DYNAMIC OPTIMIZATION: BASICS

Lecture Overview

- Static versus dynamic optimization?
- What applications?
- Why should optimization apply in human movement?
- How can dynamic optimization be implemented?
- What objective functions are relevant?
- How are dynamic optimization problems solved?
- Solution procedures - Calculus of variation
- Solution procedures - Optimal control
- Solution procedures - Static optimization
- Evaluation criteria
- Generalized performance criterion

STATIC VERSUS DYNAMIC OPTIMIZATION?

Optimization – is the process of minimizing or maximizing the costs/benefits of some action.

Static Optimization – refers to the process of minimizing or maximizing the costs/benefits of some objective function for one instant in time only.

Dynamic Optimization – refers to the process of minimizing or maximizing the costs/benefits of some objective function over a period of time. Sometimes called optimal control.

In both cases equality and inequality constraints can be enforced.

WHAT APPLICATIONS?

Optimization has many applications:-

- Economics
- Mechanical Designs
- Geographical Models
- Providing models with a nervous system
  - Jumping
  - FES
  - Locomotion
  - Reaching tasks
- and many others

WHY SHOULD OPTIMIZATION APPLY IN HUMAN MOVEMENT?
(Repeat)

Examples of Optimized Movement

- Cotes and Meade (1960) examining horizontal treadmill walking showed that the stride frequency selected by the walkers required the least energy expenditure, compared with walking at stride frequencies less than or greater than the selected frequency.

- Cavanagh and Williams (1982) examining running at a constant speed showed subjects selected a stride length which minimized the energy cost for running at that speed.

- McMahon, Valiant, and Frederick (1987) presented evidence to suggest that the preferred style of running also requires the lowest energy cost.

- Brett (1965) presented evidence to suggest that swimming salmon also select the most efficient style.

- Consider natural selection.

HOW CAN DYNAMIC OPTIMIZATION BE IMPLEMENTED?

There are four components of optimization process:-

1. A dynamic system to be controlled
2. A performance criterion which defines movement goal
3. A controller which acts on the system
4. An algorithm which implements the control

HOW CAN DYNAMIC OPTIMIZATION BE IMPLEMENTED?

The task is the following: *produce a movement with specified beginning and end positions which minimizes the muscular stresses squared.*

Objective Function

$$Minimize \int_{t_{start}}^{t_{end}} \sum_{i=1}^{n=7} \left(\frac{F_i}{CSA_i} \right)^2$$

Inequality constraints

$$\left| \theta_j(t_{end}) - \theta_j^{/}(t_{end}) \right| \leq errortol$$

$$\left| \dot{\theta}_j(t_{end}) - \dot{\theta}_j^{/}(t_{end}) \right| \leq errortol$$

Where

n - number of muscles in the model

F_i - moment produced by the i[th] muscle

CSA_i - i[th] muscle cross-sectional area

$\theta_j, \dot{\theta}_j$ - joint angles and angular velocities

$\theta^{/}{}_j, \dot{\theta}^{/}{}_j$ –joint angles and angular velocities predicted by the model

$errortol$ - measure of the error in the data, constrains how close final model joint data should be to the measured data.

WHAT OBJECTIVE FUNCTIONS ARE RELEVANT?

Equivalent Question: How does CNS control human movement?

Jerk (e.g. arm movements)

$$Minimize \int_{t_{start}}^{t_{end}} \left(\dddot{x}^2 + \dddot{y}^2\right) dt$$

Muscle Stress (e.g. walking)

$$Minimize \int_{t_{start}}^{t_{end}} \sum_{i=1}^{NM} \left(\frac{F_i}{CSA_i}\right)^2$$

Mechanical Energy (e.g. Jumping)

$$Maximise \Rightarrow J\left(\theta, \dot{\theta}, t_f\right) = Y_{CM}\left(t_f\right) + \dot{Y}_{CM}^2\left(t_f\right) / 2.g$$

There are many others….

HOW ARE DYNAMIC OPTIMIZATION PROBLEMS SOLVED?

There are a variety of ways in which dynamic optimization problems can be solved and it depends on the nature of the objective function.

Calculus of Variation (e.g. Akhiezer, 1962)
Permits some problems to be simplified, others can be solved analytically

Optimal Control (e.g. Polak and Mayne, 1975)
There are software packages which allow the solution of optimal control problems.

Static Optimization (e.g. Goh and Teo, 1988)
It is sometimes possible to use static optimization techniques to solve dynamic optimization problems.

SOLUTION PROCEDURES
Calculus of Variation

[Permits some problems to be simplified, others can be solved analytically.]

Consider the problem of moving the load shown schematically

The resistance to motion is a function of the mass (**M**), the degree of damping (**D**), and the stiffness (**K**). The task is to move the mass from **x=0** to **x=X** in time **T**.

SOLUTION PROCEDURES
Calculus of Variation

If the equations of motion of the system are written and the objective function formulated an analytical solution can be obtained.

Minimize Velocity $Minimize \int_o^T \left(\dot{x}^2\right) dt$

Gives
$$\dot{x} = c_1$$
$$x = c_1.t + c_o$$

Where c_1, c_o are constants depndent on intitial and ifnal conditions.

Minimize Acceleration $Minimize \int_o^T \left(\ddot{x}^2\right) dt$

Gives
$$\ddot{x} = 2.c_2 + 6.c_3.t$$
$$\dot{x} = 2.c_2.t + 3.c_3.t^2$$
$$x = c_2.t^2 + c_3.t^3$$

Where $c_0, c_1, c_2, c_3,$ are constants dependent on intitial and final conditions.

SOLUTION PROCEDURES
Calculus of Variation

Minimize Jerk $$Minimize \int_{o}^{T} \left(\dddot{x}^2\right) dt$$

Gives $$x = \sum_{i=0}^{5} c_i . t^i$$

Similar simple equations are available for the analytical solution to movement problems commonly accounted in human movement (e.g. Flash and Hogan, 1985).

SOLUTION PROCEDURES
Optimal Control

- There are software packages which allow the solution of optimal control problems. The theory behind these techniques is similar to those for static optimization (Polak and Mayne, 1975; Teo and Moore, 1978).

- The objective function is often simplified using calculus of variation. (see Bryson and Ho, 1975).

- These techniques are generally time consuming, for example Anderson et al. (1995) estimated to simulate a vertical jump using a four segment, eight muscle model took three days of CPU time. Consider an 8 segment, 14 degree of freedom model, actuated by 46 muscle models comprising 120 first-order differential equations. One iteration of an optimal control algorithm takes the following times

IRIS	*21.36 hours*
Cray	*0.77 Hours*
128 Parallel Processors	*0.88 hours*

- It was estimated that 100 iterations would be required for convergence.

SOLUTION PROCEDURES
Static Optimization

- Goh and Teo, (1988) proposed solving optimal control problems using static optimization algorithms.

- The optimal control problem is converted into a parameter optimization approach by dividing up the control history (e.g. muscle activations) into a series of knots from which the full time history can be determined by interpolation.

- This approach normally gives equivalent results to traditional optimal control solutions, but the algorithms used to implement these procedures are generally numerically more robust. Another advantage is that the solutions are normally attained using less processing time (Goh and Teo, 1988).

SOLUTION PROCEDURES
Static Optimization

- Therefore the static optimization techniques can be used, for example
  - Direct Search Technique - Hooke and Jeeves (1961)
  - Downhill Simplex - Nelder and Mead, (1965)
  - Gradient Search - Powell, 1970).

- This approach is particularly well suited to problems which assume a singular bang-bang control for the muscles.

EVALUATION CRITERIA

Two Principle Methods

EMG - Use EMG signal to indicate when the muscles compare this with the activity of the muscles as predicted by the optimization procedure.

Performance Measures – does the predicted movement correspond with actual movement, and/or is the movement outcome the same as with real subjects (e.g. movement time, jump height).

GENERALIZED PERFORMANCE CRITERION

Zajac and Winters (1990) proposed the following objective function which they termed a “generalized performance criterion”

$$Minimize = J = J_{TK} + J_{NM} + J_{BJ}$$

Where the three components are

<u>T</u>ask specific <u>K</u>inematics – track a given trajectory, make hand follow straight line, minimize jerk, etc.

<u>N</u>euro<u>M</u>uscular – minimize muscle stresses, neural effort, etc.

<u>B</u>one <u>J</u>oint – minimize contact forces, avoid certain ranges of motion, etc.

REVIEW QUESTIONS

1) What is meant by dynamic optimization?

2) Why is dynamic optimization considered more physiologically realistic than static optimization?

3) Give some examples of objective functions which can be used for dynamic optimization (name source of function). What are their application?

4) How can dynamic optimization problems be solved?

5) What are the evaluation criteria for dynamic optimization problems?

6) Present a "generalized performance criteria" that you think could be applicable to some aspect of human movement (name the movement).