



Define an array of equations giving the expansion about the point of evaluation for temperatures at the four nearest points. MathCad has trouble propagating actual derivative expressions through the symbolic algebra operations, so we fake it with a variable named $dTdx$. I'm not going to operate directly on elements of this array, because MathCad again becomes confused about their contents, due to undefined variables. This is just a convenient source for cut and paste operations in what follows

$$\left[T_0 - T_E + \sum_{n=1}^4 \frac{\Delta x^n \cdot dTdx^n}{n!} \right]$$

$$\left[\begin{array}{l}
T_o - T_{EE} + \sum_{n=1}^4 \frac{(2\Delta x)^n \cdot dTdx^n}{n!} \\
T_o - T_W + \sum_{n=1}^4 \frac{(-\Delta x)^n \cdot dTdx^n}{n!} \\
T_o - T_{WW} + \sum_{n=1}^4 \frac{(-2\Delta x)^n \cdot dTdx^n}{n!} \\
T_o - T_N + \sum_{n=1}^4 \frac{\Delta y^n \cdot \frac{d^n}{dy^n} T(x,y)}{n!} \\
T_o - T_{NN} + \sum_{n=1}^4 \frac{(2\Delta y)^n \cdot \frac{d^n}{dy^n} T(x,y)}{n!} \\
T_o - T_S + \sum_{n=1}^4 \frac{(-\Delta y)^n \cdot \frac{d^n}{dy^n} T(x,y)}{n!} \\
T_o - T_{SS} + \sum_{n=1}^4 \frac{(-2\Delta x)^n \cdot \frac{d^n}{dy^n} T(x,y)}{n!}
\end{array} \right]$$

Now multiply the x expansions by coefficients A and B, and sum them. Look at the coefficients of the derivative terms using the "coeffs" symbolic operator. Note that the coefficients of A and B were obtained by copy and paste from the expansions above.

$$\begin{aligned}
 & A \cdot \left[T_o - T_E + \sum_{n=1}^6 \frac{\Delta x^n \cdot dTdx^n}{n!} \right] \dots \\
 & + B \cdot \left[T_o - T_W + \sum_{n=1}^6 \frac{(-\Delta x)^n \cdot dTdx^n}{n!} \right] \dots \\
 & + C \cdot \left[T_o - T_{EE} + \sum_{n=1}^6 \frac{(2\Delta x)^n \cdot dTdx^n}{n!} \right] \dots \\
 & + D \cdot \left[T_o - T_{WW} + \sum_{n=1}^6 \frac{(-2\Delta x)^n \cdot dTdx^n}{n!} \right]
 \end{aligned}
 \quad \text{coeffs, dTdx} \rightarrow
 \left(\begin{array}{l}
 -D \cdot T_{WW} + D \cdot T_o - C \cdot T_{EE} + C \cdot T_o + A \cdot T_o - A \cdot T_E + B \cdot T_o - B \cdot T_W \\
 -2 \cdot D \cdot \Delta x + A \cdot \Delta x - B \cdot \Delta x + 2 \cdot C \cdot \Delta x \\
 \frac{1}{2} \cdot B \cdot \Delta x^2 + \frac{1}{2} \cdot A \cdot \Delta x^2 + 2 \cdot D \cdot \Delta x^2 + 2 \cdot C \cdot \Delta x^2 \\
 \frac{-4}{3} \cdot D \cdot \Delta x^3 + \frac{4}{3} \cdot C \cdot \Delta x^3 - \frac{1}{6} \cdot B \cdot \Delta x^3 + \frac{1}{6} \cdot A \cdot \Delta x^3 \\
 \frac{2}{3} \cdot C \cdot \Delta x^4 + \frac{1}{24} \cdot B \cdot \Delta x^4 + \frac{2}{3} \cdot D \cdot \Delta x^4 + \frac{1}{24} \cdot A \cdot \Delta x^4 \\
 \frac{4}{15} \cdot C \cdot \Delta x^5 - \frac{4}{15} \cdot D \cdot \Delta x^5 - \frac{1}{120} \cdot B \cdot \Delta x^5 + \frac{1}{120} \cdot A \cdot \Delta x^5 \\
 \frac{4}{45} \cdot C \cdot \Delta x^6 + \frac{4}{45} \cdot D \cdot \Delta x^6 + \frac{1}{720} \cdot A \cdot \Delta x^6 + \frac{1}{720} \cdot B \cdot \Delta x^6
 \end{array} \right)$$

The coefficient of the first derivative must be zero. Paste that into the equation solution step below. The coefficient of the second derivative is taken to be minus one to ease interpretation of the final results. Hit "Ctrl Shift ." and insert the "solve" symbolic operation followed by a comma and the list of variables for

$$\left(\begin{array}{l} A \cdot \Delta x - B \cdot \Delta x + 2 \cdot C \cdot \Delta x - 2 \cdot D \cdot \Delta x = 0 \\ \frac{1}{2} \cdot A \cdot \Delta x^2 + \frac{1}{2} \cdot B \cdot \Delta x^2 + 2 \cdot C \cdot \Delta x^2 + 2 \cdot D \cdot \Delta x^2 = -1 \\ \frac{1}{6} \cdot A \cdot \Delta x^3 - \frac{1}{6} \cdot B \cdot \Delta x^3 + \frac{4}{3} \cdot C \cdot \Delta x^3 - \frac{4}{3} \cdot D \cdot \Delta x^3 = 0 \\ \frac{1}{24} \cdot A \cdot \Delta x^4 + \frac{1}{24} \cdot B \cdot \Delta x^4 + \frac{2}{3} \cdot C \cdot \Delta x^4 + \frac{2}{3} \cdot D \cdot \Delta x^4 = 0 \end{array} \right) \text{solve, } \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \rightarrow \left(\begin{array}{cccc} -4 & -4 & 1 & 1 \\ 3 \cdot \Delta x^2 & 3 \cdot \Delta x^2 & 12 \cdot \Delta x^2 & 12 \cdot \Delta x^2 \end{array} \right)$$

Unfortunately the operation above doesn't actually assign the resulting values to A and B. The next two assignments do that for you.

$$A := \frac{-4}{(3 \cdot \Delta x^2)} \quad B := \frac{-4}{(3 \cdot \Delta x^2)} \quad C := \frac{1}{(12 \cdot \Delta x^2)} \quad D := \frac{1}{(12 \cdot \Delta x^2)}$$

Now I copy and paste the coefficient array from above, select it, and hit "Ctrl ." to do a symbolic evaluation, including replacement of values of A and B. The array to the right contains final values of coefficients in the summed Taylor expansions. The first element is the approximation for the second derivative. the last element is the coefficient of the sixth derivative, indicating the approximation is fourth order.

$$\left(\begin{array}{l}
 A \cdot T_o - A \cdot T_E + B \cdot T_o - B \cdot T_W + C \cdot T_o - C \cdot T_{EE} + D \cdot T_o - D \cdot T_{WW} \\
 A \cdot \Delta x - B \cdot \Delta x + 2 \cdot C \cdot \Delta x - 2 \cdot D \cdot \Delta x \\
 \frac{1}{2} \cdot A \cdot \Delta x^2 + \frac{1}{2} \cdot B \cdot \Delta x^2 + 2 \cdot C \cdot \Delta x^2 + 2 \cdot D \cdot \Delta x^2 \\
 \frac{1}{6} \cdot A \cdot \Delta x^3 - \frac{1}{6} \cdot B \cdot \Delta x^3 + \frac{4}{3} \cdot C \cdot \Delta x^3 - \frac{4}{3} \cdot D \cdot \Delta x^3 \\
 \frac{1}{24} \cdot A \cdot \Delta x^4 + \frac{1}{24} \cdot B \cdot \Delta x^4 + \frac{2}{3} \cdot C \cdot \Delta x^4 + \frac{2}{3} \cdot D \cdot \Delta x^4 \\
 \frac{4}{15} \cdot C \cdot \Delta x^5 - \frac{4}{15} \cdot D \cdot \Delta x^5 + \frac{1}{120} \cdot A \cdot \Delta x^5 - \frac{1}{120} \cdot B \cdot \Delta x^5 \\
 \frac{4}{45} \cdot C \cdot \Delta x^6 + \frac{4}{45} \cdot D \cdot \Delta x^6 + \frac{1}{720} \cdot A \cdot \Delta x^6 + \frac{1}{720} \cdot B \cdot \Delta x^6
 \end{array} \right) \text{factor} \rightarrow \left[\begin{array}{l}
 \frac{-1}{12} \cdot \frac{(30 \cdot T_o - 16 \cdot T_E - 16 \cdot T_W + T_{EE} + T_{WW})}{\Delta x^2} \\
 0 \\
 -1 \\
 0 \\
 0 \\
 0 \\
 \frac{1}{90} \cdot \Delta x^4
 \end{array} \right]$$