violent in their language as the Bernoullis sometimes were. One of these ripe old chestnuts may be retailed again as it is one of the comparatively early authentic instances of a story which must be at least as old as ancient Egypt, and of which we daily see variants pinned onto all sorts of prominent characters from Einstein down. Once when travelling as a young man Daniel modestly introduced himself to an interesting stranger with whom he had been conversing: “I am Daniel Bernoulli.” “And I,” said the other sarcastically, “am Isaac Newton.” This delighted Daniel to the end of his days as the sincerest tribute he had ever received.

CHAPTER NINE

Analysis Incarnate

EULER

History shows that those heads of empires who have encouraged the cultivation of mathematics, the common source of all the exact sciences, are also those whose reigns have been the most brilliant and whose glory is the most durable.

—MICHEL CHASLES

“Euler calculated without apparent effort, as men breathe, or as eagles sustain themselves in the wind” (as Arago said), is not an exaggeration of the unequalled mathematical facility of Léonard Euler (1707–1783), the most prolific mathematician in history, and the man whom his contemporaries called “analysis incarnate.” Euler wrote his great memoirs as easily as a fluent writer composes a letter to an intimate friend. Even total blindness during the last seventeen years of his life did not retard his unparalleled productivity; indeed, if anything, the loss of his eyesight sharpened Euler’s perceptions in the inner world of his imagination.

The extent of Euler’s work was not accurately known even in 1956, but it has been estimated that sixty to eighty large quarto volumes will be required for the publication of his collected works. In 1909 the Swiss Association for Natural Science undertook the collection and publication of Euler’s scattered memoirs, with financial assistance from many individuals and mathematical societies throughout the world—rightly claiming that Euler belongs to the whole civilized world and not only to Switzerland. The careful estimates of the probable expense (about $80,000 in the money of 1909) were badly upset by the discovery in St. Petersburg (Leningrad) of an unsuspected mass of Euler’s manuscripts.

Euler’s mathematical career opened in the year of Newton’s death. A more propitious epoch for a genius like that of Euler’s could not have been chosen. Analytic geometry (made public in 1637) had been in use ninety years, the calculus about fifty, and Newton’s law of uni-
universal gravitation, the key to physical astronomy, had been before the mathematical public for forty years. In each of these fields a vast number of isolated problems had been solved, with here and there notable attempts at unification; but no systematic attack had yet been launched against the whole of mathematics, pure and applied, as it then existed. In particular the powerful analytical methods of Descartes, Newton, and Leibniz had not yet been exploited to the limit of what they were then capable, especially in mechanics and geometry.

On a lower level algebra and trigonometry were then in shape for systematization and extension; the latter particularly was ready for essential completion. In Fermat’s domain of Diophantine analysis and the properties of the common whole numbers no such “temporary perfection” was possible (it is not even yet); but even here Euler proved himself the master. In fact one of the most remarkable features of Euler’s universal genius was its equal strength in both of the main currents of mathematics, the continuous and the discrete.

As an algorist Euler has never been surpassed, and probably never even closely approached, unless perhaps by Jacobi. An algorist is a mathematician who devises “algorithms” (or “algorisms”) for the solution of problems of special kinds. As a very simple example, we assume (or prove) that every positive real number has a real square root. How shall the root be calculated? There are many ways known; an algorist devises practicable methods. Or again, in Diophantine analysis, also in the integral calculus, the solution of a problem may not be forthcoming until some ingenious (often simple) replacement of one or more of the variables by functions of other variables has been made; an algorist is a mathematician to whom such ingenious tricks come naturally. There is no uniform mode of procedure—algorists, like facile rhymesters, are born, not made.

It is fashionable today to despise the “mere algorist”; yet, when a truly great one like the Hindu Ramanujan arrives unexpectedly out of nowhere, even expert analysts hail him as a gift from Heaven: his all but supernatural insight into apparently unrelated formulas reveals hidden trails leading from one territory to another, and the analysts have new tasks provided for them in clearing the trails. An algorist is a “formalist” who loves beautiful formulas for their own sake.

Before going on to Euler’s peaceful but interesting life we must mention two circumstances of his times which furthered his prodigious activity and helped to give it a direction.

In the eighteenth century the universities were not the principal centers of research in Europe. They might have become such sooner than they did but for the classical tradition and its understandable hostility to science. Mathematics was close enough to antiquity to be respectable, but physics, being more recent, was suspect. Further, a mathematician in a university of the time would have been expected to put much of his effort on elementary teaching; his research, if any, would have been an unprofitable luxury, precisely as in the average American institution of higher learning today. The Fellows of the British universities could do pretty well as they chose. Few, however, chose to do anything, and what they accomplished (or failed to accomplish) could not affect their bread and butter. Under such laxity or open hostility there was no good reason why the universities should have led in science, and they did not.

The lead was taken by the various royal academies supported by generous or farsighted rulers. Mathematics owes an undischargeable debt to Frederick the Great of Prussia and Catherine the Great of Russia for their broadminded liberality. They made possible a full century of mathematical progress in one of the most active periods in scientific history. In Euler’s case Berlin and St. Petersburg furnished the sinews of mathematical creation. Both of these foci of creativity owed their inspiration to the restless ambition of Leibniz. The academies for which Leibniz had drawn up the plans gave Euler his chance to be the most prolific mathematician of all time; so, in a sense, Euler was Leibniz’ grandson.

The Berlin Academy had been slowly dying of brainlessness for forty years when Euler, at the instigation of Frederick the Great, shocked it into life again; and the St. Petersburg Academy, which Peter the Great did not live to organize in accordance with Leibniz’ program, was firmly founded by his successor.

These Academies were not like some of those today, whose chief function is to award membership in recognition of good work well done; they were research organizations which paid their leading members to produce scientific research. Moreover the salaries and perquisites were ample for a man to support himself and his family in decent comfort. Euler’s household at one time consisted of no fewer than eighteen persons; yet he was given enough to support them all adequately. As a final touch of attractiveness to the life of an academician in the
eighteenth century, his children, if worth anything at all, were assured of a fair start in the world.

This brings us to a second dominant influence on Euler's vast mathematical output. The rulers who paid the bills naturally wanted something in addition to abstract culture for their money. But it must be emphasized that when once the rulers had obtained a reasonable return on their investment, they did not insist that their employees spend the rest of their time on "productive" labor; Euler, Lagrange, and the other academicians were free to do as they pleased. Nor was any noticeable pressure brought to bear to squeeze out the few immediately practical results which the state could use. Wiser in their generation than many a director of a research institute today, the rulers of the eighteenth century merely suggested occasionally what they needed at once, and let science take its course. They seem to have felt instinctively that so-called "pure" research would throw off as byproducts the instantly practical things they desired if given a hint of the right sort now and then.

To this general statement there is one important exception, which neither proves nor disproves the rule. It so happened that in Euler's time the outstanding problem in mathematical research chanced also to coincide with what was probably the first practical problem of the age—control of the seas. That nation whose technique in navigation surpassed that of all its competitors would inevitably rule the waves. But navigation is ultimately an affair of accurately determining one's position at sea hundreds of miles from land, and of doing it so much better than one's competitors that they can be outsailed to the unfavorable only for them, of a naval battle. Britannia, as everyone knows, rules the waves. That she does so is due in no small measure to the practical application which her navigators were able to make of purely mathematical investigations in celestial mechanics during the eighteenth century.

One such application concerned Euler directly—if we may anticipate slightly. The founder of modern navigation is of course Newton, although he himself never bothered his head about the subject and never (so far as seems to be known) planted his shoe on the deck of a ship. Position at sea is determined by observations on the heavenly bodies (sometimes including the satellites of Jupiter in really fancy navigation); and after Newton's universal law had suggested that with sufficient patience the positions of the planets and the phases of the Moon could be calculated for a century in advance if necessary, those who wished to govern the seas set their computers on the nautical almanac to grinding out tables of future positions.

In such a practical enterprise the Moon offers a particularly vicious problem, that of three bodies attracting one another according to the Newtonian law. This problem will recur many times as we proceed to the twentieth century; Euler was the first to evolve a calculable solution for the problem of the Moon ("the lunar theory"). The three bodies concerned are the Moon, the Earth, and the Sun. Although we shall defer what little can be said here on this problem to later chapters, it may be remarked that the problem is one of the most difficult in the whole range of mathematics. Euler did not solve it, but his method of approximative calculation (superseded today by better methods) was sufficiently practical to enable an English computer to calculate the lunar tables for the British Admiralty. For this the computer received £5000 (quite a sum for the time), and Euler was voted a bonus of £300 for the method.

Léonard (or Leonhard) Euler, a son of Paul Euler and his wife Marguerite Brucker, is probably the greatest man of science that Switzerland has produced. He was born at Basle on April 15, 1707, but moved the following year with his parents to the nearby village of Riechen, where his father became the Calvinist pastor. Paul Euler himself was an accomplished mathematician, having been a pupil of Jacob Bernoulli. The father intended Léonard to follow in his footsteps and succeed him in the village church, but fortunately made the mistake of teaching the boy mathematics.

Young Euler knew early what he wanted to do. Nevertheless he dutifully obeyed his father, and on entering the University of Basle studied theology and Hebrew. In mathematics he was sufficiently advanced to attract the attention of Johannes Bernoulli, who generously gave the young man one private lesson a week. Euler spent the rest of the week preparing for the next lesson so as to be able to meet his teacher with as few questions as possible. Soon his diligence and marked ability were noticed by Daniel and Nicolaus Bernoulli, who became Euler's fast friends.

Léonard was permitted to enjoy himself till he took his master's degree in 1724 at the age of seventeen, when his father insisted that he abandon mathematics and put all his time on theology. But the
father gave in when the Bernoullis told him that his son was destined to be a great mathematician and not the pastor of Riechen. Although the prophecy was fulfilled Euler’s early religious training influenced him all his life and he never discarded a particle of his Calvinistic faith. Indeed as he grew older he swung round in a wide orbit toward the calling of his father, conducting family prayers for his whole household and usually finishing off with a sermon.

Euler’s first independent work was done at the age of nineteen. It has been said that this first effort reveals both the strength and the weakness of much of Euler’s subsequent work. The Paris Academy had proposed the masting of ships as a prize problem for the year 1727; Euler’s memoir failed to win the prize but received an honorable mention. He was later to recoup this loss by winning the prize twelve times. The strength of the work was the analysis—the technical mathematics—it contained; its weakness the remoteness of the connection, if any, with practicality. The last is not very surprising when we remember the traditional jokes about the nonexistent Swiss navy. Euler might have seen a boat or two on the Swiss lakes, but he had not yet seen a ship. He has been criticized, sometimes justly, for letting his mathematics run away with his sense of reality. The physical universe was an occasion for mathematics to Euler, scarcely a thing of much interest in itself; and if the universe failed to fit his analysis it was the universe which was in error.

Knowing that he was a born mathematician, Euler applied for the professorship at Basle. Failing to get the position, he continued his studies, buoyed up by the hope of joining Daniel and Nicolaus Bernoulli at St. Petersburg. They had generously offered to find a place for Euler in the Academy and kept him well posted.

At this stage of his career Euler seems to have been curiously indifferent as to what he should do, provided only it was something scientific. When the Bernoullis wrote of a prospective opening in the medical section of the St. Petersburg Academy, Euler flung himself into physiology at Basle and attended the lectures on medicine. But even in this field he could not keep away from mathematics: the physiology of the ear suggested a mathematical investigation of sound, which in turn led out into another on the propagation of waves, and so on—this early work kept branching out like a tree gone mad in a nightmare all through Euler’s career.

The Bernoullis were fast workers. Euler received his call to St. Petersburg in 1727, officially as an associate of the medical section of the Academy. By a wise provision every imported member was obliged to take with him two pupils—actually apprentices to be trained. Poor Euler’s joy was quickly dashed. The very day he set foot on Russian soil the liberal Catherine I died.

Catherine, Peter the Great’s mistress before she became his wife, seems to have been a broadminded woman in more ways than one, and it was she who in her reign of only two years carried out Peter’s wishes in establishing the Academy. On Catherine’s death the power passed into the hands of an unusually brutal faction during the minority of the boy czar (who perhaps fortunately for himself died before he could begin his reign). The new rulers of Russia looked upon the Academy as a dispensable luxury and for some anxious months contemplated suppressing it and sending all the foreign members home. Such was the state of affairs when Euler arrived in St. Petersburg. Nothing was said in the confusion about the medical position to which he had been called, and he slipped into the mathematical section, after having almost accepted a naval lieutenancy in desperation.

Thereafter things went better and Euler settled down to work. For six years he kept his nose to the grindstone, not wholly because he was absorbed in his mathematics but partly because he dared not lead a normal social life on account of the treacherous spies everywhere.

In 1733 Daniel Bernoulli returned to free Switzerland, having had enough of holy Russia, and Euler, at the age of twenty six, stepped into the leading mathematical position in the Academy. Feeling that he was to be stuck in St. Petersburg for the rest of his life, Euler decided to marry, settle down, and make the best of things. The lady was Catharina, a daughter of the painter Gsell, whom Peter the Great had taken back to Russia with him. Political conditions became worse, and Euler longed more desperately than ever to escape. But with the rapid arrival of one child after another Euler felt more securely tied than before and took refuge in incessant work. Some biographers trace Euler’s unmatched productivity to this first sojourn in Russia; common prudence forced him into an unbreakable habit of industry.

Euler was one of several great mathematicians who could work anywhere under any conditions. He was very fond of children (he had thirteen of his own, all but five of whom died very young), and would often compose his memoirs with a baby in his lap while the older chil-
dren played all about him. The ease with which he wrote the most difficult mathematics is incredible.

Many legends of his constant outflow of ideas have survived. Some no doubt are exaggerations, but it is said that Euler would dash off a mathematical paper in the half hour or so before the first and second calls to dinner. As soon as a paper was finished it was laid on top of the growing stack awaiting the printer. When material to fill the transactions of the Academy was needed, the printer would gather up a sheaf from the top of the pile. Thus it happened that the dates of publication frequently ran counter to those of composition. The crazy effect was heightened by Euler’s habit of returning many times to a subject in order to clarify or extend what he had already done, so that occasionally a sequence of papers on a given topic is seen in print through the wrong end of the telescope.

When the boy czar died, Anna Ivanovna (niece of Peter) became Empress in 1780, and so far as the Academy was concerned, things brightened up considerably. But under the indirect rule of Anna’s paramour, Ernest John de Biron, Russia suffered one of the bloodiest reigns of terror in its history, and Euler settled down to a spell of silent work that was to last ten years. Halfway through he suffered his first great misfortune. He had set himself to win the Paris prize for an astronomical problem for which some of the leading mathematicians had asked several months’ time. (As a similar problem occurs in connection with Gauss we shall not describe it here.) Euler solved it in three days. But the prolonged effort brought on an illness in which he lost the sight of his right eye.

It should be noted that the modern higher criticism which has been so effective in discrediting all the interesting anecdotes in the history of mathematics has shown that the astronomical problem was in no way responsible for the loss of Euler’s eye. But how the scholarly critics (or anyone else) come to know so much about the so-called law of cause and effect is a mystery for David Hume’s (a contemporary of Euler) ghost to resolve. With this caution we shall tell once more the famous story of Euler and the atheistic (or perhaps only pantheistic) French philosopher Denis Diderot (1713–1784). This is slightly out of its chronological order, as it happened during Euler’s second stay in Russia.

Invited by Catherine the Great to visit her Court, Diderot earned his keep by trying to convert the courtiers to atheism. Fed up, Cathe-

erine commissioned Euler to muzzle the windy philosopher. This was easy because all mathematics was Chinese to Diderot. De Morgan tells what happened (in his classic Budget of Paradoxes, 1872): “Diderot was informed that a learned mathematician was in possession of an algebraical demonstration of the existence of God, and would give it before all the Court, if he desired to hear it. Diderot gladly consented. ... Euler advanced toward Diderot, and said gravely, and in a tone of perfect conviction:

“Sir, \( \frac{a + b^n}{n} = x \), hence God exists; reply!”

It sounded like sense to Diderot. Humiliated by the unrestrained laughter which greeted his embarrassed silence, the poor man asked Catherine’s permission to return at once to France. She graciously gave it.

Not content with this masterpiece, Euler in all seriousness painted his lily with solemn proofs, in deadly earnest, that God exists and that the soul is not a material substance. It is reported that both proofs passed into the treatises on theology of his day. These are probably the choicest flowers of the mathematically unpractical side of his genius.

Mathematics alone did not absorb all of Euler’s energies during his stay in Russia. Wherever he was called upon to exercise his mathematical talents in ways not too far from pure mathematics he gave the government its full money’s worth. Euler wrote the elementary mathematical textbooks for the Russian schools, supervised the government department of geography, helped to reform the weights and measures, and devised practical means for testing scales. These were but some of his activities. No matter how much extraneous work he did, Euler continued to pour out mathematics.

One of the most important works of this period was the treatise of 1736 on mechanics. Note that the date of publication lacks but a year of marking the centenary of Descartes’ publication of analytic geometry. Euler’s treatise did for mechanics what Descartes had done for geometry—freed it from the shackles of synthetic demonstration and made it analytical. Newton’s Principia might have been written by Archimedes; Euler’s mechanics could not have been written by any Greek. For the first time the full power of the calculus was directed against mechanics and the modern era in that basic science began. Euler was
to be surpassed in this direction by his friend Lagrange, but the credit for having taken the decisive step is Euler's.

On the death of Anna in 1740 the Russian government became more liberal, but Euler had had enough and was glad to accept the invitation of Frederick the Great to join the Berlin Academy. The Dowager Queen took a great fancy to Euler and tried to draw him out. All she got was monosyllables.

"Why don't you want to speak to me?" she asked.

"Madame," Euler replied, "I come from a country where, if you speak, you are hanged."

The next twenty four years of his life were spent in Berlin, not altogether happily, as Frederick would have preferred a polished courtier instead of the simple Euler. Although Frederick felt it his duty to encourage mathematics he despised the subject, being no good at it himself. But he appreciated Euler's talents sufficiently to engage them in practical problems—the coinage, water conduits, navigation canals, and pension systems, among others.

Russia never let go of Euler completely and even while he was in Berlin paid part of his salary. In spite of his many dependents Euler was prosperous, owning a farm near Charlottenburg in addition to his house in Berlin. During the Russian invasion of the March of Brandenburg in 1760 Euler's farm was pillaged. The Russian general, declaring that he was "not making war on the sciences," indemnified Euler for considerably more than the actual damage. When the Empress Elizabeth heard of Euler's loss she sent him a handsome sum in addition to the more than sufficient indemnity.

One cause of Euler's unpopularity at Frederick's court was his inability to keep out of arguments on philosophical questions about which he knew nothing. Voltaire, who spent much of his time toadying to Frederick, delighted with the other brilliant verbalists surrounding Frederick in tying the hapless Euler into metaphysical knots. Euler took it all good-naturedly and joined the others in roaring with laughter at his own ridiculous blunders. But Frederick gradually became irritated and cast about for a more sophisticated philosopher to head his Academy and entertain his Court.

D'Alembert (whom we shall meet later) was invited to Berlin to look over the situation. He and Euler had had a slight coolness over mathematics. But D'Alembert was not the man to let a personal difference cloud his judgment, and he told Frederick bluntly that it would be an outrage to put any other mathematician over Euler. This only made Frederick more stubborn and angrier than ever, and conditions became intolerable for Euler. His sons, he felt, would have no chance in Prussia. At the age of fifty nine (in 1766) he pulled up his stakes once more and migrated back to St. Petersburg at the cordial invitation of Catherine the Great.

Catherine received the mathematician as if he were royalty, setting aside a fully furnished house for Euler and his eighteen dependents, and donating one of her own cooks to run the kitchen.

It was at this time that Euler began to lose the sight of his remaining eye (by a cataract), and before long he was totally blind. The progress of his oncoming darkness is followed with alarm and sympathy in the correspondence of Lagrange, D'Alembert, and other leading mathematicians of the time. Euler himself watched the approach of blindness with equanimity. There can be no doubt that his deep religious faith helped him to face what was ahead of him. But he did not "resign" himself to silence and darkness. He immediately set about repairing the irreparable. Before the last light faded he accustomed himself to writing his formulas with chalk on a large slate. Then, his sons (particularly Albert) acting as amanuenses, he would dictate the words explaining the formulas. Instead of diminishing, his mathematical productivity increased.

All his life Euler had been blessed with a phenomenal memory. He knew Virgil's Aeneid by heart, and although he had seldom looked at the book since he was a youth, could always tell the first and last lines on any page of his copy. His memory was both visual and aural. He also had a prodigious power for mental calculation, not only of the arithmetical kind but also of the more difficult type demanded in higher algebra and the calculus. All the leading formulas of the whole range of mathematics as it existed in his day were accurately stowed away in his memory.

As one instance of his prowess, Condorcet tells how two of Euler's students had summed a complicated convergent series (for a particular value of the variable) to seventeen terms, only to disagree by a unit in the fiftieth place of the result. To decide which was right Euler performed the whole calculation mentally; his answer was found to be correct. All this now came to his aid and he did not greatly miss the light. But even at that, one feat of his seventeen blind years almost passes belief. The lunar theory—the motion of the Moon, the only problem which had ever made Newton's head ache—received its first
thorough workout at Euler's hands. All the complicated analysis was done entirely in his head.

Five years after Euler's return to St. Petersburg another disaster overtook him. In the great fire of 1771 his house and all its furnishings were destroyed, and it was only by the heroism of his Swiss servant (Peter Grimm, or Grimmmon) that Euler escaped with his life. At the risk of his own life Grimm carried his blind and ailing master through the flames to safety. The library was burned, but thanks to the energy of Count Orloff all of Euler's manuscripts were saved. The Empress Catherine promptly made good all the loss and soon Euler was back at work again.

In 1776 (when he was sixty nine) Euler suffered a greater loss in the death of his wife. The following year he married again. The second wife, Salome Abigail Gsell, was a half-sister of the first. His greatest tragedy was the failure (through surgical carelessness, possibly) of an operation to restore the sight of his left eye—the only one for which there was any hope. The operation was "successful" and Euler's joy passed all bounds. But presently infection set in, and after prolonged suffering which he described as hideous, he lapsed back into darkness.

In looking back over Euler's enormous output we may be inclined at the first glance to believe that any gifted man could have done a large part of it almost as easily as Euler. But an inspection of mathematics as it exists today soon disabuses us. For the present state of mathematics with its jungles of theories is relatively no more complicated, when we consider the power of the methods now at our disposal, than what Euler faced. Mathematics is ripe for a second

Euler's great step forward when he made mechanics analytical has already been remarked; every student of rigid dynamics is familiar with Euler's analysis of rotations, to cite but one detail of this advance. Analytical mechanics is a branch of pure mathematics, so that Euler was not tempted here, as in some of his other flights toward the practical, to fly off on the first tangent he saw leading into the infinite blue of pure calculation. The severest criticism which Euler's contemporaries made of his work was his uncontrollable impulse to calculate merely for the sake of the beautiful analysis. He may occasionally have lacked a sufficient understanding of the physical situations he attempted to reduce to calculation without seeing what they were all about. Nevertheless, the fundamental equations of fluid motion, in use today in hydrodynamics, are Euler's. He could be practical enough when it was worth his trouble.

One peculiarity of Euler's analysis must be mentioned in passing, as it was largely responsible for one of the main currents of mathematics in the nineteenth century. This was his recognition that unless an infinite series is convergent it is unsafe to use. For example, by long division we find

$$\frac{1}{x-1} = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \ldots,$$

the series continuing indefinitely. In this put $x = \frac{1}{2}$. Then

$$-2 = 2 + 2^2 + 2^3 + 2^4 + \ldots,$$

$$= 2 + 4 + 8 + 16 + \ldots.$$

The study of convergence (to be discussed in the chapter on Gauss) shows us how to avoid absurdities like this. (See also the chapter on
The curious thing is that although Euler recognized the necessity for caution in dealing with infinite processes, he failed to observe it in much of his own work. His faith in analysis was so great that he would sometimes seek a preposterous “explanation” to make a patent absurdity respectable.

But when all this is said, we must add that few have equalled or approached Euler in the mass of sound and novel work of the first importance which he put out. Those who love arithmetic—not a very “important” subject, possibly—will vote Euler a palm in Diophantine analysis of the same size and freshness as those worn by Fermat and Diophantus himself. Euler was the first and possibly the greatest of the mathematical universalists.

Nor was he merely a narrow mathematician: in literature and all of the sciences, including the biologic, he was at least well read. But even while he was enjoying his Aeneid Euler could not help seeing a problem for his mathematical genius to attack. The line “The anchor drops, the rushing keel is stay’d” set him to working out the ship’s motion under such circumstances. His omnivorous curiosity even swallowed astrology for a time, but he showed that he had not digested it by politely declining to cast the horoscope of Prince Ivan when ordered to do so in 1740, pointing out that horoscopes belonged in the province of the court astronomer. The poor astronomer had to do it.

One work of the Berlin period revealed Euler as a graceful (if somewhat too pious) writer, the celebrated Letters to a German Princess, composed to give lessons in mechanics, physical optics, astronomy, sound, etc., to Frederick’s niece, the Princess of Anhalt-Dessau. The famous letters became immensely popular and circulated in book form in seven languages. Public interest in science is not the recent development we are sometimes inclined to imagine it is.

Euler remained virile and powerful of mind to the very second of his death, which occurred in his seventy seventh year, on September 18, 1783. After having amused himself one afternoon calculating the laws of ascent of balloons—on his slate, as usual—he dined with Lexell and his family. “Herschel’s Planet” (Uranus) was a recent discovery; Euler outlined the calculation of its orbit. A little later he asked that his grandson be brought in. While playing with the child and drinking tea he suffered a stroke. The pipe dropped from his hand, and with the words “I die,” “Euler ceased to live and calculate.”

*The quotation is from Condorcet’s Eloge.