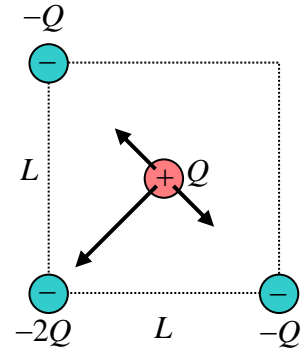


Show all your work

There are three negative charges located at the corners of a square of side L . There is a single positive charge in the center of the square. (a) Draw arrows indicating the direction of the individual forces acting on the charge at the center. Be sure you give the arrows the appropriate relative size. (b) If the value of $Q = 5.0 \text{ nC}$, and $L = 10 \text{ cm}$, determine the net force on the charge at the center and present your answer in component form. (c) What additional charge could be placed at the empty corner of the square so that the charge in the center is in equilibrium?



- (b) By symmetry, two of the forces cancel. Thus we need only find the force due to the $-2Q$ charge,

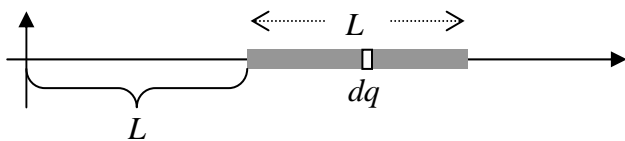
$$\vec{F} = \frac{2Q^2}{4\pi\epsilon_0(L^2/2)} \left[-\cos(45^\circ) \hat{i} - \sin(45^\circ) \hat{j} \right]$$

$$\vec{F} = \frac{-\sqrt{2}Q^2}{2\pi\epsilon_0 L^2} \left[\hat{i} + \hat{j} \right] = \boxed{-6.36 \times 10^{-5} (\hat{i} + \hat{j}) \text{ N/C}}$$

- (c) Placing another negative charge ($-2Q$) at the upper right hand corner would put the charge at the center in equilibrium.

Show all your work

A thin rod of length L is on the x -axis a distance L from the origin and contains a total charge Q spread uniformly over it. The goal is to find the electric field, \vec{E} , at the origin ($x = 0$). (a) What is the linear charge density λ ? (b) Determine the infinitesimal charge, dq . (c) Write down the expression for the infinitesimal electric field, $d\vec{E}$, at the origin. (d) Finally, calculate the integral with the correct limits of integration.



(a) $\lambda = Q/L$

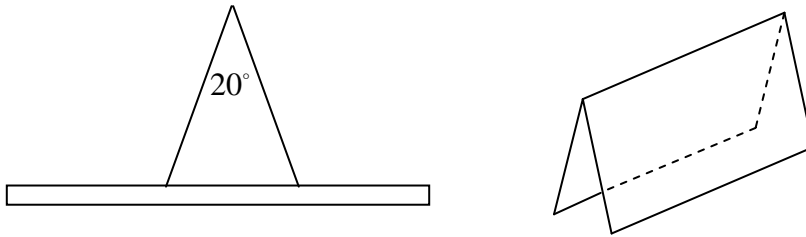
(b) $dq = \lambda dx = \frac{Qdx}{L}$

(c) $d\vec{E} = \frac{dq}{4\pi\epsilon_0 x^2} (-\hat{i})$
 $d\vec{E} = \frac{(Q/L)dx}{4\pi\epsilon_0 x^2} (-\hat{i})$

(d) $\vec{E} = \frac{-Q}{4\pi\epsilon_0 L} \int_{x=L}^{x=2L} \frac{dx}{x^2} \hat{i}$
 $\vec{E} = \frac{-Q}{4\pi\epsilon_0 L} \left(\frac{-1}{x} \right) \Big|_L^{2L} \hat{i} = \boxed{\frac{-Q}{8\pi\epsilon_0 L^2} \hat{i}}$

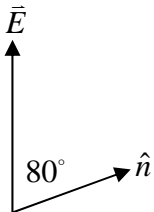
Show all your work

A sheet of paper, 22 cm by 28 cm is folded in half, with a 20° angle and set down on a flat table as shown. A uniform electric field, $\vec{E} = 50 \text{ N/C } \hat{j}$, points upwards, perpendicular to the table. Determine the electric flux coming out the top of the paper.



$\Phi_e = EA \cos \theta$, so to determine the electric flux, we need the angle between the E-field and the surface normal.

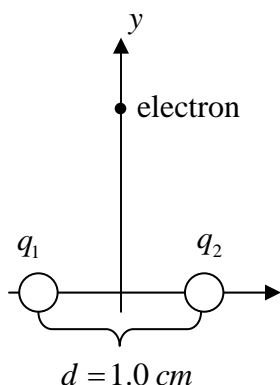
The angle between the vertical and the normal is 80° ,



Thus, $\Phi_e = (50 \text{ N/C})(0.0616 \text{ m}^2) \cos(80^\circ) = \boxed{0.535 \text{ N} \cdot \text{m}^2 / \text{C}}$

Show all your work

Consider the arrangement of charges q_1 and q_2 held in place as shown, 1.0 cm apart. An electron is located initially 5.0 cm above the origin on the y -axis. If the electron is released from rest, how fast is it moving when it is located 1.0 cm above the origin? The charge of the electron is $-1.6 \times 10^{-19}\text{ C}$ and $q_1 = q_2 = 2.0 \times 10^{-6}\text{ C}$.



Use conservation of energy:

$$\Delta K = -\Delta U$$

$$\frac{1}{2}mv_f^2 - 0 = -U_f + U_i$$

$$U_i = 2 \frac{-(1.6 \times 10^{-19})(2.0 \times 10^{-6})}{4\pi(8.85 \times 10^{-12})\sqrt{(0.05)^2 + (0.005)^2}} = -1.145 \times 10^{-13}\text{ J}$$

$$U_f = 2 \frac{-(1.6 \times 10^{-19})(2.0 \times 10^{-6})}{4\pi(8.85 \times 10^{-12})\sqrt{(0.01)^2 + (0.005)^2}} = -5.147 \times 10^{-13}\text{ J}$$

$$v_f = \sqrt{\frac{2}{9.11 \times 10^{-31}}(-1.145 \times 10^{-13} - -5.147 \times 10^{-13})}$$

$$v_f = \boxed{9.37 \times 10^8\text{ m/s}}$$

Our answer is problematic...the electron's speed actually *exceeds* the speed of light! To do things correctly, we would have to use Einstein's theory of special relativity. This is often the case when dealing with small particles – but the theory is beyond the scope of the present course.

Show all your work

Two $2.0\text{ cm} \times 2.0\text{ cm}$ metal electrodes (plates) are spaced 1.0 mm apart and connected by wires to the terminals of a 9.0 V battery. (a) What is the charge on each electrode and the potential difference between the two plates? (b) Now the plates are disconnected from the battery and insulating handles are used to separate the plates to a new spacing of 2.0 mm . Now what is the charge on each plate and the potential difference between the plates? (c) How has the stored energy changed in the process of separating the plates? (d) Where does this energy come from or go to?

(a) First we need to determine the capacitance, $C = \epsilon_0 A / d = 3.54\text{ pF}$. The potential difference is that of the battery, 9.0 V . Thus, the charge on each plate is,

$$Q = C\Delta V = (3.54\text{ pF})(9.0\text{ V}) = 31.86\text{ pC}$$

(b) When the plates are disconnected from the battery, the voltage can change but the *charge* will remain constant (it has nowhere to go). Thus, $Q = 31.86\text{ pC}$. However, the capacitance must drop in half since the spacing has doubled: $C = 1.77\text{ pF}$. To find the potential difference, we realize that the electric field will remain constant since the charge density has not changed ($E = \eta / \epsilon_0$). Thus, the potential difference must double since the spacing has doubled,

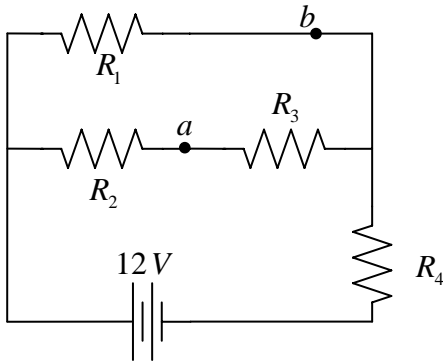
$$\Delta V = Ed = 18\text{ V}.$$

(c) The stored energy doubles as the plates are separated. With $U = \frac{1}{2}C(\Delta V)^2$, the voltage doubles but the capacitance drops in half. The overall effect is that U doubles.

(d) The increase in energy is provided by the person separating the plates (the work they must do to separate them – after all, the plates are attracted to each other).

Show all your work

Find the current through each resistor if $R_1 = 3.0 \Omega$, $R_2 = 3.0 \Omega$, $R_3 = 6.0 \Omega$, and $R_4 = 2.0 \Omega$.
 (b) Which point is at a higher potential, a or b ? Explain.



(a) Reduce R_2 and R_3 since they are in series: $R_{23} = 9.0 \Omega$. Then, this resistor is in parallel with R_1 . Reduce these down to one resistor: $R_{123} = \left(\frac{1}{9} + \frac{1}{3}\right)^{-1} = \frac{9}{4} \Omega$. Finally, this is then in series with R_4 , making the equivalent resistance: $R_{eq} = \frac{17}{4} \Omega$.

The current in this equivalent resistor is: $I = \frac{12V}{4.25 \Omega} = 2.82 A$.

Now go backwards...this is the current through R_{123} and R_4 . So, $I_{R_4} = \boxed{2.82 A}$

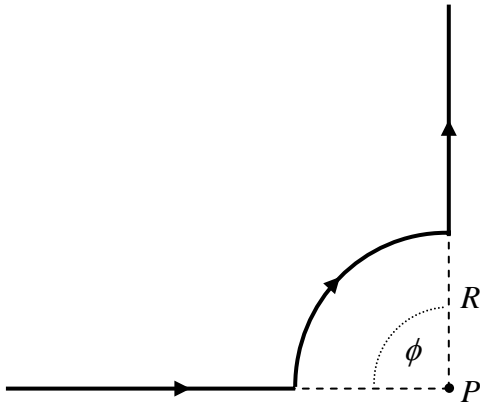
To get the current in R_1 and R_{23} we need the voltage across these. Well, this is the same as the voltage across R_{123} (since R_1 and R_{23} are in parallel). This is $\Delta V_{123} = (2.82 A)(2.25 \Omega) = 6.35 V$.

Now we can get the current through R_1 and R_{23} : $I_{R_1} = (6.35 V)/(3.0 \Omega) = \boxed{2.12 A}$,
 $I_{R_{23}} = (6.35 V)/(9.0 \Omega) = 0.71 A$. However, since R_2 and R_3 are in series they both have this current: $I_{R_2} = I_{R_3} = \boxed{0.71 A}$

(b) Point a must be at a higher potential since there is less of a voltage drop across R_2 than there is across R_1 (these resistors have the same value but the current is higher through R_1).

Show all your work

Using the Biot-Savart law, determine the magnetic field at the center of the circular arc (point P) when the angle is $\phi = \pi/2$. Show that your answer is consistent with the expression for the field of a complete circular current loop when $\phi = 2\pi$: $B = \mu_0 I / 2R$.



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^\phi \frac{d\vec{s} \times \vec{r}}{r^3}$$

In this problem, the straight parts of the current, will not contribute to the field at point P since $d\vec{s}$ is parallel or antiparallel to \vec{r} . In all cases, \vec{r} points from a current element to the point P .

For the current arc, the cross product $d\vec{s} \times \vec{r}$, points into the page, so this is the direction of the B-field at the point P . Also, the magnitude of \vec{r} is just the radius of the arc, $|\vec{r}| \equiv r = R$. Since $d\vec{s}$ is perpendicular to \vec{r} for the current arc, the angle between the two is 90° and the sine is 1. So we have,

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^\phi \frac{d\vec{s} \times \vec{r}}{r^3} = \frac{\mu_0 I}{4\pi} \int_0^\phi \frac{ds}{r^2} = \frac{\mu_0 I}{4\pi} \int_0^\phi \frac{ds}{R^2}$$

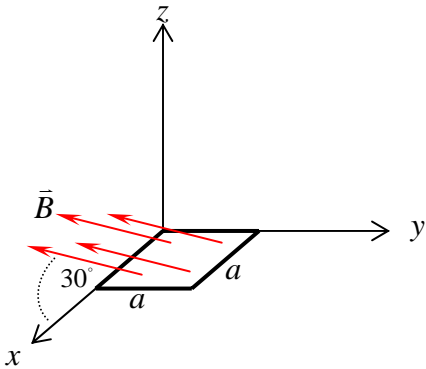
Now ds is an infinitesimal element of the arc, $ds = R d\phi$, so we may now integrate to find the field,

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{\pi/2} \frac{R d\phi}{R^2} = \frac{\mu_0 I \phi}{4\pi R} \Big|_0^{\pi/2} = \boxed{\frac{\mu_0 I}{8R}}, \text{ into the page.}$$

A complete circle would give $4 \times$ this amount, or $\mu_0 I / 2R$.

Show all your work

A uniform magnetic field $B = 2.8 T$ points at an angle of 30° to the positive x -axis. A square loop of side $a = 5.0 cm$ lies flat in the x - y plane. (a) What is the magnetic flux through the loop? (b) Now the flux drops by half in a time of $0.001 s$. What is the average induced EMF in the loop? (c) If the loop has a resistance of 0.02Ω , what is the average induced current? (d) In what direction does the current flow if looking *down* from above the x - y plane? Explain your answer.



(a) $\Phi_m = BA \cos \theta = (2.8 T)(0.05 m)^2 \cos(60^\circ) = \boxed{0.0035 T \cdot m^2}$

(b) $\varepsilon_{avg} = -\frac{\Delta \Phi_m}{\Delta t} = -\frac{-0.00175 T \cdot m^2}{0.001 s} = \boxed{1.75 V}$

(c) $I_{ind} = \frac{\varepsilon}{R} = \frac{1.75 V}{0.02 \Omega} = \boxed{87.5 A}$

(d) Since the flux upward through the loop is decreasing, a current flows in the CCW direction to send more flux upward and stop the decrease. This can also be explained by the + sign of the induced current since CCW is the positive orientation for the area normal in the upward z -direction.