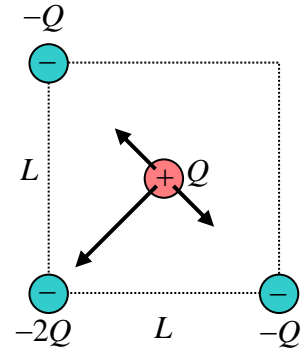


Show all your work

As shown, there are three negative charges located at the corners of a square of side L . There is a single positive charge in the center of the square. (a) Draw arrows indicating the direction of the individual forces acting on the charge at the center. Be sure you give the arrows the appropriate relative size. (b) If the value of $Q = 5.0 \text{ nC}$, and $L = 10 \text{ cm}$, determine the net force on the charge at the center and present your answer in component form. (c) What additional charge could be placed at the empty corner of the square so that the charge in the center is in equilibrium? Explain.



- (b) By symmetry, two of the forces cancel. Thus we need only find the force due to the $-2Q$ charge,

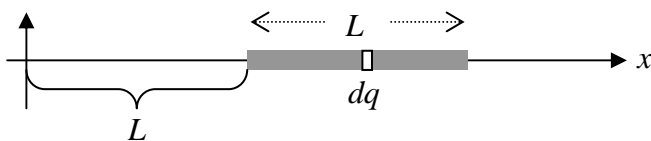
$$\vec{F} = \frac{2Q^2}{4\pi\epsilon_0(L^2/2)} \left[-\cos(45^\circ) \hat{i} - \sin(45^\circ) \hat{j} \right]$$

$$\vec{F} = \frac{-\sqrt{2}Q^2}{2\pi\epsilon_0 L^2} \left[\hat{i} + \hat{j} \right] = \boxed{-6.36 \times 10^{-5} (\hat{i} + \hat{j}) \text{ N/C}}$$

- (c) Placing another negative charge ($-2Q$) at the upper right hand corner would put the charge at the center in equilibrium since the forces would be equal and opposite and thus cancel.

Show all your work

As shown, a thin rod of length L is on the x -axis a distance L from the origin and contains a total charge Q spread uniformly over it. The goal is to find the electric field, \vec{E} , at the origin ($x = 0$). (a) What is the linear charge density λ ? (b) Determine the infinitesimal charge, dq . (c) Write down the expression for the infinitesimal electric field, $d\vec{E}$, at the origin. (d) Finally, calculate the integral with the correct limits of integration.



(a) $\lambda = Q/L$

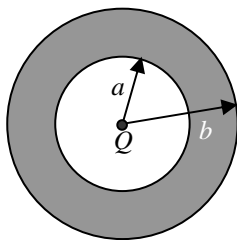
(b) $dq = \lambda dx = \frac{Qdx}{L}$

(c) $d\vec{E} = \frac{dq}{4\pi\epsilon_0 x^2} (-\hat{i})$
 $d\vec{E} = \frac{(Q/L)dx}{4\pi\epsilon_0 x^2} (-\hat{i})$

(d) $\vec{E} = \frac{-Q}{4\pi\epsilon_0 L} \int_{x=L}^{x=2L} \frac{dx}{x^2} \hat{i}$
 $\vec{E} = \frac{-Q}{4\pi\epsilon_0 L} \left(\frac{-1}{x} \right) \Big|_L^{2L} \hat{i} = \boxed{\frac{-Q}{8\pi\epsilon_0 L^2} \hat{i}}$

Show all your work

A conducting spherical shell of inner radius a and outer radius b surrounds a point charge $+Q$ (which is fixed in place). Someone puts an additional charge $+Q$ on the outer surface of the conductor (at $r = b$). (a) Draw and label the charge that must exist on each of the conductor surfaces at $r = a$ and $r = b$ in electrostatic equilibrium. (b) Using Gauss' law, determine the electric field in three regions: $r < a$, $a < r < b$, and $r > b$. (c) What is the surface charge density (η) at $r = a$ and $r = b$?



We use spherical Gaussian surfaces concentric with the center of the sphere.

- (a) $-Q$ moves to the inner surface of the cavity (at $r = a$), leaving behind $+2Q$ on the outer surface at $r = b$

(b) $r < a$: The charge enclosed is Q , so $E4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \boxed{\frac{Q}{4\pi\epsilon_0} \hat{r}}$

$a < r < b$: Here, the net charge enclosed is zero, Thus $\vec{E} = \boxed{0 \hat{r}}$

$r > b$: The charge enclosed is $Q + (-Q) + 2Q = 2Q$, so $E4\pi r^2 = \frac{2Q}{\epsilon_0} \Rightarrow \vec{E} = \boxed{\frac{2Q}{4\pi\epsilon_0} \hat{r}}$

(c) At $r = a$, the surface charge density is, $\eta_{r=a} = \frac{-Q}{4\pi a^2}$.

At $r = b$, the surface charge density is, $\eta_{r=b} = \frac{2Q}{4\pi b^2}$

Physics 212 / Summer 2009
Dr. Zimmerman
Ch. 29 Quiz

Name: ANSWER KEY

Show all your work

A proton ($q_p = 1.6 \times 10^{-19} \text{ C}$), initially at rest, is released from a metal surface that is charged to a potential of 300 V . What is the speed of the proton when it is far from the surface? The mass of a proton is $m_p = 1.67 \times 10^{-27} \text{ kg}$.

Use conservation of energy:

$$K_i + U_i = K_f + U_f$$

$$0 + q_p V_i = \frac{1}{2} m_p v_f^2 + 0$$

$$v_f = \sqrt{\frac{2q_p V_i}{m_p}} = \sqrt{\frac{2(1.6 \times 10^{-19} \text{ C})(300 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}} = \boxed{2.4 \times 10^5 \text{ m/s}}$$

Physics 212 / Summer 2009
Dr. Zimmerman
Ch. 30 Quiz

Name: ANSWER KEY

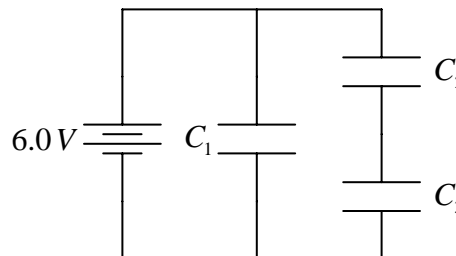
Show all your work

What are (a) the charge on and (b) the potential difference across each capacitor in the following circuit? $C_1 = 2.0 \mu F$, $C_2 = 4.0 \mu F$, $C_3 = 6.0 \mu F$

(a) C_2 and C_3 are in series so,

$$C_{23} = \left(\frac{1}{C_2} + \frac{1}{C_3} \right)^{-1} \mu F = \frac{12}{5} \mu F = 2.4 \mu F$$

Then C_1 and C_{23} are in parallel and will have the same potential difference – that of the battery.



$$Q_1 = C_1(6.0 V) = \boxed{12 \mu C}$$

$$Q_{23} = C_{23}(6.0 V) = \boxed{14.4 \mu C} = Q_2 = Q_3$$

(Because C_2 and C_3 are in series they have the same charge – that of C_{23} .)

(b) $\Delta V_1 = \boxed{6.0 V}$; it is in parallel with the battery

$$\Delta V_2 = Q_2 / C_2 = \boxed{3.6 V}$$

$$\Delta V_3 = Q_3 / C_3 = \boxed{2.4 V}$$

(The voltage across C_2 and C_3 should add to $6.0 V$ since they're in series.)

Show all your work

The electron beam inside an analog television picture tube is 0.44 mm in diameter and carries a current of $50 \mu\text{A}$. The electron beam impinges on the inside of the picture tube screen. (a) How many electrons strike the screen each second? (b) What is the current density in the electron beam? (c) The electrons move with a velocity of $4.0 \times 10^7 \text{ m/s}$. What electric field strength is needed to accelerate electrons from rest to this velocity in a distance of 5.0 mm ? (d) Each electron transfers its energy to the picture tube screen upon impact. What is the power delivered to the screen by the electron beam?

$$(a) \quad I = \frac{Q}{\Delta t} \Rightarrow Q = I\Delta t = (5.0 \times 10^{-5} \text{ A})(1.0 \text{ s}) = 5.0 \times 10^{-5} \text{ C}$$

$$N_e = 5.0 \times 10^{-5} \text{ C} / (1.6 \times 10^{-19} \text{ C/electron}) = \boxed{3.13 \times 10^{14}}$$

$$(b) \quad J = \frac{I}{A} = \frac{(5.0 \times 10^{-5} \text{ A})}{\pi(2.2 \times 10^{-4} \text{ m})^2} = \boxed{330 \text{ A/m}^2}$$

$$(c) \quad a = \frac{v_f^2}{2\Delta x} = \frac{(4.0 \times 10^7 \text{ m/s})^2}{2(0.005 \text{ m})} = 1.6 \times 10^{17} \text{ m/s}^2$$

$$a = \frac{eE}{m} \Rightarrow E = \frac{ma}{e} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.6 \times 10^{17} \text{ m/s}^2)}{1.6 \times 10^{-19} \text{ C}}$$

$$E = \boxed{9.11 \times 10^5 \text{ N/C}}$$

$$(d) \quad K = \frac{1}{2}mv^2 = 0.5(9.11 \times 10^{-31} \text{ kg})(4.0 \times 10^7 \text{ m/s})^2 = 7.288 \times 10^{-16} \text{ J}$$

Every second, $5.0 \times 10^{-5} \text{ C} / 1.6 \times 10^{-19} \text{ C} = 3.125 \times 10^{14}$ electrons strike the screen.

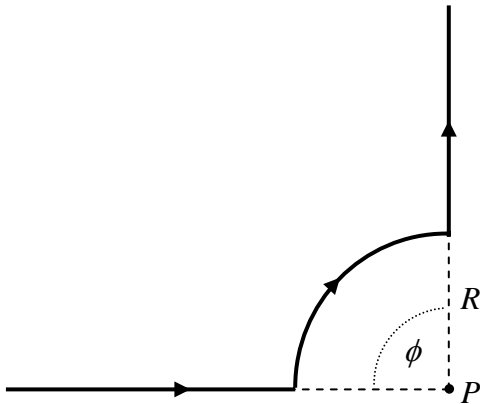
Thus, the total power delivered is,

$$P = (7.288 \times 10^{-16} \text{ J})(3.125 \times 10^{14} \text{ s}^{-1}) = \boxed{0.23 \text{ W}}$$

CHOOSE ONE OF THE TWO PROBLEMS.

Show all your work

Problem 1: Using the Biot-Savart law, determine the magnetic field at the center of the circular arc (point P) when the angle is $\phi = \pi/2$. Show that your answer is consistent with the expression for the field of a complete circular current loop when $\phi = 2\pi$: $B = \mu_0 I / 2R$.



La loi de Biot et de Savart:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

In this problem, the straight parts of the current, will not contribute to the field at point P since $d\vec{s}$ is parallel or antiparallel to \vec{r} . In all cases, \vec{r} points from a current element to the point P .

For the current arc, the cross product $d\vec{s} \times \vec{r}$, points into the page, so this is the direction of the B-field at the point P . Also, the magnitude of \vec{r} is just the radius of the arc, $|\vec{r}| \equiv r = R$. Since $d\vec{s}$ is perpendicular to \vec{r} for the current arc, the angle between the two is 90° and the sine is 1. So we have,

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^\phi \frac{d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int_0^\phi \frac{ds}{r^2} = \frac{\mu_0 I}{4\pi} \int_0^\phi \frac{ds}{R^2}$$

Now ds is an infinitesimal element of the arc, $ds = R d\phi$, so we may now integrate to find the field,

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{\pi/2} \frac{R d\phi}{R^2} = \frac{\mu_0 I \phi}{4\pi R} \Big|_0^{\pi/2} = \boxed{\frac{\mu_0 I}{8R}}, \text{ into the page.}$$

A complete circle would give $4 \times$ this amount, or $\mu_0 I / 2R$.

Problem 2: A wire bent at a 90° – angle carries a 10 A current in the direction shown. The length of each segment is 50 cm . If a uniform magnetic field of 0.10 T is applied in the direction shown, what is the magnitude and direction of the net force on the wire?

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

Consider each segment separately. For the vertical segment, the force is *into the page*.

The magnitude is,

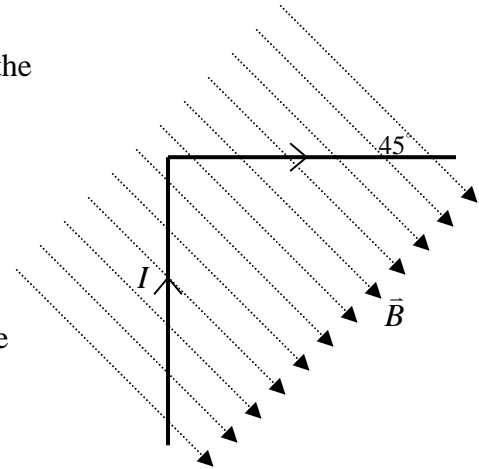
$$F_{\text{vert}} = I\ell B \sin 45^\circ = 0.35\text{ N}$$

The force on the horizontal segment is also *into the page*. The magnitude is the same,

$$F_{\text{horiz}} = I\ell B \sin 45^\circ = 0.35\text{ N}$$

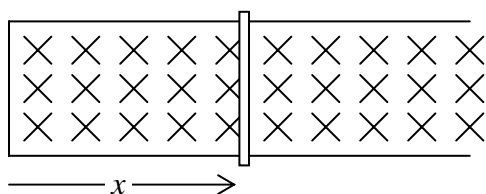
Thus, the net force is,

$$\vec{F}_{\text{net}} = \boxed{0.71\text{ N} \otimes}$$



Show all your work

In the figure shown, a 25 cm – long conducting bar is in contact with conducting rails and forms a closed circuit. The magnetic field is uniform, having a magnitude of 0.65 T . (a) If the circuit resistance is 5.0 Ω , and the position of the bar is given by $x = 0.5t^2$, use Faraday’s law to determine the magnitude and direction of the current flowing in the bar at $t = 2.0$ s . (b) What is the force on the bar at $t = 2.0$ s ?



We choose the normal to the loop area to be *into the page*, in the same direction of the field. This fixes the positive orientation of the loop to be *clockwise*.

$$\varepsilon = -\frac{d\Phi_m}{dt} = -\frac{d(B\ell x)}{dt}$$

$$\varepsilon = -B\ell \frac{d(0.5t^2)}{dt} = -B\ell t$$

$$\varepsilon(t = 2.0 \text{ s}) = -(0.65 \text{ T})(0.25 \text{ m})(2.0 \text{ s}) = -0.325 \text{ V}$$

$$I(t = 2.0 \text{ s}) = \frac{-0.325 \text{ V}}{5.0 \Omega} = \boxed{-0.065 \text{ A}}$$

The minus sign of our answer indicates the current flows *counterclockwise*; that is, the current flows *up the page* through the conducting bar.

The force is given by:

$$\vec{F} = I\vec{\ell} \times \vec{B} = (0.065 \text{ A})(0.25 \text{ m})(0.65 \text{ T})(-\hat{i})$$
$$\vec{F} = \boxed{-0.011 \hat{i} \text{ N}}$$

Show all your work

A series RLC circuit is connected to an AC voltage of amplitude $\varepsilon_0 = 24 V$ and frequency $440 Hz$. The phase angle at this frequency is 30° . If $L = 12 mH$ and $C = 33 \mu F$, what is the circuit resistance, R ? (b) What is the impedance of the circuit? (c) Determine the RMS current in the circuit, I_{rms} . (d) Calculate the RMS voltage across the capacitor, $V_{C(rms)}$.

$$(a) \tan \phi = \frac{X_L - X_C}{R}$$

$$X_L = 2\pi fL = 33.2 \Omega, X_C = 1/2\pi fC = 11.0 \Omega$$

$$R = \frac{X_L - X_C}{\tan \phi} = \boxed{38.5 \Omega}$$

$$(b) Z = \sqrt{(X_L - X_C)^2 + R^2} = \boxed{44.4 \Omega}$$

$$(c) I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{24 V}{\sqrt{2}(44.4 \Omega)} = \boxed{0.38 A}$$

$$(d) V_{C(rms)} = I_{rms} X_C = (0.38 A)(11 \Omega) = \boxed{4.2 V}$$