ASTRO504
Extragalactic Astronomy

3. Luminosity function
Fig. 3.50 *Left panel:* galaxy luminosity function as obtained from 13 clusters of galaxies. For the **solid circles**, cD galaxies have also been included. **Right panel:** a schematic plot of the Schechter function.

Source (*left panel*): P. Schechter 1976, *An analytic expression for the luminosity function for galaxies*, ApJ 203, 297, p. 300, Fig. 2. ©AAS. Reproduced with permission
Fig. 3.51 The luminosity function for different Hubble types of field galaxies (top) and galaxies in the Virgo cluster of galaxies (bottom). Dashed curves denote extrapolations. In contrast to Fig. 3.50, the more luminous galaxies are plotted towards the left. The Schechter luminosity function of the total galaxy distribution is the sum of the luminosity functions of individual galaxy types which can deviate significantly from the Schechter function. One can see that in clusters the major contribution at faint magnitudes comes from the dwarf ellipticals (dEs), and that at the bright end ellipticals and S0s contribute much more strongly to the luminosity function than they do in the field. This trend is even more prominent in regular clusters of galaxies. Source: B. Binggeli et al. 1988, The luminosity function of galaxies, ARA&A 26, 509, Fig. 1, p. 542. Reprinted, with permission, from the Annual Review of Astronomy & Astrophysics, Volume 26 ©1988 by Annual Reviews www.annualreviews.org
Fig. 3.52  Left panel: The luminosity function of galaxies, i.e., the number density of galaxies as a function of absolute r-band magnitude. The total luminosity function is shown as the grey histogram, with the smooth curve being a fit with a double-Schechter function (3.59). Also shown are the luminosity function of early-type galaxies, split according to the Sérsic index $n$ into concentrated and less concentrated ones (red and orange histograms, respectively), and late-type galaxies shown in blue. The early-types with $n \leq 2$ are totally subdominant for all $L$, and contribute substantially to the early-type population only for very low luminosities, in agreement with what is seen in Fig. 3.39. Right panel: The stellar mass function of galaxies, with the same galaxy populations as in the left-hand panel. The total mass function is again fit with a double-Schechter function. Source: M.R. Blanton & J. Moustakas 2009, Physical Properties and Environments of Nearby Galaxies, ARA&A 47, 159, p. 166, Fig. 3. Reprinted, with permission, from the Annual Review of Astronomy & Astrophysics, Volume 47 ©2009 by Annual Reviews www.annualreviews.org
Eddington (1913)

On a Formula for Correcting Statistics, etc. 359


In astronomical investigations it often happens that we construct a table to exhibit the number of stars having successive values of a certain measured property, e.g., the number of stars of successive magnitudes, or the number between successive limits of proper motion. In general the observations are subject to a probable error, which is at least approximately known. The error is in general small, otherwise our table would not be of much value, but it must have some effect on the numbers of the table. It is not, however, usual to take account of the probable error, probably because it is not generally realised that it can be eliminated and an improved table formed in a very simple way.

To avoid the awkwardness of general terms, suppose we are concerned with counts of stars between given limits of magnitude, and, knowing the average accidental error of our determinations of magnitude, we wish to apply corrections to our counts to eliminate these errors.

Let \( w(n) \) be the observed number of stars between magnitudes \( n \) and \( n + dnm \).

Let the probable error of the observed magnitudes be \( 0.177/k \), so that the frequency of an error \( x \) is proportional to \( e^{-x^{2}} \).

We have
\[
\frac{w(n)}{\sqrt{2\pi}e^{-x^{2}}dx}
\]
for, of the stars having a true magnitude \( m + x \), the proportion
\[
\frac{m}{\sqrt{2\pi}e^{-x^{2}}}
\]
will have an error of measurement \( -x \), and will therefore be observed as of magnitude \( m \).

By the symbolic form of Taylor's theorem
\[
\frac{e^{x}}{\sqrt{2\pi}e^{-x^{2}}dx} = \frac{1}{\sqrt{2\pi}e^{-x^{2}}} \cdot \nu(m),
\[
\therefore
w(n) = \sqrt{\frac{\pi}{2}} \int_{n}^{\infty} e^{-x^{2}/2} \cdot \nu(m) dx.
\]

The integral is of the well-known form
\[
\int_{-\infty}^{\infty} e^{-x^{2}/2} dx = \sqrt{\pi}
\]
and
\[
\therefore
w(n) = \sqrt{\frac{\pi}{2}} \cdot \frac{1}{\sqrt{2\pi}e^{-x^{2}}dx} = \frac{1}{\sqrt{2\pi}e^{-x^{2}}} \cdot \nu(m);
\]

therefore
\[
\nu(m) = \exp \left( -\frac{1}{2\pi} \frac{1}{\sqrt{2\pi}e^{-x^{2}}dx} \right) \cdot \nu(n) = \nu(m) - \frac{1}{\sqrt{2\pi}e^{-x^{2}}} \cdot \nu^{2}(m).
\]

The result so far is accurate.

For a small accidental error, confining ourselves to the first two terms, the result can be given in a form convenient for computation as follows:

The tabular second difference for intervals \( n \)
\[
\frac{w(n + a) + w(n - a) - 2w(n) - w^{2}(n)}{2a}
\]
approximately.

Also
\[
\frac{1}{2a} \approx 1.246 \times \text{probable error}
\]

Thus the approximate correction is
\[
\left( 1.246 \times \text{probable error}^{2} \right) \times \text{tabular second difference}
\]

Note on the Convergence of the Series.—The Astronomer Royal has pointed out to me that the series in some typical cases is divergent, e.g., \( \nu(m) - (1 + m^{2})^{-1} \). The operator \( \frac{d^{2}}{dx^{2}} \) introduces a factor of order \( 2m^{4} \), whilst the divisor is only \( m \). Apparently, however, in these cases the expansion is asymptotic; and it seems certain that the first few terms give the approximate correction quite correctly. The divergence arises from using the Taylor expansion beyond its range of convergence.

The difficulty does not really arise in practice. In a table with a finite number of entries, we are actually dealing with a polynomial, in which case all the series terminate, and no question of divergence arises. Thus, if \( \nu \) is the tabular interval, the tabular values of \( \nu(m) \) may be represented from \( m - m_{a} \) to \( m + m_{a} \) by a polynomial of the \( m_{a}^{4} \) degree, say \( \nu_{a}(m) \). By taking \( m_{a} \) sufficiently great the whole range of \( \nu \), which contributes appreciably to \( u(m) \), can evidently be included. Beyond the limits \( m \pm m_{a} \), the divergence between \( \nu_{a}(m) \) and \( \nu(m) \) will generally increase rapidly; it can be shown, however, that
\[
\int_{m_{a}}^{m_{b}} e^{-x^{2}/2} dx
\]

tends to zero as \( m_{a} \) is increased, and hence the part of \( \nu_{a}(m) \) beyond the limits \( m \pm m_{a} \) will (when \( m_{a} \) is great) make no contribution to \( u(m) \).

Thus the polynomial and the true function agree for the \( 2m_{a} + 1 \) tabular entries \( m - m_{a} \) to \( m + m_{a} \), and beyond these limits they both make contributions to \( u(m) \), which tend to zero. Hence the polynomial can be used for our purpose, and a terminating series results.

Substituting the polynomial only requires that we should deduce the differential coefficients from the tabular differences—a procedure which we should naturally adopt in any case.