1. Estimating Muscle Forces

Chao and An (1978)

"However, in examining the voluntary functions of an anatomical structure, there is a striking ability for it to produce consistent functional activities. Such unique ability must be controlled by certain physiological laws based on which proper solutions to the redundant problem may be obtained. Consequently, it would be important for the biomechanicians to quantitate these laws, not only for the purpose of seeking basic understanding of how musculoskeletal systems function but also to establish workable models and theories for the analysis of abnormal functions in the hope of providing valuable clinical information." (page 159).

This idea is a common in biomechanics. One attempt to quantify these laws has been to assume that muscle forces are estimated so that some measure of load on the system is minimized. For example Penrod et al. (1974) proposed that muscles are recruited to satisfy a function which minimizes the sum of the muscle forces, which they claimed was equivalent to minimizing the muscular effort. They claimed that this function, "....is intuitively appealing and may represent an accurate picture for a normal, healthy system familiar with the loads it sustains." (page 128)

Crowninshield and Brand (1981) proposed that minimizing the muscle stress effectively increased endurance time, so was the function via which muscle forces are selected.

2. Optimization Theory

There is a well established set of mathematical techniques which permit the minimization of a function (static optimization). The minimization of a function is set-up in the following way,

**Objective Function**

Minimize \[ U(X_1, X_2, \ldots, X_n) \]

subject to

**Equality Constraints**

\[ \Phi_k(X_1, X_2, \ldots, X_n) = 0 \quad k = 1, e \]

**Inequality Constraints**

\[ \Phi_I(X_1, X_2, \ldots, X_n) \geq 0 \quad I = 1, f \]

Where

- \( n \) - number of independent variables
- \( X_i \) - independent variables
- \( e \) - number of equality constraints,
- \( f \) - number of inequality constraints.

Imagine a two muscles system which are generating a moment of 30 N.m at a joint and the muscles have moment arms of 0.01 m and 0.03 m at this joint, and can produce a maximum forces of 1500 and 1000 N respectively. If the moment is produced by these muscles in such a way that the sum of the muscle forces squared is minimized, the objective function is,
Objective Function

\[ \text{Minimize} \Rightarrow \sum F_m^2 = F_{M_1}^2 + F_{M_2}^2 \]

The equality constraint would be to make sure the net moment produced by the muscles is equal to the moment at the joint,

Equality Constraint

\[ 0.01 \times F_{M_1} + 0.03 \times F_{M_2} = 30.0 \]

The inequality constraints should be such that the muscles only produce positive forces, and do not produce forces which exceed their maximums,

Inequality constraints

\[ F_{M_1} \geq 0 \text{N} \quad F_{M_2} \geq 0 \text{N} \]
\[ F_{M_1} \leq 1500 \text{N} \quad F_{M_2} \leq 1000 \text{N} \]

3. MATLAB

The M-file `muscle_force` permits the estimation of muscle forces using a number of different criteria. It uses an analytical solution for this class of problems (Challis and Kerwin, 1993). This software can be downloaded from the following webpage,

http://www.personal.psu.edu/faculty/j/h/jhc10/KINES574/Data_Sources.htm

The function is called using the following format,

\[ [F_m] = \text{muscle_force}(M, r, p, A, F_{\text{max}}) \]

The inputs are,

- \( M \) - moment at joint to be satisfied.
- \( r \) - array of moment arms of muscles
- \( p \) - power of objective function
- \( A \) - array of divisors for objective function [optional]
- \( F_{\text{max}} \) - array of maximum force muscles can produce [optional]

The output is

- \( F_m \) - array of muscle forces, which satisfy the constraints and minimize the objective function.

For example for the data and objective function given in section 2,

\[ [F_m] = \text{muscle_force}(30, [0.01 \ 0.03], 2, [1 \ 1], [1500 \ 1000]); \]

This gives a muscle force of 300 N for muscle 1 and 900 for muscle 2. Note as there is no divisor in the objective function array \( A \) is set to all ones.

4. Objective Functions

The following six objective functions are popular in biomechanics for the estimation of individual muscles forces,

**Muscle Force**

\[ U = \sum_{i=1}^{NM} F_i \]

\[ U = \sum_{i=1}^{NM} F_i^3 \]

**Muscle Stress**

\[ U = \sum_{i=1}^{NM} \left( \frac{F_i}{CSA_i} \right) \]

\[ U = \sum_{i=1}^{NM} \left( \frac{F_i}{CSA_i} \right)^3 \]

**Relative Muscle force**

\[ U = \sum_{i=1}^{NM} \left( \frac{F_i}{F_{\text{MAX}i}} \right) \]

\[ U = \sum_{i=1}^{NM} \left( \frac{F_i}{F_{\text{MAX}i}} \right)^3 \]
5. Example Data

The following table gives the properties of the three major elbow flexors, when the elbow is at 130 degrees of flexion (180 degrees is full extension). The maximum muscle force is based on the force-length properties of the muscles.

<table>
<thead>
<tr>
<th>Muscle</th>
<th>Moment Arm (r)</th>
<th>Muscle Cross Sectional Area (A)</th>
<th>Maximum Muscle Force (Fmax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biceps</td>
<td>0.036 m</td>
<td>5.8 cm²</td>
<td>556.3 N</td>
</tr>
<tr>
<td>Brachialis</td>
<td>0.021 m</td>
<td>7.4 cm²</td>
<td>878.8 N</td>
</tr>
<tr>
<td>Brachioradialis</td>
<td>0.054 m</td>
<td>2.0 cm²</td>
<td>92.5 N</td>
</tr>
</tbody>
</table>

The above data are used to estimate the muscle forces required to produce for flexion moments at the elbow joint varying from 1 to 40 N.m.

Objective Function: \[ U = \sum_{i=1}^{NM} F_i^3 \]

Inequality constraints: \[ F_i \geq 0 N \]

There are no upper constraints. The following graph illustrates the results.

6. Caveat

There is no study which thoroughly validates static optimization for the estimation of muscle forces. It is a popular approach, often used after resultant joint moments have been
computed using inverse dynamics. As this approach has not been validated it should be used with caution. Studies should be compared with caution as they often use different objective functions to estimate muscle forces for the same task.

7. References

