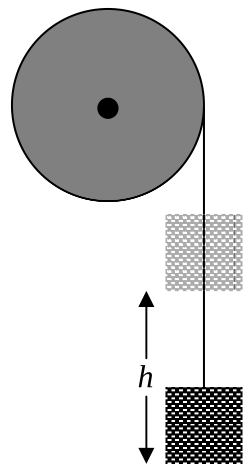
Physics 211 / Spring 2009 **Name: Answer Key**
Dr. Zimmerman

Review Worksheet 2

(1) Initially at rest, a mass $m = 5.0\ kg$ is attached to a rope that is wound around a pulley wheel as shown (a solid disk of mass $M = 10kg$ and radius $R = 0.25m$). (a) Using conservation of energy, determine the angular speed of the wheel, when the mass has fallen a distance of $h = 2.5m$. (b) What is the linear speed of the hanging mass at this instant? (c) What is the angular momentum of the disk (including the direction)? (d) Using kinematics and Newton's 2nd law, determine the tension in the string while the mass is falling.



(a) $E_i = mgh$

$E_f = \tfrac{1}{2} I_{disk}\omega^2 + \tfrac{1}{2} mv^2$

$mgh = \tfrac{1}{2} I_{disk}\omega^2 + \tfrac{1}{2} mv^2$

$$\omega = \sqrt{\frac{2mgh}{\left(\frac{M}{2} + m\right)R^2}} = \boxed{19.8\ rad/s}$$

(b) $v = R\omega = (0.25\ m)(19.8\ rad/s) = \boxed{4.95\ m/s}$

(c) $L = I_{disk}\omega = \boxed{6.19\ kg \cdot m^2/s}$ {into the page}

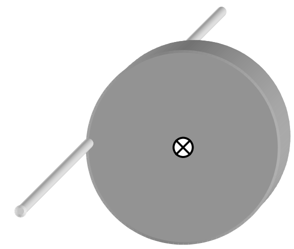
(c) $v_f^2 = 2ah$

$a = 4.9\ m/s^2$

$mg - T = ma$

$T = m(g - a) = \boxed{24.5\ N}$

(2) A $5.0\,kg$, $60\,cm$ – diameter disk rotates on a frictionless axle through one edge. The axle is parallel to the floor. The cylinder is held with the center of mass at the same height as the axle, then released. (a) What is the cylinder's initial angular acceleration? (b) What is the cylinder's angular velocity when the center of mass is directly below the axle? (c) For small angles, at what frequency (in Hertz) will the cylinder oscillate about this axle? The moment of inertia about the center of mass is $I_{cm} = 0.5MR^2$.



The torque form of Newton's 2nd law gives,

$$\tau_{net} = I\alpha$$

If CCW is positive we have,

$$-\cancel{M}g\cancel{R} = (\tfrac{1}{2}\cancel{M}R^{\cancel{2}} + \cancel{M}R^{\cancel{2}})\alpha$$

(a) $\alpha = \dfrac{-2g}{3R} = \boxed{-21.8\,rad/s^2}$

Use conservation of energy,

$$E_i = MgR = \tfrac{1}{2}I\omega^2 = E_f$$

$$\cancel{M}g\cancel{R} = \tfrac{1}{2}(\tfrac{3}{2}\cancel{M}R^{\cancel{2}})\omega^2$$

(b) $\omega = \sqrt{\dfrac{4g}{3R}} = \boxed{6.6\,rad/s}$

$$-\cancel{M}g\cancel{R}\sin\theta = (\tfrac{1}{2}\cancel{M}R^{\cancel{2}} + \cancel{M}R^{\cancel{2}})\frac{d^2\theta}{dt^2}$$

$$-\frac{2g}{3R}\sin\theta = \frac{d^2\theta}{dt^2}$$

Using the small angle approximation ($\sin\theta \approx \theta$) we have,

$$\frac{d^2\theta}{dt^2} + \left(\frac{2g}{3R}\right)\theta = 0$$

$$\Rightarrow \omega = 2\pi f = \sqrt{\frac{2g}{3R}}$$

(c) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{2g}{3R}} = \boxed{0.74\,Hz}$

(3) A mass M oscillates on a frictionless horizontal surface with frequency $2.5\ Hz$ and amplitude $0.20\ m$ when connected to a spring of constant $250\ N/m$. (a) Determine the value of M and its maximum speed. (b) Another mass $M/2$ is dropped directly from above and sticks onto the first mass when the spring is fully extended. What is the maximum speed of the two masses when they move through the equilibrium position? (c) Why does the maximum speed change but not the amplitude?

(a) $$2\pi f = \sqrt{\frac{k}{m}}$$

$$M = \frac{k}{4\pi^2 f^2} = \frac{250\ N/m}{4\pi^2 (2.5\ Hz)^2} = \boxed{1.0\ kg}$$

$$v_{max} = A\omega = (0.20\ m)(15.7\ rad/s) = \boxed{3.14\ m/s}$$

(b) $$E_i = \tfrac{1}{2}kA^2 = 0.5(250\ N/m)(0.20\ m)^2 = 5.0\ J$$

$$E_f = \tfrac{1}{2}(1.5M)v^2$$

We may apply conservation of energy,

$$\tfrac{1}{2}(1.5M)v^2 = 5.0\ J$$

$$0.5(1.5\ kg)v^2 = 5.0\ J$$

$$v = \boxed{2.58\ m/s}$$

(c) The amplitude does not change because the total energy is unchanged. The maximum speed is reduced because the frequency is reduced by the same factor. Another way to think about it is that the greater mass moves at a lower speed since the energy is consta

(4) Space shuttle astronauts can weigh themselves (measure their mass, really) by sitting in an oscillating chair and measuring the period of oscillation. When the $20\ kg$ – chair is empty, the period of oscillation is $T = 1.98\ s$. (a) Determine the spring constant. (b) Now Annie the astronaut sits in the chair and finds the period has changed to $3.90\ s$, what is her mass? (c) If the amplitude of the chair's oscillation is $30\ cm$ when Annie is seated in it, what is the maximum speed of the chair?

(a) $k = \frac{4\pi^2 m}{T^2} = \boxed{201.4\ N/m}$

(b) $m + M_{Annie} = \frac{kT^2}{4\pi^2} = 77.6\ kg$

$M_{Annie} = \boxed{57.6\ kg}$

(c) $v_{max} = A\omega = (0.3\ m)(1.61\ rad/s) = \boxed{0.48\ m/s}$