Time Level Selection

Explicit

$$(\rho_j^{n+1} - \rho_j^n) / \Delta t + \nabla_j \cdot (\rho^n V^n) = 0 ; \qquad (1)$$

Implicit

$$(\rho_j^{n+1} - \rho_j^n) / \Delta t + \nabla_j \cdot (\rho^{n+1} V^{n+1}) = 0 ; \qquad (2)$$

Semi-Implicit

$$(\rho_j^{n+1} - \rho_j^n) / \Delta t + \nabla_j \cdot (\rho^n V^{n+1}) = 0 ; \qquad (3)$$

Crank Nicholson

$$(\rho_j^{n+1} - \rho_j^n) / \Delta t + .5 \; [\nabla_j \cdot (\rho^n V^n) + \nabla_j \cdot (\rho^{n+1} V^{n+1})] = 0 . \qquad (4)$$

Here The finite volume divergence operator is:

$$\nabla_j \cdot (\rho V) = (A_{j+1/2} \langle \rho V \rangle_{j+1/2} - A_{j-1/2} \langle \rho V \rangle_{j-1/2}) / vol_j . \qquad (5)$$

Common Spatial Averages

Donor Cell or Upwind

$$\langle \rho V \rangle_{j+1/2} = \begin{cases} \rho_j V_{j+1/2}\ , & V_{j+1/2} \geq 0 \\ \rho_{j+1} V_{j+1/2}\ , & V_{j+1/2} < 0\ . \end{cases} \tag{6}$$

Central Difference

$$\langle \rho V \rangle_{j+1/2} = 0.5\ (\rho_j + \rho_{j+1})\ V_{j+1/2} \tag{7}$$

Full Semi-Implicit Equations

Motion

$$(V^{n+1}_{j+1/2} - V^{n}_{j+1/2})\ /\ \Delta t + V^{n}_{j+1/2}\ \nabla_{j+1/2}\ V^{n} + \frac{1}{\langle \rho \rangle^{n}_{j+1/2} \Delta x_{j+1/2}} (p^{n+1}_{j+1} - p^{n+1}_{j}) + K^{n}_{j+1/2}\ V^{n+1}_{j+1/2}\ \left| V^{n}_{j+1/2} \right| = 0\ ; \tag{8}$$

$$V_{j+1/2}\ \nabla_{j+1/2}\ V = \begin{cases} V_{j+1/2}\ (V_{j+1/2} - V_{j-1/2})\ /\ \Delta x_{j+1/2}\ , & V_{j+1/2} \geq 0 \\ V_{j+1/2}\ (V_{j+3/2} - V_{j+1/2})\ /\ \Delta x_{j+1/2}\ , & V_{j+1/2} < 0\ , \end{cases} \tag{9}$$

Mass

$$(\rho^{n+1}_{j} - \rho^{n}_{j})\ /\ \Delta t + \nabla_j \cdot (\rho^{n} V^{n+1}) = 0\ ; \tag{10}$$

Energy

$$(\rho^{n+1}_{j}\ e^{n+1}_{j} - \rho^{n}_{j}\ e^{n}_{j})\ /\ \Delta t + \nabla_j \cdot (\rho^{n} e^{n} V^{n+1}) + p^{n+1}_{j}\ \nabla_j \cdot (V^{n+1}) = q_j \tag{11}$$

Fully Conservative Energy Equation

$$\frac{\partial[\rho(e + 1/2\ V^2)]}{\partial t} + \nabla \cdot [\rho(e + p/\rho + 1/2\ V^2)\vec{V}] = q_j \qquad \textbf{(12)}$$

Finite Difference References

Patrick J. Roache, Computational Fluid Dynamics (Hermosa Publishers)

R. D. Richtmyer and K. W. Morton, Difference Methods For Initial Value Problems (Interscience Publishers)