

I. Determine whether the function has a maximum or minimum. If so, state where and find the extremum value of the function at that point.

(1)  $f(x) = e^{-x^2}$      $x = 0$ ;  $f''(0) = -2 \Rightarrow \text{max}$

(2)  $f(x) = (x^2 - 1)e^{-2x^2}$      $x = 0, \pm\sqrt{3/2}$ ;  $f''(0) > 0 \Rightarrow \text{min}$ ;  $f''(\pm\sqrt{3/2}) < 0 \Rightarrow \text{max}$

(3)  $f(x) = \sin^2(2x - 1)$      $x = \frac{1}{2}, \frac{1}{2} \pm \frac{\pi}{2}, \frac{1}{2} \pm \pi, \frac{1}{2} \pm \frac{3\pi}{2}, \dots$ ;  
 $f''(0) < 0, f''(\frac{1}{2} \pm \frac{\pi}{2}) > 0, f''(\frac{1}{2} \pm \pi) < 0, \dots \Rightarrow \text{max, min, max, ...}$

II. Using the chain rule, find the first derivative of each of the following:

(4)  $f(x) = \left[ (x^4 - 3x^3)^2 - x^5 \right]^2$      $f'(x) = 2 \left[ (x^4 - 3x^3)^2 - x^5 \right] \left[ 2(x^4 - 3x^3)(4x^3 - 9x^2) - 5x^4 \right]$

(5)  $f(x) = 5(x - \sin(x^2))^3$      $f'(x) = 15 \left[ x - \sin(x^2) \right]^2 \left[ 1 - 2x \cos(x^2) \right]$

(6)  $f(x) = \left[ (x^2 - 3(x^3 - x)^{-2})^3 \right]^4$      $f'(x) = 12 \left[ (x^2 - 3(x^3 - x)^{-2})^3 \right]^3 (x^2 - 3(x^3 - x)^{-2})^2 (2x + 18x^2(x^3 - 1)^{-3})$

(7)  $f(x) = \sqrt{x^{-2} + (x^2 - \sin(x))^3}$      $f'(x) = \frac{[-2x^{-3} + 3(x^2 - \sin(x))(2x - \cos(x))]}{2\sqrt{x^{-2} + (x^2 - \sin(x))^3}}$

(8)  $f(x) = \frac{x(3\sin^3(x) - x)^2}{\sqrt{(x - 3e^x)^3}}$

$$f'(x) = \frac{\sqrt{(x - 3e^x)^3} \left[ 2x(3\sin^3(x) - x)(9\sin^2(x) - 1) \right] - x(3\sin^3(x) - x)^2 \frac{1}{2} \left[ (x - 3e^x)^3 \right]^{-\frac{1}{2}} 3(x - 3e^x)^2 (1 - 3e^x)}{(x - 3e^x)^3}$$

III. Find the linear approximation to the following:

(9)  $f(0.09)$ ; where  $f(x) = \sin(x)$        $\sin(x) \approx x$ ;  $\boxed{\sin(0.09) \approx 0.09}$

(10)  $f(0.05)$ ; where  $f(x) = 1 - \cos^3(x)$        $\cos(x) \approx 1 - \frac{x^2}{2}$ ;  $\left(1 - \frac{x^2}{2}\right)^3 \approx 1 - \frac{3x^2}{2}$ ;

$$\boxed{1 - \cos^3(.05) \approx \frac{3x^2}{2} = 0.00375}$$

(11)  $f(x) = x - \sqrt{x^2 - d^2}$ ;  $\boxed{f(x) \approx x - x\sqrt{1 - \frac{d^2}{x^2}} \approx x - x\left(1 - \frac{d^2}{2x^2}\right) = \frac{d^2}{2x}}$

(12)  $f(30m/s)$ ; where  $f(v) = \frac{c}{\sqrt{c^2 - v^2}}$  and  $c = 3 \times 10^8 m/s$        $\boxed{f(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1 + \frac{v^2}{2c^2} = 1.0000000000000001}$

(13)  $K = (\gamma - 1)mc^2$ , where  $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$        $\boxed{K \approx \left(1 + \frac{v^2}{2c^2} - 1\right)mc^2 = \frac{mv^2}{2}}$