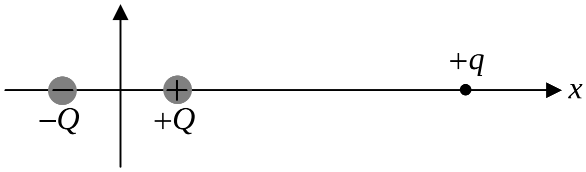
Physics 212 / Summer 2008 Name: ANSWER KEY
Dr. Zimmerman
Exam 1

Material Covered: Chapters 25 – 28 in *Physics for Scientists & Engineers with Modern Physics (1/e)* by Randall Knight.

Problem	Score/Possible
(1)	/ 25
(2)	/ 25
(3)	/ 25
(4)	/ 25
Total →	/ 100

Show all your work.
Exam is closed book.
Partial Credit will be given.
You may use a calculator.
Please don't cheat.

(1) A linear dipole consists of two charges, $-Q$ and $+Q$, located at $x=-a$ and $x=+a$, respectively, as shown. Located at a point on the positive *x*-axis ($x>a$), there is another charge $+q$. (a) Find the force on this charge due to the other two. Express your answer in terms of unit vectors. (b) Using the binomial approximation, show that the force decreases as $1/x^3$ for $x \gg a$. (c) Given your answer for the force, determine the *electric field* of the dipole for these large values of *x*.



(a) $$\vec{F} = \boxed{\frac{qQ}{4\pi\varepsilon_0}\left[\frac{-1}{(x+a)^2}+\frac{1}{(x-a)^2}\right]\hat{i}}$$

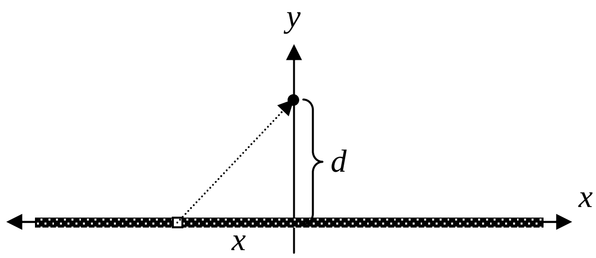
$$\simeq \frac{qQ}{4\pi\varepsilon_0 x^2}\left[-(1+a/x)^{-2}+(1-a/x)^{-2}\right]$$

(b) $$= \frac{qQ}{4\pi\varepsilon_0 x^2}\left[-1+2a/x+1+2a/x\right]$$; includes only first two terms of Binomial series.

$$\vec{F} = \boxed{\frac{qQa}{\pi\varepsilon_0 x^3}\hat{i}}$$

(c) $$\vec{E} = \boxed{\frac{Qa}{\pi\varepsilon_0 x^3}\hat{i}}$$, since $\vec{E}=\vec{F}/q$

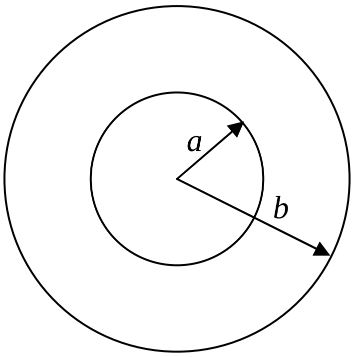
(2) A uniform charge density λ_0 exists along the entire *x*-axis. You are located at a point on the *y*-axis at $y = d$. (a) Using the basic steps as we have outlined in class, determine the differential electric field, $d\vec{E}$, at this point, due to an infinitesimal charge element of the charge distribution (with unit vectors). I have located the charge element for you. (b) Set up, *but don't solve* the integral required to find the total field at the point of interest. You may use symmetry arguments in simplifying/eliminating part of your answer. Make sure you clearly define all relevant quantities (i.e., show your work!)



(a) $$d\vec{E} = \frac{\lambda_0 dx}{4\pi\varepsilon_0 \left(x^2 + d^2\right)} \left[\frac{x}{\sqrt{x^2 + d^2}} \hat{i} + \frac{d}{\sqrt{x^2 + d^2}} \hat{j} \right]$$

(b) $$\vec{E} = \frac{\lambda_0 d}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{dx}{\left(x^2 + d^2\right)^{3/2}} \hat{j}$$, since horizontal components cancel by symmetry

(3) You are given two conducting, concentric spherical shells of radius *a* and *b* as shown below. There is charge on the shells but nowhere else. Note: these shells have no thickness so there are only *three* regions to consider. (a) If the electric field in the region between the shells ($a < r < b$) is given as $\vec{E} = (-2Q/3\pi\varepsilon_0 r^2)\,\hat{r}$, how much charge resides on the inner sphere? (b) If outside, at $r > b$, the total electric flux is $3Q/\varepsilon_0$, how much charge resides on the outer shell?



(a)

$$E 4\pi r^2 = q_{in}/\varepsilon_0$$

$$(-2Q/3\pi\varepsilon_0 r^2)(4\pi r^2)(\varepsilon_0) = q_{in}$$

$$q_{in} = \boxed{-8Q/3} = q_{inner\ shell}$$

(b)

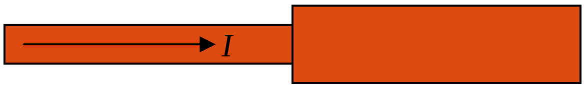
$$\Phi_e = q_{in}/\varepsilon_0$$

$$3Q/\varepsilon_0 = q_{in}/\varepsilon_0$$

$$q_{in} = 3Q = q_{inner\ shell} + q_{outer\ shell}$$

$$q_{outer\ shell} = 3Q - (-8Q/3) = \boxed{17Q/3}$$

(4) Two copper wires are connected end-to-end as shown. One has a diameter of $2.0\ mm$, the other one, twice that. The current flowing in the smaller wire is $3.5\ A$. (a) Find the current density in the larger diameter wire. (b) If the number of density of copper is $8.5\times10^{28}\ m^{-3}$, what is the drift velocity of electrons in each wire? (c) How do the electric fields in the two wires compare?



(a) *The current is the same in both wires so*: $J = \dfrac{3.5\ A}{\pi(0.002\ m)^2} = \boxed{2.79\times10^5\ A/m^2}$

(b) $\sigma = \dfrac{ne^2\tau}{m}$ $\quad J = \sigma E \quad$ $v_d = \dfrac{e\tau}{m}E$

Taking the 3rd equation above and multiplying both sides by ne we have,

$$nev_d = \frac{ne^2\tau}{m}E$$
$$nev_d = \sigma E$$
$$nev_d = J$$
$$v_d = \frac{J}{ne}$$
$$v_{d(small)} = \frac{3.5\ A}{(8.5\times10^{28}\ m^{-3})(1.6\times10^{-19}\ C)\pi(0.001\ m)^2} = \boxed{8.2\times10^{-5}\ m/s}$$

$$v_{d(large)} = v_{d(small)}/4 = \boxed{2.1\times10^{-5}\ m/s}$$

(c) $E \propto J$ and $J \propto \dfrac{1}{A}$. Thus, the electric field is inversely proportional to area.

$$E_{small} = \boxed{4E_{large}}$$