THE PERUVIAN QUIPU
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MAN, having curiosity as one of his basic characteristics, has long desired to unlock the secrets of those who have gone before him in time. Thus, he has embarked on archeological expeditions, has studied history books, has consulted ancient sources of information, and has generally set about looking for the answers to his questions.

Mathematicians, possessing that same curiosity, have been no different from their fellow beings. They, too, have searched and traveled, and from their efforts have emerged hundreds of books written on the historical foundations of modern mathematics. Especially has the greatest effort been concentrated in the eastern hemisphere (i.e., Europe, Africa, and Asia). The logical question, then, concerns the apparent lack of books containing information as to the nature of the mathematics of the Americas—that is, of the original inhabitants of North and South America. The answer is twofold. First, there really was a lack of specialized mathematics in these geographical areas; and secondly, many of the secrets of ancient American mathematics have been, and are being, carefully guarded.

Such is apparently the situation when one inquires into the nature of the mathematical system of the Incas of Peru. Many allusions to the subject have been made by a great number of the Spanish chroniclers whose works are still preserved today. Yet we find that no distinct literary work has been completely dedicated to the subjects of ancient Peruvian mathematics or the place of the Incan quipu (a mnemonic device used to keep record of computations) within that system. All that the inquirer finds are generalities, which do not pinpoint specific information.

However, using some of the hints given to us by the aforementioned chroniclers, research studies have been carried out by several men in an effort to discover what mathematical knowledge was known to the Incas and what importance the Peruvian quipu enjoyed in that mathematics. Two such scientists are Leland Locke and Erland Nordenskiöld.

The remainder of this discussion will be centered around the results of these studies. The reader, however, should note that many of the conclusions reached by these men (especially by Nordenskiöld) are nothing more than beliefs. There is no definite way to prove the truth of them, even though they are based on long and rigorous studies.

Before discussing the intricacies of the Incan mathematical system, it is necessary to note that, in comparison with other cultural achievements of the civilization, the field of mathematics was relatively barren. For example, the Incas lacked the complex computational system of the Mayans of Mexico; being somewhat like the ancient Babylonians, they were primarily interested in keeping records, rather than developing elaborate methods of computation.

Thus, we arrive at the subject of our discussion: the Peruvian quipu. The quipu was the device used by the Incas to record
results of various kinds of mathematical problems. We have proof that quipus were used not only in Peru but also in other areas of South America.\(^1\) Most of those that we now use for study were found in dry graves along the coastal areas of Peru. This is especially true of those used by Mr. Locke and Mr. Nordenskiöld.

The Incas were not especially adroit in advanced mathematical computation. They were much more interested in recording the results of the yearly census and in keeping an account of the number of sheep they had in a herd. Their mathematical achievements were not great, but their method of keeping these statistics, which they apparently valued so highly, was ingenious and still intrigues many twentieth-century scholars.

But exactly how did they compute the results they recorded on the quipus? An abacus was used, but not of the same type as came into such widespread use, during the time of the Roman Empire, in the Mediterranean region of the world. Instead of the wires and beads of the more conventional one, the Incan abacus consisted of a rectangular slab of stone into which were cut a number of rectangular and square compartments so that a free octagonal space was left in the middle, and two opposite corner rectangles were raised. Another two sections were mounted on the originally raised portion such that there were now three levels represented. An overhead view would thus be depicted by Figure 1\(^2\) with the darkened areas denoting raised portions.

Pebbles were used to keep the accounts, and their positions within the various rectangles and squares that composed the instrument yielded the total. For example, a pebble in a small square was one unit; when it was put into a rectangular one, its value was doubled; and when it was put into the central octagonal area, its value was tripled. The value of the pebble would be multiplied by six if put on the first of the upper levels; if it were placed on the uppermost level, its value would be multiplied by twelve. The color of the pebbles used indicated the nature of the objects being counted. Since this method of computation was efficient in arriving at results but did not yield a permanent record, the Incas (who most likely borrowed this abacus from the Canaris of Ecuador) felt a need for another instrument, which could be used not as a calculating device but as a numerical record keeper.\(^3\)

The quipu fulfilled this need. A first look at a quipu might yield the impression that it is nothing more than a knotted cord with no special significance, but this is not the case. It is true that a quipu is a collection of cords in which knots have been tied, but these are definitely and systematically arranged to have special significance to whoever embarks on the task of deciphering it.

The quipu consists of a main cord to which branches (other cords) are attached like a fringe. These branches have been named H cords by Locke. They are fastened to the main cord in groups, and many times these groups contain equal numbers of H cords. To the H cords are attached still other cords, these being called B cords. Any additional ones are


\[^2\] Ibid., p. 125.

\[^3\] Ibid., pp. 124–25.
labeled B cords of the second order (see Fig. 2).

**Fig. 2.—Quipu**

The knots themselves are seldom present on the main cord. Instead, they are concentrated on the H and B cords. As to the nature of the knots, it has been noted by Mr. Locke that there are three main types of knots. These represent different values, depending on the particular knot used in a particular position on the cord in question. The Incas used a decimal arithmetical system much like ours of today. It is interesting to note that, like ours, their system was based on ten and was a place system. Thus, on the quipu, each type of knot that was used had a specific decimal value. There were so-called overhand knots, simple single knots in the cord. These represented tens, hundreds, thousands, and ten thousands. The Incas rarely used numbers greater than ten thousand. Then there were “Flemish knots,” S-shaped configurations used to denote “one.” All other integers were depicted by “long knots” in which the specific value could be determined by counting the number of times the cord was wound before tying the knots. “Long knots” were also used at times to represent tens and hundreds. In such cases the position of the knots on the cord would denote the value. The units were placed nearest the bottom, the tens being placed immediately above them, then the hundreds, thousands, and ten thousands.

Let us illustrate exactly how the Incas used the quipu by the following example. A native desires to keep an account of the number of animals in his herds. He uses his abacus and arrives at the fact that he has 10 sheep, 30 llamas, 3 dogs, and 100 goats, giving him a total of 143 animals. To construct his quipu he would begin with a horizontal cord (main cord) from which he would suspend five H cords (see Fig. 3). On the first (left) cord he would place one overhand knot for the 10 sheep he has; on the second, three overhand knots would tally the number of llamas; the third would boast a long knot with three twists for the 3 dogs; and the fourth cord would have one overhand knot tied in it and placed in the appropriate spot to denote 100 goats. The fifth cord is the totalizer cord, and it would contain knots representing the total number of animals in the herd, 143.

The colors of the cords are significant and indicate that the 10 registered on the quipu stands for sheep and not llamas. Thus, each thing to be counted was assigned a color to help avoid confusion. But what was done when the Incas ran out of colors? It is evident that colors had to be reused but with different significance. As a result, the deciphering of the quipu became an intricate art, and a specially trained individual was needed to do the job. He was the Quipucamayoc (keeper of

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the quipus), and his job was the construction, preservation, and decoding of the quipu.5

The purposes of the quipu were numerous, with authorities agreeing and disagreeing vehemently concerning specific uses.

Mr. Mason asserts that the Incas used the quipu to keep an account of their history and traditions. In short, he says, "... they never developed any system of writing—pictographic, ideographic, hieroglyphic, or alphabetic."6 Their history was not written down; it was tied into the many knots that composed the quipu.

In accord with this view of the purpose of the quipu, we find an interesting reference in the Comentarios Reales written by El Inca Garcilaso de la Vega. He suggested that the quipu was indeed used as a replacement for writing because it formed a basic part of the Incan postal system. He says,

Otros recaudos llevaban, no de palabra, sino por escrito, digámoslo así, aunque hemos dicho que no tuvieron letras, las cuales eran nudos dados en diferentes hilos de diversos colores... Los nudos y los colores de los hilos significaban el número de gente, armas, o vestidos, o bastimiento, o cualquiera otra cosa que se hubiese de hacer, enviar o aprestar. A estos hilos a?udados llamaban los indios quipu. . . .

[Other messages were carried not by word but in writing. We say this although we have said that they (the Incas) had no writing. This writing was composed of knots of different colors on different cords... The knots and the colors of the cords signified the number of men, arms, dresses, supplies, or whatever that they had to make, send, or prepare. These knotted cords were called quipu...]

Another interesting use of the quipu was proposed by Herrera. He says that after the conquest of the Incas by the Spaniards and the conversion to Catholicism, there still remained a communication problem between the two groups because of the difference in language. As a result, when an Inca wanted to confess to a Spanish priest, he did so by means of a quipu on which were written all of his infringements of church law.8

It has also been suggested that one of the purposes of the quipu was the registering of astronomical and magic numbers. Erland Nordenskiöld's extensive research has resulted in the promulgation of this theory. The quipus he used for his research were solely those that came from dry coastal graves; and all were in excellent condition, the majority being perfect specimens. It should be noted before discussing the issue any further that Nordenskiöld’s opinions are not wholly accepted by modern authorities, but they do plant seeds of speculation in the reader's mind.

Nordenskiöld believed, even before consulting the quipus he used for his study, that they were not accounts of historical events or compilations of figures that corresponded to the yearly census. He based this presumption on the fact that primitive peoples usually believe that entombing anything containing information about the living is likened to entombing the living themselves. As a result, he conjectured that the numbers recorded on the quipu might be connected with magic of some sort. This conviction was further strengthened when he stumbled upon Bertoino's statement of the preoccupation of the Incas with magic and counting.

Adivinar tomando un puñado de algo y contando después los granos. Contar una almohada de papas para saber si el año será buen. Es gran superstición.9

[Foretell future events by taking a handful of something and counting the grains. Count a sack of potatoes to know if it will be a good year or not.]

Upon examination of a group of quipus, he did discover a predominance of the number 7, not as a number present on the

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6 Ibid., p. 226.

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cords themselves, but as a divisor of the totals registered on single cords and as the last digit of a number that was a divisor of the totals registered on neighboring cords. Since this number appeared so often in these divisors and since Bertonio stated that the Incas connected magic with counting, Nordenskiöld concluded that the number 7 must have been considered as possessing mystical value.

The quipus studied also were shown to contain astronomical numbers. An example is the first quipu Nordenskiöld examined. It, like the others, showed a predominance of numbers that contained 7 or numbers whose divisors ended in 7. But it also contained a total equal to 1,172, which in turn equals $2 \times 586$. Nordenskiöld reasoned that these numbers represented days and that this total was to denote the period of two synodic revolutions of Venus, which is equal to $2 \times 584$ days. The four-day discrepancy was explained away as being added so that the difference between three of our solar years ($3 \times 365$) and two synodical revolutions of Venus would be a magical number—in this case 77.

This particular example is one of the reasons why experts believe Nordenskiöld has looked for relationships and therefore forced a discovery of what he desired. His findings cannot be completely refuted, though, because other quipus he studied did show forth a prevalence of astronomical numbers.

The Spanish chroniclers themselves have stated the relationship between the quipus and the Incas’ interest in heavenly bodies.

More exactly, Molina states:

No obstante que usavan de una quenta muy subtil de unas ebras de lana de dos huidos y puesta lana de colores en los nudos los cuales llaman quipos entendanse y entiendense tanto por esta quenta que dan razón de más de quinientos años de todas las cosas que en esta tierra en este tiempo han pasado: tienen yndios yndustríados y maestros de dichis quipus y quentas y estos yban de generacion en generacion mostrando lo pasado y empapándolo en la memoria a los que avian de entrar, que por maravilla se olvida-van cosa por pequeña que fuese tenían en estos quipus que casi son a modo de parvillos con que las hiedas recan en nuestra España salvo ser ramales tentan tanta quenta en los años, meses, y luna de tal suerte que no avia lunar, luna, año, ni mes aunque no con tanta pulcia como después que Ynga Yupanqui empezo a señorear y conquistar esta tierra; porque hasta entonces los ygas no avian salido de los alrededores del Cusco. Como por la relación que Va. S. Roma. tiene, parece este ynga fue el primero que empezo a poner quenta y razón en todos los cosas y el que quito cultos y discultos y ceremonias que en cada uno dellos hacen, porque no obstante que antes que reynasen sus antecesores, tenían meses y años por sus quipus, no se reglan con tanto concierto como despues que este fue señor que se reglan por los yuvernos y yeranos.

[Notwithstanding the fact that they used in a very subtle manner some strings of wool with two knots and that they put colored wool in the knots that they called quipus, having used them and using them now in acknowledging for more than 500 years all of the things that have happened in this land, some industrious and wise Indians had some quipus that they passed from generation to generation. These contained every little thing that had happened and were such good accounts of the years, months, and moons that there was not a single moon, year or month that was not included. But after Ynga Yupanqui began to conquer the land, he began to construct accounts of everything. And even though before he became ruler, they had recorded months and years on their quipus, now they included winters and summers too.]

Armed with the above information, it is no wonder that Nordenskiöld felt certain that some of the numbers registered on the quipus he studied must have been astronomical—especially when he did encounter them himself, too.

One quipu studied by Nordenskiöld was of an odd circular shape. It yielded numbers having to do with the types of months the Incas used. He notes that the sum of the numbers depicted on this specimen equals 3,481 or $118 \times 29.5$. Also, the total of the numbers on all the cords of one color results in a number divisible by 30. The total on the remaining colors is divisible by 29.5. Over and over again, examination of various other specimens yields the same results: years of 365 days
divided into months of either 29.5 or 30 days each.\textsuperscript{12}

Therefore, based on the previous discussion, Nordenskiöld concluded that the quipus used by the Incas and examined by him contained: (1) the number 7 combined in various forms and often represented, (2) numbers that expressed days, and (3) astronomical numbers.

In conclusion, one should fully be aware of the fact that the secrets of the Peruvian quipus have not yet been completely unlocked. This is clearly demonstrated when we note that Nordenskiöld's study occurred in 1925, and little progress has been made since then. In summation, we do know that the Incas counted in groups of ten, used a decimal placement system, and figured on the abacus. The results of

\textsuperscript{12} Nordenskiöld, Calculations with Years and Months in the Peruvian Quipus (Sweden: Comparative Ethnographical Studies, 1925), II, Part 2, 5–34.

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**A TEXT ON TRIGONOMETRY**

**BY LEVI BEN GERSON (1288–1344)**

By Pamela H. Espenshade, Princeton Day School, Princeton, New Jersey

LEVÍ ben Gerson (1288–1344), a prominent Hebrew writer who lived in Provence, is most noted for his religious and philosophical writings. Nonetheless, his astronomical works, although largely unstudied, are quite imposing and voluminous. His chief astronomical work, contained in Wars of the Lord, Book V, Part 1 (136 chapters), was translated into Latin by the Augustinian hermit Peter of Alexandria in 1342 and dedicated to Pope Clement VI. Since, however, Clement VI was elected May 7, 1342, it is probable that the work was undertaken for Benedict XIII (1334–1342).\textsuperscript{1} The Latin version of the table of contents was published by Renan in 1843.\textsuperscript{2}

A section of these astronomical writings was circulated separately under the title of De Sinibus, Chordis et Arcibus, item

\textsuperscript{1} L. Thorndike, A History of Magic and Experimental Science, pp. 309–11.