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SOME FACTS ABOUT KURT GÖDEL

HAO WANG

The text of this article was done together with Gödel in 1976 to 1977 and was approved by him at that time. The footnotes and section headings have been added much later.

§1. Education and doctoral dissertation. Gödel was born on April 28, 1906 at Brno (or Brünn in German), Czechoslovakia (at that time part of the Austro-Hungarian Monarchy). After completing secondary school there, he went, in 1924, to Vienna to study physics at the University. His interest in precision led him from physics to mathematics and to mathematical logic. He enjoyed much the lectures by Furtwangler on number theory and developed an interest in this subject which was, for example, relevant to his application of the Chinese remainder theorem in expressing primitive recursive functions in terms of addition and multiplication. In 1926 he transferred to mathematics and coincidentally became a member of the M. Schlick circle. However, he has never been a positivist, but accepted only some of their theses even at that time.¹ Later on, he moved further and further away from them. He completed his formal studies at the University before the summer of 1929. He also attended during this period philosophical lectures by Heinrich Gomperz whose father was famous in Greek philosophy.

At about this time he read the first edition of Hilbert-Ackermann (1928) in which the completeness of the (restricted) predicate calculus was formulated and posed as an open problem. Gödel settled this problem and wrote up the result as his doctoral dissertation which was finished and approved in the autumn of 1929. The degree was granted on February 6, 1930. A somewhat revised version of the dissertation was published in 1930 in the Monatshefte. It contains an acknowledgement that ‘I am indebted to Professor H. Hahn for several valuable suggestions that

¹On November 4, 1976, Gödel mentioned some details on this point. At that time (1926 and soon after), he agreed with them that the existing state of philosophy was poor (not necessary but just a historical accident), and, in substance, agreed only with their method of analysis of philosophical and scientific concepts (using mathematical logic). He disagreed with their negation of objective reality and their thesis that metaphysical problems are meaningless. In discussions with them, Gödel took the nonpositivistic position.

In one of their publications before 1930 (a one time publication by H. Hahn and O. Neurath devoted to the Wiener Kreis), Gödel was listed as a member. Of Carnap’s Logische Aufbau, Gödel said that it treats only the relation between sense perception and physical objects logically and leaves out psychological aspects. I do not know whether he meant to contrast this with Husserl’s writings of which he thought well certainly in the 1970s.
were of help to me in the execution (Durchf"uhrung) of this paper'. Godel tells me that ‘in the execution’ should be replaced by ‘regarding the formulation for the publication’.

\section{The incompleteness theorems}

In the summer of 1930, G"odel began to study the problem of proving the consistency of analysis. He found it mysterious that Hilbert wanted to prove directly the consistency of analysis by finitist method. He believes generally that one should divide the difficulties so that each part can be overcome more easily. In this particular case, his idea was to prove the consistency of number theory by finitist number theory, and prove the consistency of analysis by number theory, where one can assume the truth of number theory, not only the consistency. The problem he set for himself at that time was the relative consistency of analysis to number theory; this problem is independent of the somewhat indefinite concept of finitist number theory.

He represented real numbers by formulas (or sentences) of number theory and found he had to use the concept of truth for sentences in number theory in order to verify the comprehension axiom for analysis. He quickly ran into the paradoxes (in particular, the Liar and Richard’s) connected with truth and definability. He realized that truth in number theory cannot be defined in number theory and therefore his plan of proving the relative consistency of analysis did not work. He went on to draw the conclusion that in suitably strong systems such as that of Principia mathematica (type theory) and that of set theory (‘Zermelo-Fraenkel’), there are undecidable propositions. (See §7 of the notes on G"odel’s 1934 lectures, The Undecidable (M. Davis, Editor), 1965, pp. 63–65).

At this time, G"odel represented symbols by natural numbers, sentences by sequences of numbers, and proofs by sequences of sequences of numbers. All these notions and also the substitution function are easily expressible even in small finitary subsystems of type theory or set theory. Hence there are undecidable propositions in every system containing such a system. The undecidable propositions are finitary combinatorial in nature.

In September 1930, G"odel attended a meeting at K"onigsberg (reported in the second volume of Erkenntnis) and announced his result. R. Carnap, A. Heyting, and J. von Neumann were at the meeting. Von Neumann was very enthusiastic about the result and had a private discussion with G"odel. In this discussion, von Neumann asked whether number-theoretical undecidable propositions could also be constructed in view of the fact that the combinatorial objects can be mapped onto the integers and expressed the belief that it could be done. In reply, G"odel said, ‘Of

\footnotetext[2]{G"odel had completed his dissertation before showing it to Hahn. This was longer than the published paper and accepted for the degree in its original form. It was in preparing the shorter version for publication that G"odel made use of Hahn’s suggestions.}

\footnotetext[3]{Or ‘propositional functions’ according to a prominent tradition at that time.}

\footnotetext[4]{At the same meeting F. Waismann also gave a lecture entitled The nature of mathematics: Wittgenstein’s standpoint, but the text has not been published (see Erkenntnis, vol. 2, 91 ff. where the other three lectures of the session by Carnap, Heyting, and von Neumann are printed). Compare also Wittgenstein and the Vienna Circle (B. McGuiness, Editor), 1979, pp. 19–21.}

Hilbert gave his address Logic and nature at the general session. G"odel went to hear it, and this was the only time when G"odel saw Hilbert.
course undecidable propositions about integers could be so constructed, but they would contain concepts quite different from those occurring in number theory like addition and multiplication". Shortly afterward Gödel, to his own astonishment, succeeded in turning the undecidable proposition into a polynomial form preceded by quantifiers (over natural numbers). At the same time but independently of this result, Gödel also discovered his second theorem to the effect that no consistency proof of a reasonably rich system can be formalized in the system itself.

An abstract stating these results was presented on October 23, 1930 to the Vienna Academy of Sciences by Hans Hahn. Shortly afterwards Gödel received a letter from von Neumann suggesting the theorem on consistency proofs as a consequence of Gödel's original result. The full celebrated paper was received for publication by the Monatshefte on November 17, 1930 and published early in 1931. In a note dated January 22, 1931 (K. Menger, Kolloquium, vol. 3, pp. 12–13), Gödel gave a more general presentation of his theorems using Peano arithmetic rather than type theory as the basic system. The major paper was also Gödel's Habilitationsschrift.

§3. Positions held, health, and personal contacts. Gödel served as Privatdozent at the University of Vienna for the years 1933–1938. He visited the Institute for Advanced Study in 1933–1934, lecturing on his incompleteness results in the spring of 1934 (compare The undecidable, op. cit. for the notes from these lectures). (Hans Hahn died in 1934.) He visited the Institute again in the autumn of 1935. During 1936 he was sick and very weak.

From autumn of 1938 until spring of 1947, he received annual appointments at the Institute and stayed there all the time except for the spring term of 1939 when he lectured at Rotterdam and the autumn term of 1939 when he was back in Vienna.5 He became a permanent member of the Institute in 1947 and a professor in 1953. He received the Einstein Award in 1951 and the science medal in 1975. He has received honorary degrees from Yale (1951), Harvard (1952), Amherst (1967), and Rockefeller (1972). He is a Fellow of the American Academy of Arts and Sciences, American Philosophical Society, and the National Academy of Sciences. He is a Foreign Fellow of the British Academy and the Royal Society.

During 1930–1933, Gödel continued his study of logic and mathematics (including the foundation of geometry and the beautiful subject of functions of complex variables). He read the proof sheets of the book by Hahn on real functions and learned the subject. He attended Hahn's seminar on set theory and took part in Menger’s colloquium. A number of Gödel's results from this period were reported in the proceedings of this colloquium. He also wrote a fairly large number of reviews for the Zentralblatt.

Gödel visited Göttingen in 1932 and saw Siegel, Gentzen, Noether, and probably also Zermelo there. He never met Herbrand but only corresponded with him. His second letter did not reach Herbrand who had died in the meantime. In the earlier years at Princeton, he met with Church and Kleene more often than with Rosser.

Gödel’s health is generally poor and for certain periods has prevented him from

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5Gödel married Adele Pockert on September 20, 1938.
serious work. He had rheumatic fever when he was eight or nine years old and this has had a bad effect on his heart. He has had serious trouble with his digestion during almost all his adult years. Since 1947 he has had an infection of the kidneys which makes him very sensitive to colds and he does not recover from colds without using antibiotics. In particular, his health was exceptionally poor in 1936, in 1961 and in 1970.

§4. Constructible sets. It must have been in 1930 when Gödel first heard about Hilbert’s proposed outline of a proof of the continuum hypothesis in axiomatic set theory (say ZF). This was the time when Gödel began to think about the continuum problem. He felt that one should not build up the hierarchy in a constructive way and it is not necessary to do so for a proof of consistency of CH, and, therefore, one does not have to construct the ordinals except for a prejudice against the objectivist point of view. The ramified hierarchy came to Gödel’s mind. Gödel observes that Hilbert did not believe CH to be decidable in ZF, since Hilbert adds a tacit axiom stating that every set can be defined and speaks of his proposed proof as a great triumph of his proof theory. Moreover, according to Hilbert’s claim, he did much more than proving CH, he also proved the consistency of ZF on the way. To prove so much, one must expect a very difficult proof.

It must have been in 1935 when Gödel first came up with his consistency proof of the axiom of choice, with a ramified hierarchy. He was then sick in 1936. It was either in the autumn of 1937 or in the spring of 1938 when Gödel lectured on this result in Vienna; the series of lectures was entitled Axiomatik der Mengenlehre. In the summer of 1938, Gödel extended his result to the extent familiar today. He introduced the axiom of constructibility and proved the relative consistency of GCH. An announcement of these and related results was submitted to Proceedings of the National Academy of Sciences USA and published before the end of 1938. At the same time, viz. in the autumn term of 1938–1939, Gödel lectured in detail on the main proofs at the Institute, resulting in the monograph that appeared in 1940. A rather detailed and more intuitive outline was communicated to Proceedings of the National Academy of Sciences USA on February 14, 1939 and published shortly afterwards.

Professor Wang Sian-jun of Beijing University wrote to me on February 28, 1978:

Soon after I arrived at Vienna in February 1937, I went to Gödel’s home to see him. During the three terms when I was in Vienna (spring and autumn of 1937, spring of 1938), Gödel gave only one course entitled Axiomatik der Mengenlehre, probably in the autumn of 1937. It was not a seminar. I remember constructible sets being considered in the lectures. Not many people took the course, probably only five or six.

Professor Wang also remembers that Gödel recommended to him Herbrand’s Thèse, Fraenkel’s and Hausdorff’s Mengenlehre, as well as Hilbert and Ackermann’s textbook.

Probably in 1937, Gödel told Professor Wang that after the incompleteness theorems more work in the general area of mathematical logic would not make much difference: “Jetzt, Mengenlehre”. For an accidental reason, Gödel elaborated the history of these results in May and June of 1977 and I was asked to write up his observations. The result follows:

In this connection his main achievement really is that he first introduced the concept of constructible sets into set theory defining it as in his Proceedings paper of 1939, proved that the axioms of set theory (including the axiom of choice) hold for it, and conjectured that the continuum
§5. Work in logic in 1940–1943. Gödel’s health was relatively good during this period. He worked mostly in logic. In 1941 he obtained a general consistency proof of the axiom of choice by metamathematical considerations. Gödel has agreed recently to reconstruct this proof when his health is better. Gödel believes it highly likely that the proof goes through even when large cardinals are present. After so much development in set theory, it will indeed be a remarkable event to see a result obtained thirty-five years ago open up large new horizons.

Gödel’s famous interpretation of intuitionistic number theory by primitive recursive functionals was obtained in 1942. Shortly afterwards he lectured on these results at Princeton and Yale. E. Artin was present at the Yale lecture. The result was published in German in 1958 on the occasion of Bernays’ seventieth birthday. An English translation was completed before 1970 by Gödel himself with a note explaining why the proof is not circular but uses something more evident to interpret intuitionist number theory. The paper was already in proofs when Gödel fell ill. He is now willing to have it published.

It was in 1943 when Gödel arrived at a proof of the independence of the axiom of choice in the framework of (finite) type theory. The idea of the proof makes it clear why the proof works. For that reason alone, it would be of interest to reconstruct the proof. It uses intensional considerations. The interpretation of the logical connectives is changed. A special topology has to be chosen.

The method looked promising toward getting also the independence of CH. But Gödel developed a distaste for the work and did not enjoy continuing it. In the first place, it seemed at that time he could do everything in twenty different ways and it was not visible which was better. In the second place, he was at that time more interested in philosophy. Looking back with the results of Paul J. Cohen (1963) it becomes clear that the method can also establish the independence of the continuum hypothesis. He now regrets that he did not continue the work. If he had continued with it, he would probably have gotten the independence of CH by 1950, and the development of set theory would have progressed faster.

hypothesis also will hold. He told these things to von Neumann during his stay at Princeton in the autumn of 1935. The discovery of the proof of this conjecture on the basis of his definition is not too difficult. Gödel gave the proof (also for the GCH) not until three years later because he had fallen ill in the meantime. This proof was using a submodel of the constructible sets in the lowest case countable, similar to the one commonly given today.

On June 9, 1976, Gödel recalled that in 1941 or 1942 he wrote his paper on Russell’s mathematical logic. He described the paper as a history of logic with special reference to the work of Russell.

Judging from Russell’s reply to his critics (dated July 1943), it seems likely that Gödel wrote the paper mainly in 1942 to 1943. The note says in part: ‘Dr. Gödel’s most interesting paper on my mathematical logic came into my hands after my replies had been completed, and at a time when I had no leisure to work on it. . . . His great ability, as shown by his previous work, makes me think it highly probable that many of his criticisms of me are justified’.

Gödel’s paper concludes with some optimistic claims by Leibniz on the future possibilities of mathematical logic. Gödel told me that during the war he was interested in Leibniz but could not get hold of the manuscripts of Leibniz. When these manuscripts finally came in after the war, his interest had shifted to other directions.

Apparently Gödel continued to work on trying to apply his method to the continuum hypothesis for quite some time. It seems likely that his philosophical paper on the continuum problem
He was interested in Leibniz, particularly the universal characteristic, and in the relation between Kant’s philosophy and relativity theory.\textsuperscript{10}

\textbf{§6. Philosophy.} Einstein used to reside near Gödel and for many years they went home together almost every day. But Gödel’s interest in relativity theory came from his interest in Kant’s philosophy of space and time rather than his talks with Einstein.

Gödel worked on GTR during 1947 to 1950 or 1951. He then spent one year working on his Gibbs Lecture.\textsuperscript{11}

It caused Gödel a good deal of trouble to have agreed to write a paper on Carnap to prove that mathematics is not syntax. He went through many versions during 1953 to 1955 or 1956. Finally he did not publish the paper, because he thinks that a more convincing refutation could be found.

Gödel started to read Husserl in 1959.

In philosophy Gödel has never arrived at what he looked for: to arrive at a new view of the world, its basic constituents and the rules of their composition.

\hspace{1em} (published toward the end of 1947) was meant in part as a sort of conclusion of his own mathematical work on this problem. It could be interpreted as a summary of his thoughts on this problem and an invitation to others to continue where he left off. At any rate, looking back now we may conjecture that between 1943 and 1947 a transition occurred from Gödel’s concentration on mathematical logic to other theoretical interests which are primarily philosophical.

In this connection we may also mention Gödel’s 1946 lecture Remarks before the Princeton Bicentennial Conference on problems in mathematics (published in The undecidable (M. Davis, Editor), 1965, pp. 84–88). Gödel, encouraged by the successful analysis of the invariant concept of computability, suggests looking for invariant notions of definability and provability. From both this lecture and the 1947 paper one gets the clear impression that Gödel was interested only in really basic advances.

Undoubtedly the transition was gradual. The distaste for the work on the independence of CH presumably began in 1944 and grew stronger in the next year or two. The Princeton lecture and the philosophical article on the continuum problem served as an interlude. By Gödel’s own account, he began working on relativity theory in 1947.

\textsuperscript{10}In fact, Gödel wrote an article (in typescript of 28 pages) entitled: Some observations about the relationship between theory of relativity and Kantian philosophy. Probably this came from this period. It seems likely the paper was written in 1947 or before. The paper published in the volume honoring Einstein was completed before February 1949 but says much less about Kantian philosophy.

\textsuperscript{11}Since the lecture was given at the end of 1951, this probably means that Gödel essentially spent the year 1951 preparing for it. I have seen a handwritten manuscript of the paper. I attended the lecture and have preserved a printed announcement. The title is Some basic theorems on the foundations of mathematics and their philosophical implications. It was given at Alumnae Hall, Pembroke College in Brown University, December 26, 1951, 8 p.m. I remember Gödel read from a manuscript at high speed, including quotations in French from Hermite. Gödel told me of some of the ideas of the lecture which are reported in my From mathematics to philosophy, 1974.

This book of mine also contains other contributions by Gödel. Two long letters from him are included which explain how his philosophical view helped him in making his major discoveries in logic. His mature thoughts on objectivism, new axioms of set theory, and the contrast between mind and machine are reported in formulations approved by him.

From 1971 to 1977 I had frequent conversations with Gödel and wrote up some parts of them which were then discussed with him. It is my present intention to try to work over these and other notes from the conversations in order that some of his unpublished views will be known more broadly.
Several philosophers, in particular Plato and Descartes, claim to have had at certain moments in their lives an intuitive view of this kind totally different from the everyday view of the world.\textsuperscript{12}

\textsuperscript{12}Gödel thought that Husserl probably had a similar experience of revelation some time from 1909 to 1910.

In June 1976 Gödel said that his work at the Institute had been split in three ways: institute work, mathematics, and philosophy. He was very conscientious about his work at the Institute especially with regard to the evaluation of applicants. Hassler Whitney reported on this in his lecture on March 3, 1978.

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