Chapter 2 / 9 Problem Answers (assigned)

(2.4) $v_i = \boxed{13.2 m/s}$ (2.5) $R \geq \boxed{12.5 km}$

(2.11) Answer given in text. (2.15) Answer given in text.

(2.25a) $F_N = mg\left(1+\frac{2h}{R}\right)$ (b) $F_N = mg\left(\frac{2h}{R}+\frac{3}{\sqrt{2}}-2\right)$ (c) $v_{block} = \sqrt{2g\left(h - R\left(1-1/\sqrt{2}\right)\right)}$

(d) $x = \left(\sqrt{2}-1\right)R + h + \sqrt{h^2 - \frac{3}{2}R^2 + \sqrt{2}R^2}$

(e) $y(x)$ describes the shape of the track since $U(x) = mgy(x)$

(2.27) $W = \boxed{0.1764 J}$

(2.32) $\theta = \sin^{-1}\left[\frac{1 \pm \mu_K\sqrt{3+4\mu_K^2}}{2\left(1+\mu_K^2\right)}\right]$

(2.34a) $y = \frac{-m}{\alpha}\left[v + \frac{mg}{\alpha}\ln\left(1-\frac{\alpha v}{mg}\right)\right]$ (b) $y = \frac{-m}{2\beta}\ln\left(1-\frac{\beta v^2}{mg}\right)$

(2.37) $F(x) = \frac{-m\alpha^2}{x^3}$

(2.39a) $v = \frac{-1}{\beta}\ln\left[\frac{\alpha\beta t}{m} + e^{-\beta v_0}\right]$ (b) $t = \frac{m}{\alpha\beta}\left(1-e^{-\beta v_0}\right)$ (c) $x = \frac{m}{\alpha\beta}\left[\frac{1}{\beta} - e^{-\beta v_0}\left(v_0 + \frac{1}{\beta}\right)\right]$

(2.52a) $F(x) = \frac{-4U_0 x}{a^2}\left[1-\frac{x^2}{a^2}\right]$ (b) stable at $x = 0$; unstable at $x = \pm a$

(c) $\omega = \sqrt{\frac{4U_0}{ma^2}}$ (d) $v = \sqrt{\frac{2U_0}{m}}$ (e) $x = a\left[\frac{e^{t\sqrt{\frac{8U_0}{ma^2}}}-1}{e^{t\sqrt{\frac{8U_0}{ma^2}}}+1}\right]$

(2.54a) $v = \boxed{980 m/s}$ (b) $v = \boxed{679.7 m/s}$

(9.54) $\frac{m}{m_0} = \boxed{e^{-1}}$

(9.60) $t = \boxed{24.9 s}$ (9.63a) $h = \boxed{1,837 km}$ (b) $h = \boxed{598 km}$

Chapter 3 Answers

(2a) $\beta = 0.0693 s^{-1}$ (b) $\nu_1 - \nu_0 = 3.8\times 10^{-5}\,Hz$ (c) $e^{\beta\tau_1} = 1.007$

(4a) $\langle T\rangle_{time} = \dfrac{m\omega_0^2 A^2}{4} = \langle U\rangle_{time}$ (b) $\langle T\rangle_{space} = \dfrac{m\omega_0^2 A^2}{3} = 2\langle U\rangle_{space}$

(6) $\omega = \sqrt{\dfrac{k}{\mu}}$; $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ {called the reduced mass}

(7) Answer given in question.

(12) $\theta(t) = \theta_0\left(1+\omega_0\right)e^{-\omega_0 t}$ {critically damped motion}

(18) $E_{total} = \frac{1}{2} m\omega_0^2 A^2$; $E_{loss/cycle} = -2\pi m\beta\omega_0 A^2$

(28) $F(t) = \frac{4}{\pi}\sin(\omega t) + \frac{4}{3\pi}\sin(3\omega t) + \frac{4}{5\pi}\sin(5\omega t) + \ldots$

(39) $F(t) = \frac{1}{\pi} + \frac{1}{2}\sin(\omega t) + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{1-4n^2}\right)\cos(2n\omega t)$

(45) $Q \approx 187$

Chapter 5 Answers

(2) $\rho(r) = \dfrac{c}{2\pi r G}$; c is a constant

(7) $\Phi(R) = \dfrac{-GM}{L} \ln\left[\dfrac{L/2 + \sqrt{L^2/4 + R^2}}{-L/2 + \sqrt{L^2/4 + R^2}}\right]$

(13) $\vec{F} = \dfrac{-4\pi Gm}{3}\left[\left(\rho_1 - \rho_2\right)R_1^3 + \rho_2 R_0^3\right]\hat{r}$

(15) $t = 5069s$ $(84.5min)$

(16) $\vec{F} = 2\pi\rho_s Gm\ \hat{j}$

(17) $h = \dfrac{3GM_m R^2}{2gD^3}$

(18) Answer in question; $\dfrac{h_{moon}}{h_{sun}} = 2.2$

Chapter 6 Answers

(2) $y = ax + b$ {of course!}

(4) $z = a\phi + b$ {ϕ is the azimuthal angle measured from x}

(6) $t = \pi\sqrt{\frac{a}{g}}$ {as given in question}

(10) $R = H/2$

Chapter 7 Answers

(3) $\mathscr{L} = \boxed{\tfrac{1}{2}m(R-\rho)^2\dot{\theta}^2 + \tfrac{1}{2}I\dot{\phi}^2 - mg\left[R - (R-\rho)\cos\theta\right]}$

$f(\theta,\phi) = \boxed{(R-\rho)\theta - \rho\phi = 0}$

$\boxed{\ddot{\theta} + \left[\frac{5g}{7(R-\rho)}\right]\sin\theta = 0}$; $\boxed{\ddot{\phi} + \frac{5g}{7\rho}\sin\left(\frac{\rho}{R-\rho}\phi\right) = 0}$

$\omega = \boxed{\sqrt{\frac{5g}{7(R-\rho)}}}$

(7) $\mathscr{L} = \boxed{m\ell^2\left[\dot{\phi}_1^2 + \tfrac{1}{2}\dot{\phi}_2^2 + \dot{\phi}_1\dot{\phi}_2\cos(\phi_1-\phi_2)\right] + mg\ell\left(2\cos\phi_1 + \cos\phi_2\right)}$

$\boxed{2\ddot{\phi}_1 + \ddot{\phi}_2\cos(\phi_1-\phi_2) + \dot{\phi}_2^2\sin(\phi_1-\phi_2) + \frac{2g}{\ell}\sin\phi_1 = 0}$

$\boxed{\ddot{\phi}_2 + \ddot{\phi}_1\cos(\phi_1-\phi_2) + \dot{\phi}_1^2\sin(\phi_1-\phi_2) + \frac{g}{\ell}\sin\phi_2 = 0}$

(10) (a) $x(t) = \boxed{\frac{gt^2}{4}}$ (b) $x(t) = \boxed{\frac{M\ell}{m}\left[\cosh(\alpha t) - 1\right]}$

(11) $\mathscr{L} = \tfrac{1}{2}m\left[R^2\omega^2 + R^2\left(\dot{\phi}+\omega\right)^2 + 2R^2\omega\left(\dot{\phi}+\omega\right)\cos\phi\right]$, where ϕ is the angle the particle makes with a radius of the circle; the diameter makes an angle of ωt with a fixed horizontal axis. Then, the equation of motion becomes: $\boxed{\ddot{\phi} + \omega^2\sin\phi = 0}$

(12) $r(t) = \boxed{r_0 \cosh(\alpha t) + \frac{g}{2\alpha^2}\left[\sin(\alpha t) - \sinh(\alpha t)\right]}$

(13) $\boxed{\ddot{\theta} + \frac{g}{b}\sin\theta - \frac{a}{b}\cos\theta = 0}$

$T = \boxed{\frac{2\pi\sqrt{b}}{(a^2 + g^2)^{1/4}}}$

Hint: Find θ_0 in terms of a and g by setting $\ddot{\theta} = 0$; Taylor expand $\sin\theta$ and $\cos\theta$ about θ_0; pick off the angular frequency from the differential equation of motion.

(15) $\boxed{\ddot{\ell} - \ell\dot{\theta}^2 + \frac{k}{m}(\ell - b) - g\cos\theta = 0}$

$\boxed{\ddot{\theta} + \frac{2\dot{\ell}\dot{\theta}}{\ell} + \frac{g}{\ell}\sin\theta = 0}$

(22) $\mathscr{L} = \boxed{\frac{m\dot{x}^2}{2} - \frac{ke^{-t/\tau}}{x}}$; $\mathscr{H} = \boxed{\frac{p_x^2}{2m} + \frac{ke^{-t/\tau}}{x}}$

(23) $\mathscr{H} = \boxed{\frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) + U(x, y, z)}$

(24) $\mathscr{L} = \boxed{\frac{m}{2}\left(\ell^2\dot{\theta}^2 + \alpha^2\right) + mg\ell\cos\theta}$

$\mathscr{H} = \boxed{\frac{p_\theta^2}{2m\ell^2} - \frac{m\alpha^2}{2} - mg\ell\cos\theta}$; in this case the Hamiltonian is *not* the total energy.

Chapter 8 Answers

(4) $\langle U \rangle = \boxed{-k/a}$; $\langle T \rangle = \boxed{k/2a}$

(8) Using the potential given, integrate to find $\theta(r)$. Hint: make sure to include an integration constant and specify it to be equal to $\pi/4$ for simplicity. You should then show that $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a^2 = \frac{\alpha'}{\varepsilon' + 1}$, $b^2 = \frac{\alpha'}{\varepsilon' - 1}$.

Here, $\alpha' = \ell^2/\mu E$ and $\varepsilon' = \sqrt{1 + \frac{\ell^2 k}{\mu E^2}}$

(9a) The boost is radial, $\ell_f/\ell_i = \boxed{1}$

(b) $U(r) = \boxed{-\frac{GM_e m_s}{r}}$; $T(r) = \boxed{\frac{GM_e m_s}{r}}$; $V(r) = \boxed{-\frac{GM_e m_s}{r} + \frac{\ell^2}{2\mu r^2}}$; $E(r) = \boxed{0}$

(10) Use the virial theorem ($T = -\frac{1}{2}U$) and show that $E_f = 0$ meaning orbit is parabolic.

(14) $F(r) = \boxed{-\frac{\ell^2}{\mu}\left[\frac{6k}{r^4} + \frac{1}{r^3}\right]}$

(18) Start with Kepler's elliptic orbit equation and show that $F(r) = \boxed{-\frac{k}{r^2}}$

(24a) $d = \boxed{1.59 \times 10^6 m}$ (b) $d = \boxed{1.9 \times 10^6 m}$

(25a) $r_a = \boxed{287 km}$ (b) $v_a = \boxed{7.719 km/s}$ (c) $\tau = \boxed{1.49 hr}$

(27) $v_a = \boxed{1.608 km/s}$

(30) $v_{esc} = \boxed{11.019 km/s}$ {requires a boost of $3.327 km/s$ from orbital speed of $7.792 km/s$}

(37) $\Delta v_{\min} = \boxed{1.02 km/s}$

(40) Crashing into the sun: $\Delta v = \boxed{-26.9 km/s}$; Escaping from solar system: $\Delta v = \boxed{13.9 km/s}$

(41a) $\Delta v = \boxed{3.95 km/s}$ (b) $T_t \sim \boxed{5\ days}$

(42) $\Delta E = \boxed{2.57 \times 10^{11} J}$ (43) $\frac{v_{t1}}{v_1} = \boxed{\frac{2\sqrt{3}}{3}}$ (47) $\tau = \boxed{1.17 \times 10^8 yr}$

Chapter 9 Answers

(2) $\vec{R} = \boxed{(0, 0, h/4)}$

(9a) $\vec{v}_1 = \boxed{\left(0, -\sqrt{\dfrac{(2m_2 - m_1)E_0}{(m_1^2 + m_1 m_2)}}\right)}$ $\quad \vec{v}_2 = \boxed{\left(\sqrt{\dfrac{(m_1 + m_2)E_0}{m_2^2}}, \sqrt{\dfrac{(2m_1 m_2 - m_1^2)E_0}{(m_1 m_2^2 + m_2^3)}}\right)}$

(b) $\dfrac{m_1}{m_2} = \boxed{2}$

(12a) $v = \boxed{2.02 m/s}$; he will run out of gas… (b) $v = \boxed{9 m/s}$, relative to his ref. frame

(20) $v = \boxed{\sqrt{ga}}$

(22a) Two possibilities: $v_n = \boxed{5.18 km/s}$, $v_d = \boxed{14.44 km/s}$ *or* $v_n = \boxed{19.79 km/s}$, $v_d = \boxed{5.12 km/s}$

(b) Two possibilities: $\zeta = \boxed{74.84^\circ}$ *or* $\zeta = \boxed{5.16^\circ}$ (c) $\psi_{\max} = \boxed{30^\circ}$

(23) $\dfrac{\Delta T}{T_0} = \boxed{\dfrac{-m_2}{m_1 + m_2}}$

(30) $\langle \vec{F} \rangle = \dfrac{\vec{J}}{\Delta t} = \boxed{(-127, -9)N}$

(34) $v_1 = \boxed{\dfrac{u_1}{\sqrt{2}}}$ $\quad v_2 = \boxed{\dfrac{u_1}{\sqrt{2}}}$

(42) $v_f = \boxed{4.27 m/s}$ $\quad \theta = \boxed{35.8^\circ}$

(43) $\zeta = \boxed{55.8^\circ}$

Chapter 10 Answers

(6) The shape of the curve is a paraboloid: $z(r)=\frac{\omega^2}{2g}r^2+c$

(9) The assumption is that the vertical *z*-motion is unaffected, so time of flight is calculated using this and then the deflection distance may be calculated.

(11a) projectile misses by $190km$ (b) projectile misses by $124km$

(12) The line of the plumb will follow the effective force. You must use the good approximation that $\tan\varepsilon \simeq \varepsilon$ for small angles.

(17) The surface of the lake will be perpendicular to the effective gravitational acceleration vector. Since this points at an angle, one can find the deviation over a lake of radius 162km.
I find the deviation to be much too large ($\sim 278m$). The answer key gives $7m$, which is reasonable. Though we have the same solution, their angular deviation is much, much smaller and I don't see how they can possible get that number based on the same solution. Even if one accounts for the curvature of the earth (about $5m$ per $8km$), the answer is still unreasonable.

(18) Displacement is $\sim 271m$ eastward of intended target.

(21) Water is about $1mm$ higher on the west bank.