STATIC OPTIMIZATION: BASICS

Lecture Overview

- What is optimization?
- What applications?
- How can optimization be implemented?
- How can optimization problems be solved?
- Why should optimization apply in human movement?
- How can muscle forces be estimated?
- Example I
- What objective functions are relevant?
- Evaluation Criteria
- Example II
- Example III

WHAT IS OPTIMIZATION?

Optimization – is the process of minimizing or maximizing the costs/benefits of some action.

Example 1.

If you have money to invest you would try and optimize your return by maximizing the interest you get on your money.

Example 2.

You have money to invest, but the higher interest accounts involve risk, so have two competing functions to balance to obtain maximum return (interest and risk).

[**Static Optimization** – refers to the process of minimizing or maximizing the costs/benefits of some action for one instant in time only.]

WHAT APPLICATIONS?

Optimization has many applications:-

- Economics
- Mechanical Designs
- Geographical Models
- Parameter Estimation
- Analysis of Human Movement
- and many others

HOW CAN OPTIMIZATION BE IMPLEMENTED?

There are well established mathematical techniques for solving static optimization problems. These problems are formulated in the following way

Objective Function

Minimize $U(X_1, X_2, \ldots\ldots, X_n)$

subject to

Equality constraints $\Phi_k(X_1, X_2, \ldots\ldots, X_n) = 0 \quad k = 1, e$

Inequality constraints $\Phi_I(X_1, X_2, \ldots\ldots, X_n) \geq 0 \quad I = 1, f$

Where

n - number of independent variables
X_i - independent variables
e - number of equality constraints,
f - number of inequality constraints.

HOW CAN OPTIMIZATION BE IMPLEMENTED?

Minimization versus Maximization

If it is a function U, then

$$\textbf{\textit{Maximize}}\ U(X_1, X_2, \ldots\ldots, X_n)$$

is equivalent to

$$\textbf{\textit{Minimize}}\ -1 \times U(X_1, X_2, \ldots\ldots, X_n)$$

Global Minimum – minimum possible value of a function.

Local Minimum – minimum possible value of a function in a given neighborhood.

HOW CAN OPTIMIZATION PROBLEMS BE SOLVED?

There are many algorithms for solving optimization problems.

Linear Objective Functions

If the objective function is linear the commonest technique is the Simplex algorithm (Dantzig, 1963).

Limitations

The nature of the solution to these linear problems requires that the number of non-zero variables is less than the total number of constraints (equality and inequality) plus one. Therefore without the imposition of equality constraints a linear objective functions will only maximize/minimize one variable.

HOW CAN OPTIMIZATION PROBLEMS BE SOLVED?

There are a variety of numerical techniques for solving non-linear optimization problems.

Techniques include:-

- The direct search technique of Hooke and Jeeves (1961). This procedure does not account for constraints, therefore another routine has to be run in conjunction with it so that a penalty function constrains the solution (Fiacco and McCormick, 1968).

- Another popular technique is the Downhill Simplex (Nelder and Mead, 1965).

- The most powerful class of these techniques requires evaluation of the function and its derivatives, gradient search techniques (e.g. Powell, 1970).

- The determination of a global minimum for non-linear objective functions cannot be guaranteed using numerical techniques (Siddall, 1982).

HOW CAN OPTIMIZATION PROBLEMS BE SOLVED?

It is possible using Lagrangian multipliers and forming the resultant Lagrangian Function to find an analytical solution to the problem to be minimized for certain classes of objective functions (Bertsekas, 1976).

Example

The general problem can be defined as finding the minimum of

$$U(F_{Mi}) = \sum_{i=1}^{n} (F_{Mi})^{P} \qquad P > 1$$

The minimum must be found subject to the constraint

$$g(F_{Mi}) = M - \sum_{i=1}^{n} r_i . F_{Mi} = 0$$

The Lagrangian Function is:-

$$L(F_{Mi}, \lambda) = U(F_{Mi}) + \lambda . g(F_{Mi})$$

Where λ is a Lagrangian multiplier.

HOW CAN OPTIMIZATION PROBLEMS BE SOLVED?

Solution of the Lagrangian Function for the force in the j^{th} muscle gives

$$F_J = M.\left[r_j . \sum_{i=1}^{n} \left\{ \frac{r_i}{r_j} \right\}^{\left(\frac{P}{P-1} \right)} \right]^{-1}$$

This is analytical solution to how the muscle forces would be shared to satisfy a given resultant joint moment when the task is to minimize the sum of the all the muscle forces producing the moment.

WHY SHOULD OPTIMIZATION APPLY IN HUMAN MOVEMENT?

Chao and An (1978)

"However, in examining the voluntary functions of an anatomical structure, there is a striking ability for it to produce consistent functional activities. Such unique ability must be controlled by certain physiological laws based on which proper solutions to the redundant problem may be obtained. Consequently, it would be important for the biomechanicians to quantitate these laws, not only for the purpose of seeking basic understanding of how musculoskeletal systems function but also to establish workable models and theories for the analysis of abnormal functions in the hope of providing valuable clinical information." (page 159).

WHY SHOULD OPTIMIZATION APPLY IN HUMAN MOVEMENT?

Examples of Optimized Movement

- Cotes and Meade (1960) examining horizontal treadmill walking showed that the stride frequency selected by the walkers required the least energy expenditure, compared with walking at stride frequencies less than or greater than the selected frequency.

- Cavanagh and Williams (1982) examining running at a constant speed showed subjects selected a stride length which minimized the energy cost for running at that speed.

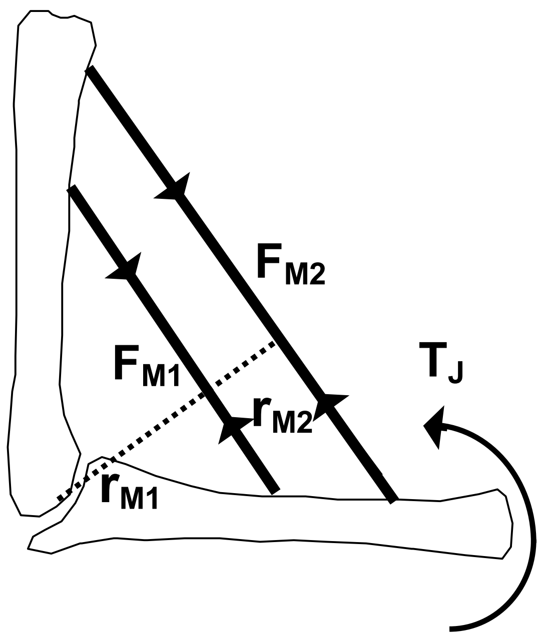
- McMahon, Valiant, and Frederick (1987) presented evidence to suggest that the preferred style of running also requires the lowest energy cost.

- Brett (1965) presented evidence to suggest that swimming salmon also select the most efficient style.

- Consider natural selection.

HOW CAN MUSCLE FORCES BE ESTIMATED?

Consider the following case



Two muscles crossing joint with moment arms

$$r_{M1} = 0.01m \qquad\qquad r_{M2} = 0.03m$$

$$T_J = 30\,N.m$$ (flexion)

$$T_J = \mathbf{30} = \mathbf{0.01} \times F_{M1} + \mathbf{0.03} \times F_{M2}$$

Solution (?) $F_{M1} = 0 \rightarrow 3000\,N$

$$F_{M2} = 0 \rightarrow 1000\,N$$

HOW CAN MUSCLE FORCES BE ESTIMATED?

Objective Function Minimize $U(X_1, X_2, \ldots\ldots, X_n)$

$$Minimize = \sum F_{Mi}^2 = F_{M1}^2 + F_{M2}^2$$

Equality constraints $\Phi_k(X_1, X_2, \ldots\ldots, X_n) = 0 \quad k = 1, e$

$$(0.01 \times F_{M1} + 0.03 \times F_{M2}) - 30 = 0.0$$

Inequality constraints $\Phi_I(X_1, X_2, \ldots\ldots, X_n) \geq 0 \quad I = 1, f$

1. Muscle can only generate force therefore

$$F_{M1} \geq 0N \qquad F_{M2} \geq 0N$$

2. Muscles has limited maximum muscle force, assume both muscle have same cross-sectional area, therefore

$$F_{M1} \leq 800N \qquad F_{M2} \leq 800N$$

Solution $F_{M1} = 600N$ $F_{M2} = 800N$

WHAT OBJECTIVE FUNCTIONS ARE RELEVANT?

- MacConaill (1967)
  "Principle of Minimal Total Muscular Force states that 'no more total muscular force is used than is both necessary and sufficient for the task to be performed, whether this be one of supporting some weight or carrying out a movement, the resistance to which may vary from zero upwards.'" (page 413).

- Penrod, Davy, and Singh (1974) proposed that muscles are recruited to satisfy a function which minimizes the sum of the muscle forces, which they claimed was equivalent to minimizing the muscular effort. They claimed that this function
  "....is intuitively appealing and may represent an accurate picture for a normal, healthy system familiar with the loads it sustains." (page 128)

- Crowninshield and Brand (1981) proposed that minimizing the muscle stress effectively increased endurance time, so was relevant.

WHAT OBJECTIVE FUNCTIONS ARE RELEVANT?

Muscle Force

$$U = \sum_{i=1}^{NM} F_i \qquad\qquad U = \sum_{i=1}^{NM} F_i^3$$

Muscle Stress

$$U = \sum_{i=1}^{NM} \left(\frac{F_i}{CSA_i} \right) = \sum_{i=1}^{NM} \sigma_i \qquad\qquad U = \sum_{i=1}^{NM} \left(\frac{F_i}{CSA_i} \right)^3 = \sum_{i=1}^{NM} \sigma_i^3$$

Relative Muscle force

$$U = \sum_{i=1}^{NM} \left(\frac{F_i}{F_{MAXi}} \right) = \sum_{i=1}^{NM} F_{RELi} \qquad U = \sum_{i=1}^{NM} \left(\frac{F_i}{F_{MAXi}} \right)^3 = \sum_{i=1}^{NM} F_{RELi}^3$$

EVALUATION CRITERIA

Two Principle Methods

EMG - Use EMG signal to indicate when the muscles compare this with the activity of the muscles as predicted by the optimization procedure.

Direct Measurement – compare optimization predicted muscle forces with directly measured muscle forces.

EXAMPLE II

Objective Function

$$U = \sum_{i=1}^{NM} F_i$$

Based on *Principle of Minimal Total Muscular Force* (MacConaill, 1967).

Muscle Data

Muscle	Moment Arm	Maximum Force	Maximum Moment
Biceps	0.036 m	600.6 N	21.6 N.m
Brachialis	0.021 m	1000.9 N	21.0 N.m
Brachioradialis	0.054 m	262.2 N	13.9 N.m

Results

Joint Moment	Biceps Force	Brachialis Force	Brachioradialis Force
5	0.0 N	0.0 N	92.6 N
10	0.0 N	0.0 N	185.2 N
15	0.0 N	0.0 N	277.8 N
20	0.0 N	0.0 N	370.4 N

EXAMPLE II

Objective Function $U = \sum_{i=1}^{NM} F_i$

Question: What is effect of imposing constraints?

Biceps force	<	600.6 N
Brachialis force	<	1000.9 N
Brachioradialis force	<	262.2 N

Note – Basmajian and Latif (1957) Integrated actions and functions of the chief flexors of the elbow: A detailed electromyographic analysis***. Journal of Bone and Joint Surgery*** 39A, 1106-1118.

EXAMPLE III

Source: Herzog, W., and Leonard, T.R. (1991) Validation of optimization models that estimate the forces exerted by synergistic muscles. ***Journal of Biomechanics*** 24, Suppl. 1, 31-39.

Functions Evaluated

1. $\Rightarrow U = \sum_{i=1}^{NM} End.Time_i$

2. $\Rightarrow U = \sum_{i=1}^{NM} \left(\frac{F_i}{CSA_i} \right)^3$

3. $\Rightarrow U = \sum_{i=1}^{NM} F_i^2$

4. $\Rightarrow U = \sum_{i=1}^{NM} \left(\frac{F_i}{F_{MAXi}} \right)^3$

5. $U = \sum_{i=1}^{NM} \left(\frac{F_i}{CSA_i} \right) = \sum_{i=1}^{NM} \sigma_i$

REVIEW QUESTIONS

1) What is meant by optimization?

2) Write an objective function, equality constraint(s), and inequality constraint(s) which could be used to estimate muscle forces.

3) What options are available for validating optimization based routines for estimating muscle forces?

4) Suggest three objective functions which could be used in estimating muscle forces. What is the problem(s) with constraining these objective functions using each muscles maximum isometric force?

5) With reference to work you have read describe two applications for static optimization problems.