Factor $C\phi = \tilde{g}$ to $(LU)\phi = \tilde{g}$

Introduce an auxiliary vector

$v = U\phi$

thus

$Lv = \tilde{g}$

and

$U\phi = v$

Solution will contain roundoff error.

Define error $\varepsilon^{(0)} = \phi - \hat{\phi}^{(0)}$ ← Result from direct elimination

Rearrange

$\phi = \hat{\phi}^{(0)} + \varepsilon^{(0)}$

If we can find $\varepsilon^{(0)}$ we can obtain a better estimate for $\phi$ using

$\hat{\phi}^{(0)} = \hat{\phi}^{(0)} + \varepsilon^{(0)}$

How do we find $\varepsilon^{(0)}$?

Establish a residual vector

$D^{(0)} = \tilde{g} - C\hat{\phi}^{(0)}$

Since $C\phi = \tilde{g}$

Thus using $C = LU$

$(LU)\varepsilon^{(0)} = D^{(0)}$ and $U\varepsilon^{(0)} = \tilde{v}$ auxiliary vector

1st

$L\tilde{v} = D^{(0)}$ Forward substitution to find $\tilde{v}$

2nd

$U\varepsilon^{(0)} = \tilde{v}$ Backward substitution to find $\varepsilon^{(0)}$

3rd

$\hat{\phi}^{(0)} = \hat{\phi}^{(0)} + \varepsilon^{(0)}$

Repeat the procedure until a satisfactory converged solution is obtained.