MODELING MUSCLE: BASICS

Lecture Overview

• Key Properties
• Model Representation
• Example I – Alexander, (1989)
• Example II – Challis and Kerwin (1994)
• Example III – Hof (1991)
• Options
• Model Selection

“This model will be a simplification and an idealization, and consequently a falsification. It is to be hoped that the features retained for discussion are those of greatest importance in the present state of knowledge.” Alan Turing (1952)
KEY PROPERTIES

Muscle Fibers
- Passive
  - Different Length
  - Different Velocities
- Active
  - Activation Dynamics

Tendon
- Passive
  - Force/Length

Joint
- Muscle Moment Arms
- Passive Moment Profile
KEY PROPERTIES

The force produced by the muscle model \( (F_m) \) can be described using the following function

\[
F_m = a_f\cdot F_{max}\cdot F_1(L_f)\cdot F_2(V_f)
\]

Where

\( a_f \) - normalized degree of activation of muscle fibers.

\( F_{max} \) - maximum isometric force muscle can produce

\( F_1(L_f) \) - normalized force length relationship of muscle,

\( F_2(V_f) \) normalized force-velocity relationship of muscle.

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MODEL REPRESENTATION

% Max. isometric tension

Length of contractile element

100
75%
50%
25%

- Lengthen  VELOCITY  Shorten +

Maximum tension

Force

Extension

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MODEL REPRESENTATION

Potential Model Components
Models of muscle normally include some of the following components.

*Contractile Component* – normally representing (some) properties of the muscles (force-length, force-velocity, activation dynamics).

*Parallel Elastic Component* – normally a linear elastic component representing elastic material in parallel to the muscle fibers (connective tissue).

*Series Elastic Component* – normally a linear elastic component representing elastic properties of material in series with the contractile component (tendon, muscle cross-bridge elasticity).
MODEL REPRESENTATION

Schematically the muscle mode components can be represented as follows

**Contractile Component**

```
[CE] OR [ ]
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**Elastic Component**

```
[ ]
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**Damping/Viscous Component**

```
[ ]
```
MODEL REPRESENTATION

Muscle models have a varying number of model components. The more complicated representations is that of Hatze (1981). The model includes the following components,

PS – parallel sarcomere elasticity
CE – contractile element
BE – cross-bridge elasticity
SE – series elastic element
PE – Parallel element of muscle (having both elasticity and viscous damping)
EXAMPLE I


Model Components
- Force-velocity
- (Series elastic component)

Equation Inputs
- Data derived from cadavers implied relatively fixed moment arm.
- Starts at velocity of zero with maximum activation.

Model Parameters
- Determined from cadaver data and experimental observations.

Model Validation
- Comparison of model output with reality.
EXAMPLE I

The model “runs” in at a given horizontal velocity and plants its leg and then jumps. The computer model was run with different horizontal velocities and knee angles at plant, jump height was computed for each of these conditions.

Equation

\[ T = T_{MAX} \frac{\dot{\theta}_{MAX} - \dot{\theta}}{\ddot{\theta}_{MAX} + C \dot{\theta}} \]

A moment-angular velocity relationship, based on Hill’s 1938 equation.
EXAMPLE II


Model Components
Force-length
Force-velocity
Series elastic component
EXAMPLE II

The model of the force-length relationship used was that of Hatze (1981)

\[
F_I = F_{IO} \cdot \exp \left[ - \left( \frac{Q-1}{SK} \right)^2 \right]
\]

\[
Q = \frac{L_F}{L_{FO}}
\]

Where

- \( F_I \) - maximum isometric tension at a given muscle fiber length
- \( F_{IO} \) - maximum isometric force produced at the optimum length of the muscle fibers
- \( L_F \) - length of the muscle fibers
- \( L_{FO} \) - length at which the muscle fibers exert their maximum tension (optimum length)

and \( SK \) is a constant specific for each muscle where \( SK \geq 0.28 \).
EXAMPLE II

The model of the force-velocity relationship of Hill (1938) was adopted

\[ F_V = a \left( \frac{V_{MAX} - V_F}{b + V_F} \right) = \frac{b(F_I + a)}{b + V_F} - a \]

Where

- \( F_V \) - maximum possible force at a given muscle fiber velocity
- \( a, b \) - constants
- \( V_{MAX} \) - maximum speed of shortening of the fibers
- \( V_F \) - current speed of shortening of the fibers
- \( F_I \) is the maximum isometric tension possible at a given muscle fiber length.

\[ a.V_{MAX} = b.F_I \]
EXAMPLE II

Series elasticity was considered to reside only in the tendon. The model of tendon used in this study assumed the stress-strain relationship of tendon to be linear, therefore:

\[ L_T = L_{TR} \left[ 1.0 + \frac{F_M}{A_T E} \right] \]

Where

- \( L_T \) - length of the tendon
- \( L_{TR} \) - resting length of the tendon
- \( F_M \) - force exerted by the muscle on the tendon
- \( A_T \) - cross-sectional area of the tendon
- \( E \) is Young's modulus of elasticity for tendon.
EXAMPLE II

Primary Assumptions of Muscle Model

- The stress-strain relationship of tendon is linear.
- Muscle fiber elasticity was insignificant compared with tendon elasticity.
- The moment at the joint caused by the passive structures crossing the joint, and joint friction was insignificant compared with that produced by the muscles.
- During the studied activity there was no co-contraction of antagonist muscles.
- The various elements of the model are adequately represented by the equations used to describe them.
EXAMPLE II

If the muscle fibers are not pennated, and parallel components produce little force then

\[ F_T = F_M \]

As the model assumed the stiffness of the tendon was constant for all lengths of the tendon then

\[ K = \frac{dF_T}{dL_T} = \frac{dF_M}{dL_T} \]

Where \( K \) is the stiffness of the tendon.

The rate of change of the muscle force is equal to the product of tendon stiffness and the rate of change of tendon length, therefore

\[ \frac{dF_M}{dt} = \frac{dF_T}{dt} = \frac{dF_T}{dL_T} \cdot \frac{dL_T}{dt} \]
EXAMPLE II

The rate of change in tendon length is equal to the difference between muscle velocity and muscle fiber velocity

$$\frac{dL_T}{dt} = V_M - V_F$$

Re-arrangement of Hill’s equation gives

$$V_F = b(F_I - a) \left(\frac{F_M}{F_M + a}\right) - b$$

Substitution gives the following

$$\frac{dF_M}{dt} = k(V_M - V_F)$$

This ordinary differential equation can be solved using a variable step-size fifth order Runge-Kutta technique.
EXAMPLE II

Equation Inputs
• Data derived from cadavers allow determination of muscle velocity.
• If the movement starts from stationary then the velocity of the muscle fibers is known (zero).

Model Parameters
• Determined using an experimental procedure.

Model Validation
• An elbow flexion was simulated driven using the muscle model, and compared with reality.
EXAMPLE II
MUSCLE MODEL SIMULATION

\[ F_M = F_T = g(a, L_M, V_F) \]

\[ M_J = \sum_{i=1}^{NM} F_{Ti} \cdot R_i \]
EXAMPLE III

**Source:** Hof and Van Den Berg (1981) EMG to Force processing parts I-IV. *Journal of Biomechanics* 14:747-792.

**Model Components**
Force-length
Force-velocity
Series elastic component
Parallel elastic component
Activation dynamics

**Model Parameters**
- Determined using an experimental procedure.

**Model Validation**
- Comparison of model predicted ankle joint moments with reality.
OPTIONS

**Force–Length** – linearize, ascending or descending limb only, plateau

**Force–Velocity** – linearize, ignore

**SEC** – ignore, linear or exponential. Include cross-bridge or just tendon.

**Muscle PEC or Joint Elasticity.**

**Activation**
- Bang-Bang
- Known
- Estimated from EMG
- Determined from a control routine
MODEL SELECTION

- Complexity and completeness
- Problem of model parameter determination (accessibility)
- Compensating errors

**Option 1**
Start from simplest model and add complexity until model reflects reality

**Option 2**
Start from complex model and remove complexity until model no longer reflects reality, previous level of complexity was most appropriate.
REVIEW QUESTIONS

1) What are the different methods via which muscle models allow for specifying muscle activation?

2) Give the structure of a muscle model you have studied (name the author of the work). Detail what the components of the model correspond to biologically. What is the major element(s) which is missing from the model?

3) What are the locations and properties of the following
   - Contractile Element
   - Parallel Elastic
   - Series Elastic

4) What are the implications of the SEC for human movement?

5) With examples outline the relative merits of simple versus a complex muscle model.