EUCLID
Euclid is said to have been younger than the first pupils of Plato but older than Archimedes, which would place the time of his flourishing about 300 B.C. He probably received his early mathematical education in Athens from the pupils of Plato, since most of the geometers and mathematicians on whom he depended were of that school. Proclus, the Neo-Platonist of the fifth century, asserts that Euclid was of the school of Plato and “intimate with that philosophy.” His opinion, however, may have been based only on his view that the treatment of the five regular (“Platonic”) solids in Book XIII is the “end of the whole Elements.”

The only other fact concerning Euclid is that he taught and founded a school at Alexandria in the time of Ptolemy I, who reigned from 306 to 283 B.C. The evidence for the place comes from Pappus (fourth century A.D.), who notes that Apollonius “spent a very long time with the pupils of Euclid at Alexandria, and it was thus that he acquired such a scientific habit of thought.” Proclus claims that it was Ptolemy I who asked Euclid if there was no shorter way to geometry than the Elements and received as answer: “There is no royal road to geometry.” The other story about Euclid that has come down from antiquity concerns his answer to a pupil who at the end of his first lesson in geometry asked what he would get by learning such things, whereupon Euclid called his slave and said: “Give him a coin since he must needs make gain by what he learns.”

Something of Euclid’s character would seem to be disclosed in the remark of Pappus regarding Euclid’s “scrupulous fairness and his exemplary kindness towards all who advance mathematical science to however small an extent.” The context of the remark seems to indicate, however, that Pappus is not giving a traditional account of Euclid but offering an explanation of his own of Euclid’s failure to go further than he did with his investigation of a certain problem in conics.

Euclid’s great work, the thirteen books of the Elements, must have become a classic soon after publication. From the time of Archimedes they are constantly referred to and used as a basic text-book. It was recognized in antiquity that Euclid had drawn upon all his predecessors. According to Proclus, he “collected many of the theorems of Eudoxus, perfected many of those of Theatetus, and also brought to incontrovertible demonstration the things which were only loosely proved by his predecessors.” The other extant works of Euclid include: the Data, for use in the solution of problems by geometrical analysis, On Divisions (of figures), the Optics, and the Phenomena, a treatise on the geometry of the sphere for use in astronomy. His lost Elements of Music may have provided the basis for the extant Sectio Canonis on the Pythagorean theory of music. Of lost geometrical works all except one belonged to higher geometry.

Since the later Greeks knew nothing about the life of Euclid, the mediaeval
translators and editors were left to their own devices. He was usually called Megarensis, through confusion with the philosopher Euclides of Megara, Plato's contemporary. The Arabs found that the name of Euclid, which they took to be compounded from ucli (key) and dis (measure) revealed the "key of geometry." They claimed that the Greek philosophers used to post upon the doors of their schools the well-known notice: "Let no one come to our school who has not learned the Elements of Euclid," thus transferring the inscription over Plato's Academy to all scholastic doors and substituting the Elements for geometry.
BOOK ONE

DEFINITIONS

1. A point is that which has no part.
2. A line is breadthless length.
3. The extremities of a line are points.
4. A straight line is a line which lies evenly with the points on itself.
5. A surface is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A plane surface is a surface which lies evenly with the straight lines on itself.
8. A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called rectilineal.
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right and the straight line standing on the other is called a perpendicular to that on which it stands.
11. An obtuse angle is an angle greater than a right angle.
12. An acute angle is an angle less than a right angle.
13. A boundary is that which is an extremity of anything.
14. A figure is that which is contained by any boundary or boundaries.
15. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another;
16. And the point is called the centre of the circle.
17. A diameter of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.
18. A semicircle is the figure contained by the diameter and the circumference cut off by it. And the centre of the semicircle is the same as that of the circle.
19. Rectilineal figures are those which are contained by straight lines, trilateral figures being those contained by three, quadrilateral those contained by four, and multilateral those contained by more than four straight lines.
20. Of trilateral figures, an equilateral triangle is that which has its three sides equal, an isosceles triangle that which has two of its sides alone equal, and a scalene triangle that which has its three sides unequal.
21. Further, of trilateral figures, a right-angled triangle is that which has a right angle, an obtuse-angled triangle that which has an obtuse angle, and an acute-angled triangle that which has its three angles acute.
EUCLID

22. Of quadrilateral figures, a square is that which is both equilateral and right-angled; an oblong that which is right-angled but not equilateral; a rhombus that which is equilateral but not right-angled; and a rhomboid that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called trapezia.

23. Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

POSTULATES

Let the following be postulated:

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

COMMON NOTIONS

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

BOOK I. PROPOSITIONS

PROPOSITION I

On a given finite straight line to construct an equilateral triangle.

Let AB be the given finite straight line.

Thus it is required to construct an equilateral triangle on the straight line AB.

With centre A and distance AB let the circle BCD be described; [Post. 3] again, with centre B and distance BA let the circle ACE be described; [Post. 3] and from the point C, in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined. [Post. 1]

Now, since the point A is the centre of the circle CDB,

AC is equal to AB. [Def. 15]

Again, since the point B is the centre of the circle CAE,

BC is equal to BA. [Def. 15]

But CA was also proved equal to AB;

therefore each of the straight lines CA, CB is equal to AB.

And things which are equal to the same thing are also equal to one another;

therefore CA is also equal to CB. [C. N. 1]

Therefore the three straight lines CA, AB, BC are equal to one another.
Therefore the triangle $ABC$ is equilateral; and it has been constructed on the given finite straight line $AB$.

(Being) what it was required to do.

**Proposition 2**

To place at a given point (as an extremity) a straight line equal to a given straight line.

Let $A$ be the given point, and $BC$ the given straight line.

Thus it is required to place at the point $A$ (as an extremity) a straight line equal to the given straight line $BC$.

From the point $A$ to the point $B$ let the straight line $AB$ be joined; [Post. 1] and on it let the equilateral triangle $DAB$ be constructed. [1. 1]

Let the straight lines $AE$, $BF$ be produced in a straight line with $DA$, $DB$; [Post. 2] with centre $B$ and distance $BC$ let the circle $CGH$ be described; [Post. 3] and again, with centre $D$ and distance $DG$ let the circle $GKL$ be described. [Post. 3]

Then, since the point $B$ is the centre of the circle $CGH$, $BC$ is equal to $BG$.

Again, since the point $D$ is the centre of the circle $GKL$, $DL$ is equal to $DG$.

And in these $DA$ is equal to $DB$; therefore the remainder $AL$ is equal to the remainder $BG$. [C.N. 3]

But $BC$ was also proved equal to $BG$; therefore each of the straight lines $AL$, $BC$ is equal to $BG$.

And things which are equal to the same thing are also equal to one another; [C.N. 1]

therefore $AL$ is also equal to $BC$.

Therefore at the given point $A$ the straight line $AL$ is placed equal to the given straight line $BC$.

(Being) what it was required to do.

**Proposition 3**

Given two unequal straight lines, to cut off from the greater a straight line equal to the less.

Let $AB$, $C$ be the two given unequal straight lines, and let $AB$ be the greater of them.

Thus it is required to cut off from $AB$ the greater a straight line equal to $C$ the less.

At the point $A$ let $AD$ be placed equal to the straight line $C$; [1. 2] and with centre $A$ and distance $AD$ let the circle $DEF$ be described. [Post. 3]

Now, since the point $A$ is the centre of the circle $DEF$, $AE$ is equal to $AD$. [Def. 15]
But $C$ is also equal to $AD$.

Therefore each of the straight lines $AE$, $C$ is equal to $AD$; so that $AE$ is also equal to $C$.

Therefore, given the two straight lines $AB$, $C$, from $AB$ the greater $AE$ has been cut off equal to $C$ the less.

(Being) what it was required to do.

**Proposition 4**

If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.

Let $ABC$, $DEF$ be two triangles having the two sides $AB$, $AC$ equal to the two sides $DE$, $DF$ respectively, namely $AB$ to $DE$ and $AC$ to $DF$, and the angle $BAC$ equal to the angle $EDF$.

I say that the base $BC$ is also equal to the base $EF$, the triangle $ABC$ will be equal to the triangle $DEF$, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend, that is, the angle $ABC$ to the angle $DEF$, and the angle $ACB$ to the angle $DFE$.

For, if the triangle $ABC$ be applied to the triangle $DEF$,

and if the point $A$ be placed on the point $D$

and the straight line $AB$ on $DE$,

then the point $B$ will also coincide with $E$, because $AB$ is equal to $DE$.

Again, $AB$ coinciding with $DE$,

the straight line $AC$ will also coincide with $DF$, because the angle $BAC$ is equal to the angle $EDF$;

hence the point $C$ will also coincide with the point $F$, because $AC$ is again equal to $DF$.

But $B$ also coincided with $E$;

hence the base $BC$ will coincide with the base $EF$.

[For if, when $B$ coincides with $E$ and $C$ with $F$, the base $BC$ does not coincide with the base $EF$, two straight lines will enclose a space: which is impossible.

Therefore the base $BC$ will coincide with $EF$] and will be equal to it. [C.N. 4]

Thus the whole triangle $ABC$ will coincide with the whole triangle $DEF$, and will be equal to it.

And the remaining angles will also coincide with the remaining angles and will be equal to them,

the angle $ABC$ to the angle $DEF$,

and the angle $ACB$ to the angle $DFE$.

(Being) what it was required to prove.

**Proposition 5**

In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.

Let $ABC$ be an isosceles triangle having the side $AB$ equal to the side $AC$;
and let the straight lines $BD$, $CE$ be produced further in a straight line with $AB$, $AC$. [Post. 2]

I say that the angle $ABC$ is equal to the angle $ACB$; and the angle $CBD$ to the angle $BCE$.

Let a point $F$ be taken at random on $BD$; from $AE$ the greater let $AG$ be cut off equal to $AF$ the less; and let the straight lines $FC$, $GB$ be joined. [Post. 1]

Then, since $AF$ is equal to $AG$ and $AB$ to $AC$, the two sides $FA$, $AC$ are equal to the two sides $GA$, $AB$, respectively; and they contain a common angle, the angle $FAG$.

Therefore the base $FC$ is equal to the base $GB$, and the triangle $AFC$ is equal to the triangle $AGB$, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend, that is, the angle $ACF$ to the angle $ABG$, and the angle $AFC$ to the angle $AGB$. [I. 4]

And, since the whole $AF$ is equal to the whole $AG$, and in these $AB$ is equal to $AC$, the remainder $BF$ is equal to the remainder $CG$.

But $FC$ was also proved equal to $GB$; therefore the two sides $BF$, $FC$ are equal to the two sides $CG$, $GB$ respectively; and the angle $BFC$ is equal to the angle $CGB$; while the base $BC$ is common to them; therefore the triangle $BFC$ is also equal to the triangle $CGB$, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.

therefore the angle $BFC$ is equal to the angle $GCB$, and the angle $BCF$ to the angle $CBG$.

Accordingly, since the whole angle $ABG$ was proved equal to the angle $ACF$, and in these the angle $CBG$ is equal to the angle $BCF$, the remaining angle $ABC$ is equal to the remaining angle $ACB$; and they are at the base of the triangle $ABC$.

But the angle $FBC$ was also proved equal to the angle $GCB$; and they are under the base.

Therefore etc. Q.E.D.