 3D motion of an electron in a magnetic field produced by 2 parallel-oriented infinite straight-line-currents, displayed by 2 graphs (x-y motion, y-z motion). By setting one current to be zero, we get a beautiful motion displayed in Figure 9 (motion in x-y plane, perpendicular to the current). After a long enough time, we find that the electron is confined to move in a circular strip.

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Figure 9: Initial conditions (I) and x-y motion of electron in magnetic field

There is only one fixed line current at (-1, 0) going in z direction.

Figure 10 displays the motion of the electron under the condition that 2 currents are of the same magnitude and direction. Figure 11 is that 2 currents are of opposite direction and the same magnitude. All of these motions can be solved by differential equations, which will be really tough work.
Figure 10: Initial conditions (II) and x-y motion of electron in magnetic field

Figure 11: Initial conditions (III) and x-y motion of electron in magnetic field