Matching Demand and Supply to Maximize Profits from Remanufacturing

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The profitability of remanufacturing depends on the quantity and quality of product returns and on the demand for remanufactured products. The quantity and quality of product returns can be influenced by varying quality-dependent acquisition prices, i.e., by using product acquisition management. Demand can be influenced by varying the selling price. We develop a simple framework for determining the optimal prices and the corresponding profitability. We motivate and illustrate our framework using an application from the cellular telephone industry.

(Remanufacturing; Product Acquisition; Econometric Models)

1. The Economics of Product Recovery

In recoverable product environments, products are reused rather than being discarded. Product recovery options include value-added recovery (remanufacturing), material recovery (recycling), and energy recovery (incineration). Product recovery reduces the requirements for virgin materials, energy consumption, and landfill space. Perhaps most importantly, from a business perspective, these systems can significantly contribute to the overall profitability of the firm. Lund (1998) reports that there are over 70,000 remanufacturing firms in the United States with total sales of $53 billion. These firms directly employ 350,000 workers and average profit margins exceed 20% (Nasr et al. 1998).

Guide and Van Wassenhove (2001) remark that, surprisingly, there is no literature on the economic analysis of the potential profitability of product recovery. Figure 1, taken from Guide and Van Wassenhove (2001), shows the relationships between the key activities in managing product returns from a business perspective. Product acquisition is one of the few areas that management can proactively influence and, as a result, determine whether reuse activities will be economically attractive. This integrated framework forces managers to focus on the global economic benefit and the fundamental role of the product acquisition process.

Indeed, no matter what type of product recovery option is practiced, the firm must obtain used products to serve as inputs to the recovery system. Hence, a crucial aspect of our framework is the ability of a remanufacturing firm to influence quality, quantity, and timing of product returns by offering a quality-dependent price incentive for used products. Empirical evidence shows that a number of remanufacturing firms in the United States have adopted such a market-driven product-acquisition management approach (Guide 2000).

European firms, on the other hand, seem to rely on the waste stream for acquiring recoverable products. Firms using this approach passively accept all product returns. They are not involved in product
recovery for economic reasons, but because of environmental legislation. Many of the firms operating under a waste stream approach consider their product recovery system to be a cost center rather than a profit center. Returned products in the waste stream tend to be old and have a poor quality, and as a consequence, the recovery options for these products are often limited. As an example, consider the imminent European Union legislation requiring tire manufacturers to arrange for the environmentally friendly disposition of one used tire for every new tire sold. Presently garages are given large bins to place used tires in and brokers pickup the bins and sell the whole mixed batch containing tires of different brands. PneuLaurent (a Michelin subsidiary) must buy those mixed batches of unknown quality to have sufficient quantities of acceptable quality Michelin tires to retread. Given the small margin for retreaded passenger tires, the business is not profitable. If PneuLaurent could buy the right quantities in the right quality classes, the economics could be quite different.

The framework provided by Guide and Van Wassenhove (2001) is very general and provides a number of insights. Many different aspects of a product recovery system are affected by choices in product acquisition management. Some of the aspects considered are: system characteristics (machine utilization rates, process lead times, work in process), revenues and costs (material, labor, acquisition price, disposal), assets (inventory, machines, buildings), and liabilities (trade payables, accrued expenses). The discussion is in general terms, and not expressed in functional relationships. Therefore, their framework cannot be used directly for setting optimal prices. The framework is a motivation for the analysis developed here.

We will focus on the market-driven recovery system and develop an economic analysis for calculating the optimal (profit-maximizing) acquisition prices and the optimal selling price for remanufactured products. We argue that it is essential to develop formal systems to support the continued growth of closed-loop supply chain systems. The remanufacturing industry is experiencing consolidation in many segments, and global coverage may be the only way to exploit regional imbalances in supply and demand (Newman 2003). Global operations will require more sophisticated tools to exploit these global opportunities. Even in the European Union, where product take back for electronics and electrical equipment has recently been mandated for environmental reasons, the problem is now considered by major corporate executives as “…a business rather than an environmental problem…” (Hieronymi 2002).

We first present an overview of remanufacturing and closed-loop supply chain literature. We then discuss a case documenting the product recovery problem at a remanufacturer of consumer electronics goods. We present an economic model, a procedure for determining the optimal prices (under certain conditions), a practical procedure for determining near-optimal prices, and some computational results for a cellular phone remanufacturing company.

2. Literature Review

The past decade has seen an enormous increase in research on remanufacturing and, more recently, closed-loop supply chains. Guide and Van Wassenhove (2002) focus on the business aspects of developing and managing profitable closed-loop supply chains. They identify the common processes required by a closed-loop supply chain: product acquisition, reverse logistics, inspection, testing and disposition, remanufacturing, and selling and distribution. Our research focus here is on product acquisition in a remanufacturing setting.
Both Fleischmann (2001) and Guide (2000) offer comprehensive reviews of remanufacturing research. Table 1 provides an overview of the more recent research using the common processes identified by Guide and Van Wassenhove (2002) to categorize the literature. There are obvious gaps in the research literature. The areas of testing and disposition, and distribution and selling of the remanufactured products have not been addressed at all from an academic perspective. The area of product acquisition has had a very limited amount of research. The operational aspects of remanufacturing have received the most attention and there are numerous publications dealing with production planning and control (PP&C), inventory control, and materials planning.

The focus on operational issues presupposes that the basic question of profitability has been adequately addressed, or is obvious. Our experiences with global firms engaged in electronics, telecommunications, and power tools suggest the question of how to make reuse activities profitable has not been adequately addressed. A conversation with a senior manager in charge of product recovery at a global electronics firm highlights this point because “...the scope of the (product recovery) process does not include acquisition or sales of remanufactured products...” (Helbig 2003). This narrow scope was despite the fact that managers were aware of constraints on effective operations imposed by the acquisition and sales processes. The managers had no models or methods to show top management the financial impacts of separating these activities from the control of the product recovery group (Hasler 2003). Product recovery is often viewed as a narrowly focused, technical operational problem without visibility at the corporate level. At the Hewlett Packard Company, product returns were treated as a low-level divisional problem until a thorough analysis showed that the total cost of product returns was equivalent to 2% of total outbound sales (Davey 2002). Unfortunately, academic research often tends to reinforce this limited view with its narrow focus on local optimization of operational issues.

Other research efforts have considered remanufacturing from a more strategic perspective. Majumder and Groenevelt (2001) present a game-theoretic model of competition in remanufacturing. Their research suggests that incentives should be given to the original equipment manufacturer to increase the fraction of remanufacturable products available, or to decrease the costs of remanufacturing. Savaskan et al. (1999) develop a game-theoretic model that addresses the issues of channel choice and coordination of the channel. General overviews of product recovery and remanufacturing are presented by Thierry et al. (1995), Fleischmann et al. (1997), and Guide (2000). We also refer the reader to the book, edited by Guide and Van Wassenhove (2003), from the First Workshop on Business Aspects of Closed-Loop Supply Chains for a review and discussion of each of the areas in Table 1.

3. Product Acquisition Management

Product acquisition is a common problem for companies offering remanufactured products in a dynamic market, where supply and demand change rapidly and on a global scale. A surplus of used products may be available in one region, but the demand for the remanufactured products may be in a geographically
distant region. A successful remanufacturing firm must carefully manage its product acquisition process, i.e., buy the right quantities of the right qualities for the right prices, so as to maximize profits.

Our experience with firms in telecommunications equipment, computers and computer peripherals, and power tools reveals many common characteristics. Don Olson (Olson 2002) of Lucent Technologies explains that the dot-com crash resulted in a flood of used equipment on the market. At the same time, telecommunication service providers are seeking ways to reduce costs. A key problem for Lucent is to identify sufficient quantities of high-quality, used equipment to meet market demand. Olson further explains that product buy-backs are a common incentive as part of new equipment sales, and acquiring the units for the best price is an on-going concern. The cost to remanufacture telecommunications equipment is dependent on the condition (quality) and age of the equipment. A similar problem is discussed by Klausner and Hendrickson (2000) in designing a power tool takeback program for a German manufacturer.

By 2002, in the United States alone, there were over 100 million cellular telephones in use. Worldwide sales were 387 million handsets for 2001 even though the total market penetration is still as low as 12% (U.S. CIA 2002). The current replacement rate for cellular handsets is almost 80% each year. This provides enormous volumes of handsets potentially available for reuse.

Demand may be influenced by the introduction of new technology (e.g., digital and analog), price changes in cellular airtime, promotional campaigns, the opening of new markets, churn (customers leaving present airtime providers), and the number of new cellular telephones manufactured. Additionally, cellular airtime providers may limit the number of telephones supported by their system, and the dropping of a phone model by a major carrier can greatly affect a local market.

The value of a used handset is highly dependent on future market demand for that particular model either in remanufactured or as-is form. The market forces discussed earlier may cause the value of a particular model of phone to drop or rise with little warning. An additional factor is that the selling price for remanufactured phones tends to drop over time, making the used phones a perishable product.

To illustrate the importance of product acquisition management in the framework by Guide and Van Wassenhove (2001), we present the specifics at a firm that recovers mobile cellular telephone handsets and accessories. We know of at least two other firms, RS Communications and The Wireless Source, in the United States that have similar global operations. Additionally, we believe that although we use a specific company to illustrate the problem and solution methodology, the problem structure is similar for many remanufacturing firms and our findings may be generalized.

ReCellular, Inc., was founded in 1991 in Ann Arbor, MI, by Charles Newman to do business in new, used, and remanufactured cellular handsets. The company offers remanufactured and graded as-is products as a high-quality, cost effective alternative to new cellular handsets. Customer services include: grading and sorting, remanufacturing, logistics, trading, and product sourcing (all services are specific to cellular handsets and accessories). ReCellular operates globally, buying and selling in markets around the world.

The nature of product acquisitions is driven by what future demands (unknown) will be for phones. ReCellular obtains used phones from a variety of sources, including cellular airtime providers and third-party collectors. Third-party collectors are often charitable foundations that act as consolidators by collecting used handsets and accessories from individuals. Cellular airtime providers also act as consolidators by collecting used phones from customers who have returned the phones at the end of service agreements, or customers upgrading to newer technology (cellular airtime providers are also often buyers of the remanufactured products). Both these and other sources worldwide may offer a variety of handsets and accessories in varying condition for a wide range of prices and quantities. Due to the low cost (approximately $0.50 per phone using air transport) of bulk transportation of phones, using a worldwide network of suppliers of used phones is practical and cost-efficient. No individual returns are accepted because the channels required for direct
returns from the consumer have too high a cost to be effective at this time.

ReCellular buys from suppliers who have been audited earlier based on on-site visits or experience. Many of the quality dimensions of a phone are visual and can be communicated clearly and concisely to potential suppliers via photographs or graphic digital images (e.g., scratched surface, broken antenna), so these suppliers have a consistent way to grade their phones and offer them in agreed-on quality classes. ReCellular’s suppliers either operate in certain areas (e.g., charitable foundations) or they are brokers who buy used phones in bulk, grade them with a high degree of accuracy, and then offer them to companies like ReCellular. As the market matures, there will be an increasing number of these broker types of suppliers. At present, the commercial telecommunications equipment market has a large number of third-party brokers, and we expect to see the same consolidation and maturation in this market.

Coupling the price schedule for different classes with well-defined characteristics for each class, as discussed above, would motivate suppliers, and the organizations that collect phones, to sort out the phones they provide by identifiable features and to provide more phones for those classes with higher prices. At the same time, if acquisition prices went below some threshold, the supplier would not collect that class of phone and the supply would disappear.

Obtaining the best grade of used products for the best price is one of the key tasks necessary for the success of ReCellular. Deciding on a fair price to offer for the used phones is a difficult and complex task. Present state-of-the-art is based primarily on expert judgment, which is acquired by trial and error.

### 4. The Economic Model

We assume there are intermediaries between the final user returning his phone and ReCellular, and these intermediaries maintain a stock of used handsets. ReCellular buys from these intermediaries who grade their phones and sell them in different quality classes. Additionally, the information about quality grades and procurement prices is shared, common knowledge. ReCellular buys from many suppliers and the total supply available from these suppliers exceeds the total needs of ReCellular.

For the sake of simplicity and clarity we list the assumptions required by our base model in Table 2. We believe these assumptions provide for a model that allows us to capture the essence of the product acquisition problem while not trivializing the true nature of the problem.

There are $N$ quality classes, numbered $1, 2, \ldots, N$, for used products. These classes differ, for example, in preliminary testing results, physical damage, and appearance. We assume that within a certain quality class, all used products have the same associated expected remanufacturing cost, denoted by $c_i$, $i = 1, 2, \ldots, N$. For ease of notation, we will refer to a used product that falls into quality class $i$ as a return of type $i$ in the remainder of this paper.

We believe that our quality, returns, and cost to remanufacture assumptions are quite reasonable because these are based on data provided by ReCellular, and our experiences with other remanufacturing firms. Table 3 shows the average costs to remanufacture for each of the quality classes and the percentages acquired of each class by ReCellular over one year.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Percentage Acquired</th>
<th>Cost to Remanufacture ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.96</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>25.88</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>52.47</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>4.23</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>2.12</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>5.34</td>
<td>45</td>
</tr>
</tbody>
</table>

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**Table 2 Base Model Assumptions**

<table>
<thead>
<tr>
<th>Assumption</th>
</tr>
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<tbody>
<tr>
<td>Perfect testing</td>
</tr>
<tr>
<td>There exist a few, mutually exclusive, quality classes for grading purposes.</td>
</tr>
<tr>
<td>No capacity constraints</td>
</tr>
<tr>
<td>No fixed costs</td>
</tr>
<tr>
<td>Return rates are independent of sales rates.</td>
</tr>
<tr>
<td>The model is for a single period and not dependent on the product life cycle.</td>
</tr>
<tr>
<td>There are no supply or demand constraints</td>
</tr>
</tbody>
</table>

**Table 3 Quality Grade, Percentage Acquired, and Cost**
To stimulate returns, price incentives are offered. The acquisition price for a return of type $i$, $i = 1, 2, \ldots, N$, is denoted by $a_i$, and $R_i(a_i)$ denotes the corresponding return rate (function). We assume that $R_i(a_i)$ is continuous, increasing, and twice differentiable. It is defined on $[\beta_i, \gamma - c_i]$, where $\beta_i$ is the minimal acquisition price below which products of type $i$ will not be returned ($R_i(\beta_i) = 0$), and $\gamma$ is the maximum price at which a remanufactured product can be sold. Clearly, acquisition prices larger than $\gamma - c_i$ can never be profitable and are therefore not considered. We order the classes in such a way (not necessarily unique) that $\beta_i + c_i \leq \beta_2 + c_2 \leq \cdots \leq \beta_N + c_N$. This will turn out to be convenient in the analysis that follows.

The first derivative of $R_i(a_i)$ is denoted by $R'_i(a_i)$. The inverse of the return rate function $R_i(a_i)$ is denoted by $A_i(r_i)$, i.e., $A_i(R_i(a_i)) = a_i$. Our modeling of the returns implies independence of the return rates. That is, the acquisition price in one class does not influence the return rates in other classes.

Remanufactured products are sold at price $p$, and $D(p)$ denotes the corresponding demand rate (function). There is only one quality class for remanufactured handsets, because all items are remanufactured to a single set of standards. We assume that $D(p)$ is continuous, decreasing, and twice differentiable. It is defined on $[\beta_1 + c_1, \gamma]$ and ends at zero, that is, $D(\gamma) = 0$. We assume that $\gamma > \beta_1 + c_N$, because otherwise one or more types of returns could never be sold at a profitable price. The first derivative of $D(p)$ is denoted by $D'(p)$. The inverse of the demand rate function $D(p)$ is denoted by $P(d)$, i.e., $P(D(p)) = p$.

The goal is to determine the combination of acquisition prices $a_i$, $i = 1, 2, \ldots, N$, and selling price $p$ that maximize the profit rate. Because the return rates are increasing in the acquisition price and the demand rate is decreasing in the selling price, we can restrict our attention to prices for which the demand rate is equal to the total return rate, i.e.,

$$D(p) = \sum_{i=1}^{N} R_i(a_i) \quad \text{or} \quad p = P\left(\sum_{i=1}^{N} R_i(a_i)\right).$$

Note that $p$ is fully determined by $a_1, \ldots, a_N$ as a result. The optimal values for $a_1, \ldots, a_N$ and $p$ are denoted by $a_1^*, \ldots, a_N^*$ and $p^*$, respectively.

Our economic model focuses on a specific point in time, i.e., we assume that the demand and return rate functions are known. Given those functions, and the remanufacturing costs, our objective is to determine the selling price and acquisition prices that maximize the profit rate. Of course, the model can be modified at any time, re-estimating demand and return rate functions and recalculating the optimal prices. In fact, the model could be used to decide when the remanufacturing of a certain product should be initiated and terminated.

5. The Optimal Prices

There are two alternative formulations for the profit maximization problem. In terms of acquisition prices, the problem can be expressed as

$$\text{Maximize } P\left(\sum_{i=1}^{N} [R_i(a_i)]\right) \sum_{i=1}^{N} [R_i(a_i)] - \sum_{i=1}^{N} [R_i(a_i)(a_i + c_i)]. \quad (P1)$$

In terms of return quantities, the problem can be expressed as

$$\text{Maximize } P\left(\sum_{i=1}^{N} r_i\right) \sum_{i=1}^{N} r_i - \sum_{i=1}^{N} (r_i(A_i(r_i) + c_i)). \quad (P2)$$

The equivalence of these two formulations follows from the one-to-one relationship between $R_i(a_i)$ and $a_i$ (recall that $R_i(a_i)$ is a strictly increasing function). Using problem formulation (P2), it is easy to derive economically meaningful conditions for concavity of the objective function, as we will show next. After doing so, our optimization analysis will be based on problem formulation (P1).

Based on a number of examples, we know that the objective function (for (P1) or (P2)) is not necessarily concave and not even always unimodal if the demand and return rate functions have complex shapes. However, there are rather general conditions under which the objective function is concave. These are stated in Lemma 1.

**Lemma 1.** The objective function in (P2) is jointly concave in the vector $r = (r_1, r_2, \ldots, r_N)$ if the following two
conditions are satisfied:

C1. The sales revenue function $dP(d)$ is concave.
C2. The acquisition cost functions $r_iA_j(r_j)$, $j = 1, 2, \ldots, N$, are convex.

Proof. The first term in the objective function of (P2) is concave if C1 is true, because if $f(x)$ is a real-
valued concave function defined on the real line, and $g(r)$ is a real-valued linear function of the vector $r$, then $f(g(r))$ is easily shown to be concave in $r$. The second term is clearly convex under C2. It follows that the objective function is concave. Q.E.D.

We remark that concavity of the sales revenue function, i.e., diminishing marginal returns from sales, is often assumed in economics (and verified empirically). Clearly, C1 is satisfied if $P(d)$ is concave, but there are also many convex (and downward-sloping), inverse demand functions for which the condition is satisfied, e.g., $P(d) = a/(b + d)$ with $a, b > 0$. Assuming convexity of the acquisition cost functions, i.e., increasing marginal costs for acquisition, also seems reasonable. There are many inverse return functions for which C2 is satisfied, e.g., $A_j(r_j) = (r_j)^k$ for any $k > 0$.

In Lemma 2 Conditions C1 and C2 for concavity of the objective function are reformulated. The new conditions are less easy to interpret, but they will turn out to be useful in the analysis that follows.

Lemma 2. Conditions C1 and C2 can be reformulated as:

C1. The function

$$G(p) := \frac{D(p)}{D'(p)} + p$$

is strictly decreasing.

C2. The functions

$$F_j(a_j) := \frac{R_j(a_j)}{R_k(a_k)} + (a_j + c_j), \quad j = 1, 2, \ldots, N,$$

are strictly increasing.

Proof. By definition, the sales revenue function $dP(d)$ is concave if and only if its derivative is strictly decreasing. Rewriting the derivative as

$$P(d) + dP'(d) = P(d) + d \frac{1}{D'(P(d))} = P(d) + \frac{D(P(d))}{D'(P(d))},$$

and using the fact that $P(d)$ is strictly increasing, it follows that $dP(d)$ is concave if and only if $G(p)$ is strictly decreasing. So the formulations of Condition C1 in Lemmas 1 and 2 are equivalent. In a similar way, using the fact that $A_j(r_j)$ are all strictly increasing, it follows that the formulations of Condition C2 in Lemmas 1 and 2 are equivalent. Q.E.D.

The proof of Lemma 2 shows that $G(p)$ can be interpreted as the marginal revenue of selling one extra remanufactured product, and $F_j(a_j)$ as the marginal cost of buying one extra used product of class $j$, $j = 1, 2, \ldots, N$. This insight can be used to directly obtain the first-order optimality conditions (FOCs) for problem (P1):

$$F_j(a_j) = G\left(p\left(\sum_{i=1}^{N} R_j(a_i)\right)\right)\quad \text{if } a_j > \beta_j, \quad \text{and (1)}$$

$$F_j(a_j) \geq G\left(p\left(\sum_{i=1}^{N} R_j(a_i)\right)\right)\quad \text{if } a_j = \beta_j, \quad \text{(2)}$$

In Appendix A, these FOCs are derived formally.

In the remainder of this section, we will assume that Conditions C1 and C2 are satisfied, so that the FOCs are sufficient for determining the optimal prices. Before doing so, however, we first state and prove a useful lemma.

Lemma 3. If Conditions C1 and C2 are satisfied, then there is an $M \in \{1, \ldots, N\}$ such that $a^*_j > \beta_j$ for all $j = 1, \ldots, M$ and $a^*_j = \beta_j$ for all $j = M + 1, \ldots, N$.

Proof. Consider any combination of acquisition prices for which there exist $j, k \in \{1, \ldots, N\}$ so that $j < k, a_j = \beta_j$, and $a_k > \beta_k$. Clearly, the FOCs (1) and (2) can only be satisfied if $F_j(a_j) \geq F_k(a_k)$. However, $F_j(a_j)$ is increasing, and hence this would imply that $\beta_j + c_j = F_j(a_j) \geq F_k(a_k) > \beta_k + c_k$, which contradicts the assumption that $\beta_1 + c_1 \leq \beta_2 + c_2 \leq \cdots \leq \beta_N + c_N$. So the considered combination of acquisition prices (and the corresponding selling price) cannot be optimal. Q.E.D.

Lemma 3 shows that it is optimal to (only) purchase in classes $1, \ldots, M$ for some $M \in \{1, \ldots, N\}$. As the proof shows, this is a direct result of ordering the classes in such a way that the “minimum required acquisition and remanufacturing cost” $\beta_j + c_j$ is increasing in $j$. Theorem 1, which is proven in
Appendix B, determines the optimal value for \( M \) and the corresponding optimal prices.

**Theorem 1.** If Conditions C1 and C2 are satisfied, then the optimal prices are determined by the following procedure:

1. \( M := 1 \).
2. \( H^M_i(a_M) := a_M. \) If \( M = 2 \), then express \( a_i, j = 1, 2, \ldots, M - 1, \) as a function \( H^M_j(a_M) \) using \( F_j(a_i) = F_j(a_M) \).
3. Express \( p, \) as a function \( K^M(a_M) \) using \( \sum_{i=1}^M R_i(H^M_i(a_M)) = D(p) \).
4. Find the value \( a^*_M \) for \( a_M \) for which \( F_M(a_M) = G(K^M(a_M)) \).
5. If either \( M = N \) or \( F_M(a^*_M) ≤ F_{M+1}(\beta_{M+1}) \), then the optimal prices are: \( a^*_j = H^M_j(a^*_M) \) for all \( j = 1, \ldots, M; a^*_j = \beta_j \) for all \( j = M + 1, \ldots, N; p^* = K^M(a^*_M) \).
   Otherwise, \( M := M + 1 \) and repeat Steps 2-5.

**Remark.** If \( R_j(a_j), j = 1, \ldots, M, \) are “simple” functions, for instance first- or second-degree polynomials, then closed-form expressions for \( H^M_j(a_M), j = 1, \ldots, M - 1, \) and \( K^M(a_M) \) are obtained in Steps 2 and 3, respectively. In such a case, Steps 4 and 5 can be performed analytically. If it is not possible to obtain closed-form expressions in Steps 2 and 3, then Steps 4 and 5 have to be carried out numerically.

We note that in cases with counter-intuitive supplier pricing behavior, where (many of the) suppliers of higher quality returns are willing to accept a lower price than suppliers of lower quality returns, the optimal acquisition prices are not necessarily increasing with quality. Clearly, acquisition prices that are not increasing with quality are undesirable and should be modified. They will induce rational suppliers to supply higher quality phones in lower quality categories. Note that in our model, this is prevented by assuming “independence of returns.”

6. **A Heuristic for when Elasticities Are Similar**

As remarked before, Steps 4 and 5 of Theorem 1 have to be done numerically if the demand and return functions have complex shapes. Furthermore, the theorem cannot be used to determine the optimal prices if Conditions C1 or C2 are not satisfied.

Below, we restrict our attention to prices that are based on a fixed profit per product, “fixed profit prices” for short. That is, we set

\[
    a_j = \max(\beta_j, a_{\text{new}} - c_j), \quad j = 1, 2, \ldots, N, \tag{3}
\]

where \( a_{\text{new}} \) denotes the (theoretical) acquisition price that we are willing to pay for an as-good-as-new returned item that does not require remanufacturing. Good-as-new returned items are very common, especially in North America, where resellers allow customers to return items, no questions asked, for up to 90 days. By fixed profit prices, we mean that for all other quality classes, the remanufacturer will pay exactly the acquisition price that will lead to the same cost if the corresponding remanufacturing cost is added. Therefore, all products will return the same profit margin as the as-new products requiring no remanufacturing.

We will denote the associated total return rate by \( R^0(a_{\text{new}}) \) and the associated selling price (for which the demand rate is equal to the total return rate) by \( p^0(a_{\text{new}}) \). That is,

\[
    R^0(a_{\text{new}}) := \sum_{i} R_i(a_{\text{new}} - c_i) \quad \text{and} \quad p^0(a_{\text{new}}) := P(R^0(a_{\text{new}})).
\]

Rewriting (3) as

\[
    a_j + c_j = a_{\text{new}} \quad \text{if} \quad a_j > \beta_j, \quad \text{and} \quad \tag{4}
\]

\[
    a_j + c_j ≥ a_{\text{new}} \quad \text{if} \quad a_j = \beta_j, \quad \text{if} \quad a_j = \beta_j, \quad \tag{5}
\]

we observe that fixed profit prices satisfy conditions that are similar to the first-order optimality Conditions (1) and (2), but ignore the shapes of the demand and return rate functions.

Determining the optimal fixed profit prices is much easier than determining the overall optimal prices, because fixed profit prices are characterized by a single price parameter \( a_{\text{new}} \). In fact, as we will show in the remainder of this section, the optimal value for \( a_{\text{new}} \) can be determined graphically.

The total return rate \( R^0(a_{\text{new}}) \) can graphically be determined by shifting each return curve \( R_i(a_i) \) for type \( i \) returns \( c_i \) units to the right, and adding the
shifted curves. Combining the so obtained total return rate curve with the demand rate curve, the profit rate

\[ p^0(a_{\text{new}}) - a_{\text{new}} \] \[ R^0(a_{\text{new}}) \]

can be determined graphically.

However, the use of our heuristic requires some caution. In cases where the elasticities of the functions differ substantially, the quality of the solution deteriorates rather quickly. Therefore, we can only recommend the heuristic in cases where the elasticities are roughly equal. Unfortunately, there are no empirical studies available that either confirm or refute the requirement of similar elasticities. We believe more information from industry will prove invaluable for future research.

7. Optimal Prices, Heuristic Solutions, and Sensitivity Analyses

In this section, we use estimates of the model parameters derived from data provided by ReCellular, apply Theorem 1 to calculate the optimal prices, and perform a sensitivity analysis on the parameters. We also apply the heuristic presented in §6 and discuss its performance.

We only consider linear demand and return functions here, because linear functions provide a good fit (linear regression models resulted in \( r^2 \geq 0.80 \) for all fit linear functions, complete details are available on request) for the collected historical data on acquisition prices versus returns and on selling price versus demand. We remark that, for confidentiality, all data are rescaled. The slope of the return function for items of class \( i, i = 1, 2, \ldots, N \), will be denoted by \( \delta_i \), i.e., \( R_i(a_i) = \delta_i(a_i - \beta_i) \). The slope of the demand function will be denoted by \( \varepsilon \), i.e., \( D(p) = \varepsilon(p - \gamma) \). The estimates for \( \gamma \) and \( \varepsilon \) are 80 and −4, respectively. Other model parameters and optimal values are given in Table 4.

It is easy to check that \( \beta_1 + c_1 \leq \beta_2 + c_2 \leq \ldots \leq \beta_n + c_n \), and hence the classes do not have to be reordered (see §4). Applying Theorem 1, the optimal selling price and the corresponding demand are 60.8 and 76.8, respectively, and the optimal profit is 1,730.5 (the full example is shown in Appendix B after the proof of Theorem 1).

In our sensitivity analysis for the ReCellular data, each parameter in Table 4 was varied in turn, keeping all the other parameters unchanged. As expected, a higher remanufacturing cost (which implies a lower quality) for returns in a certain class leads to a lower optimal acquisition price, and hence to fewer returns in that class. This decrease in returns is partially compensated by increasing the acquisition prices for other classes, but the total number of returns decreases. Because demand equals total return, the selling price increases. Figures 2a and 2b illustrate the effects of variations in the remanufacturing cost for Class 2 (other classes give similar results).

If the value of \( \beta_i \) increases, which (for linear return functions) can be interpreted as an increase by the same amount in the price for each used product of class \( i \), then the acquisition optimal price \( a_i \) increases. However, the increase in \( a_i \) is less than the increase in \( \beta_i \), so fewer products of class \( i \) are acquired if \( \beta_i \) increases. This, in turn, leads to similar effects on the rest of the system as in Figures 2a and 2b.

If the slope of the return function for some class increases, then the acquisition price for that class decreases with more products of that class acquired. The acquisition prices for all other classes (with positive returns) also decrease, leading to fewer returns for those classes, but the total number of returns increases. Because demand equals total return, the selling price decreases. These effects are illustrated for Class 2 in Figures 3a and 3b.

An increase in \( \gamma \) (demand curve shifts to the right) or a decrease in \( \varepsilon \) (steeper demand curve), implies that (the same number of) remanufactured products can be sold at a higher price. This explains why an increase in \( \gamma \) or a decrease in \( \varepsilon \) leads to higher

<table>
<thead>
<tr>
<th>Class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanufacturing cost (( c_i ))</td>
<td>5</td>
<td>20</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td>Intercept return function (( \beta_i ))</td>
<td>17</td>
<td>13</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Slope return function (( \delta_i ))</td>
<td>1</td>
<td>5</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Optimal acquisition price</td>
<td>26.8</td>
<td>17.3</td>
<td>9.8</td>
<td>6.3</td>
<td>4.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Optimal return quantities</td>
<td>9.8</td>
<td>21.5</td>
<td>36.2</td>
<td>9.3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 2a The Effect of Variations in the Remanufacturing Cost on Product Returns

Figure 2b The Effect of Variations in the Remanufacturing Cost on Acquisition Prices

Figure 3a The Effect of Variations in the Slope of the Return Function on Product Returns

Figure 3b The Effect of Variations in the Slope of the Return Function on Acquisition Prices

acquisition prices for all classes (with positive returns) as well as a higher sales price.

The impact of these parameter changes on the objective function value is illustrated in Table 5. We define relative percentage deviation as (the problem solution value minus the optimal solution of the base ReCellular case) divided by the optimal solution value of the base ReCellular case. First, we note that profit is relatively insensitive to changes in the slope of the return function ($\delta_1$). However, as we shift
Table 5  Effect of Variations in the Model Parameters, Compared to the
Estimated Parameters for ReCellular, on the Profit Associated
with the Optimal Prices

<table>
<thead>
<tr>
<th>Class</th>
<th>Parameter Value</th>
<th>Relative % Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanufacturing cost (c_i)</td>
<td>23</td>
<td>28 33 38 43</td>
</tr>
<tr>
<td>Intercept of the return function (β_i)</td>
<td>19.0</td>
<td>7.9 0.0 4.6 5.8</td>
</tr>
<tr>
<td>Slope of the return function (δ_i)</td>
<td>1</td>
<td>3 5 10 15</td>
</tr>
<tr>
<td>Intercept of the demand function (γ)</td>
<td>-67.6</td>
<td>-39.0 0.0 49.8 110.3</td>
</tr>
<tr>
<td>Slope of the demand function (ε)</td>
<td>-1</td>
<td>-2 -4 -6 -8</td>
</tr>
</tbody>
</table>

Table 6  Performance of Heuristic Pricing Compared with
Optimal Solution

<table>
<thead>
<tr>
<th>Class</th>
<th>Parameter Value</th>
<th>Relative % Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanufacturing cost (c_i)</td>
<td>23</td>
<td>28 33 38 43</td>
</tr>
<tr>
<td>Intercept of the return function (β_i)</td>
<td>4.8</td>
<td>7.9 4.9 4.7 5.7</td>
</tr>
<tr>
<td>Slope of the return function (δ_i)</td>
<td>1</td>
<td>3 5 10 15</td>
</tr>
<tr>
<td>Intercept of the demand function (γ)</td>
<td>60</td>
<td>70 80 90 100</td>
</tr>
<tr>
<td>Slope of the demand function (ε)</td>
<td>-1</td>
<td>-2 -4 -6 -8</td>
</tr>
</tbody>
</table>

8. Conclusions and Future Research

We believe this work underscores the need for reuse to be viewed as a value-creating activity. Product acquisition management is the primary driver to determine whether reuse activities will be profitable and these activities should be controlled by the firm. By controlling the quality of the acquired products that serve as input to the remanufacturing process, a decision maker can optimize the overall profitability of a remanufacturing system. The other, much less viable option, is for firms to simply wait for the waste stream to come to them and find ways to minimize the financial loss. Remanufacturers like ReCellular have made their fortunes from recognizing the competitive advantage in obtaining the right products at the right acquisition prices which allows revenue maximization.

The current legislative environment favoring producer responsibility and the rapid maturation of the markets for goods will encourage more brokers to enter the market. These brokers will offer a tiered supply structure where remanufacturers may pick and choose from a wide range of quality classes and suppliers. This evolution will make the determination of the product acquisition strategy crucial to ensuring profitability. Our experiences with firms and conversations with managers indicate this time is rapidly approaching.

Our model is the first of its type in the literature. It operationalizes the conceptual framework of Guide and Van Wassenhove (2001) by constructing a model for a cellular telephone remanufacturer. The model has proved useful in projects with other firms, even if the assumptions must be changed somewhat to fit the specifics. We are presently working with firms on a more flexible model and framework to support their specific needs.

We also believe the model is doubly relevant because the literature on remanufacturing has been deficient from a managerial perspective with its focus on tactical issues. Much of the current research only addresses a small, isolated portion of the problem, e.g., vehicle routing for product take-back, or inventory models.
Extensions of the present research should consider additional complexities such as multiple sales quality classes (including sales as-is), fixed costs and return rates that are dependent on sales rates. Additional constraints, such as capacity, should be considered. There is also a need for empirical data to estimate the actual parameters of the curves. Extensions to our base model investigating imperfect testing and classification at each stage could be a very interesting area for future research. There is an abundance of literature in economics on contracting and quality under imperfect observability conditions that would apply to this case of imperfect grading. We believe our full information model represents the first step in this inquiry.

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Appendix A. First-Order Optimality Conditions
Consider the profit maximization problem

$$\text{Maximize} \quad \Pi(a_1, \ldots, a_N) := P \left( \sum_{i=1}^{N} R_i(a_i) \right) \sum_{i=1}^{N} R_i(a_i)$$

$$- \sum_{i=1}^{N} [R_i(a_i)(a_i + c_i)]. \quad (P1)$$

The first-order optimality conditions are

$$\frac{\partial}{\partial a_j} \Pi(a_1, \ldots, a_N) = 0 \quad \text{for all } j \in \{1, 2, \ldots, N\}$$

such that $$a_j > \beta_j,$$ and (6)

$$\frac{\partial}{\partial a_j} \Pi(a_1, \ldots, a_N) \leq 0 \quad \text{for all } j \in \{1, 2, \ldots, N\}$$

such that $$a_j = \beta_j. \quad (7)$$

Differentiating the objective function with respect to $$a_j, \quad j = 1, 2, \ldots, N,$$ gives

$$\frac{\partial}{\partial a_j} \Pi(a_1, \ldots, a_N) = \left[ \frac{\partial}{\partial a_j} P \left( \sum_{i=1}^{N} R_i(a_i) \right) \right] \sum_{i=1}^{N} R_i(a_i)$$

$$+ P \left( \sum_{i=1}^{N} R_i(a_i) \right) R_i(a_i) - R_i(a_i) - R_i'(a_i)(a_i + c_i).$$

It is easy to see that

$$\frac{\partial}{\partial a_j} P \left( \sum_{i=1}^{N} R_i(a_i) \right) = \frac{R_i'(a_i)}{D(P(\sum_{i=1}^{N} R_i(a_i)))}, \quad (8)$$

and hence, we get

$$\frac{\partial}{\partial a_j} \Pi(a_1, \ldots, a_N) = \frac{R_i'(a_i)}{D(P(\sum_{i=1}^{N} R_i(a_i)))} + P \left( \sum_{i=1}^{N} R_i(a_i) \right) R_i(a_i)$$

$$- R_i(a_i) - R_i'(a_i)(a_i + c_i)$$

$$= R_i'(a_i) \left( \frac{\sum_{i=1}^{N} R_i(a_i)}{D(P(\sum_{i=1}^{N} R_i(a_i)))} \right) + P \left( \sum_{i=1}^{N} R_i(a_i) \right)$$

$$- \frac{R_i(a_i)}{R_i'(a_i)} - (a_i + c_i).$$

Using those expressions, recalling that $$R_i(\beta_j) = 0,$$ and that

$$\sum_{i=1}^{N} R_i(a_i) = D \left( P \left( \sum_{i=1}^{N} R_i(a_i) \right) \right),$$

we can rewrite (6) and (7) as

$$\frac{R_i(a_i)}{R_i'(a_i)} + (a_i + c_i) = \frac{D(P(\sum_{i=1}^{N} R_i(a_i)))}{D(P(\sum_{i=1}^{N} R_i(a_i)))} + P \left( \sum_{i=1}^{N} R_i(a_i) \right)$$

if $$a_j > \beta_j,$$ and (9)

$$(a_j + c_i) \geq \frac{D(P(\sum_{i=1}^{N} R_i(a_i)))}{D(P(\sum_{i=1}^{N} R_i(a_i)))} + P \left( \sum_{i=1}^{N} R_i(a_i) \right)$$

if $$a_j = \beta_j. \quad (10)$$

Appendix B. Proof of Theorem 1
Applying Lemma 1, it follows that there is an $$M \in \{1, \ldots, N\}$$ such that $$a^*_j > \beta_j$$ for all $$j = 1, \ldots, M$$ and $$a^*_j = \beta_j$$ for all $$j = M + 1, \ldots, N.$$ The search for the optimal acquisition prices (and the corresponding selling price) can therefore be restricted to $$a_1, \ldots, a_M$$ that satisfy $$a_j > \beta_j$$ for all $$j = 1, \ldots, M$$ and $$a_j = \beta_j$$ for all $$j = M + 1, \ldots, N$$ for some $$M \in \{1, \ldots, N\}.$$ These are exactly the prices that the procedure considers, starting with $$M = 1$$ in Step 1.

In Steps 2–4, for some fixed value of $$M \in \{1, \ldots, N\},$$ the prices that satisfy (1) are determined. Because $$F_j(a_i), \quad j = 1, 2, \ldots, M - 1$$ is increasing, $$F_j(\beta_j) = \beta_j + c_j \leq \beta_M + c_M = F_M(a_M)$$ and $$\lim_{a_i \to \infty} F_j(a_i) = \infty,$$ it follows that for each value of $$a_M \geq \beta_M,$$ there is a unique value $$a_j \in [\beta_j, \infty)$$ such that $$F_j(a_j) = F_M(a_M).$$ Hence, $$a_j, \quad j = 1, 2, \ldots, M - 1,$$ can be expressed as a function $$H^M(a_M),$$ as is done in Step 2. Furthermore, that function $$H^M(a_M)$$ is increasing because both $$F_j(a_i)$$ and $$F_M(a_M)$$ are increasing.

Similarly, it follows that $$D(p)$$ can be expressed as a decreasing function $$K^M(a_M),$$ as is done in Step 3. Because $$F_M(a_M)$$ is increasing, $$G(p)$$ is increasing (see Lemma 2), and $$K^M(a_M)$$ is decreasing, there is only a single value $$a_M^*$$ for $$a_M$$ for which $$F_M(a_M) = G(K^M(a_M)).$$ This value is determined in Step 4. Step 5 checks whether the corresponding prices satisfy (2). If so, then these prices satisfy both (1) and (2) and are optimal. If not, the procedure tries the next value for $$M.$$
Figure 4 Graphical Solution Using Shifted Total Return $R^p(\bar{a}_{\text{new}})$ and Demand $D(p)$

Note. The exact profit is $(p^0(\bar{a}_{\text{new}}) - \bar{a}_{\text{new}})R^p(\bar{a}_{\text{new}}) = (61.12 - 39.33)/75.33 = 1,645.76.$

EXAMPLE. Let $N = 6$, $c_1 = 5$, $c_2 = 20$, $c_3 = 30$, $c_4 = 35$, $c_5 = 40$, $c_6 = 45$, $R_1(a_i) = a_i - 17$, $R_2(a_i) = 5(a_i - 13)$, $R_3(a_i) = 20(a_i - 8)$, $R_4(a_i) = 30(a_i - 6)$, $R_5(a_i) = 20(a_i - 4)$, $R_6(a_i) = 30(a_i - 2)$, and $D(p) = -4(p - 80)$. It easily follows that $F_1(a_i) = 2a_i - 12$, $F_2(a_i) = 2a_i + 7$, $F_3(a_i) = 2a_i + 22$, $F_4(a_i) = 2a_i + 29$, $F_5(a_i) = 2a_i + 36$, $F_6(a_i) = 2a_i + 43$, and $G(p) = 2p - 80$. Applying the procedure in Theorem 1 gives:

Step 1. $M := 1$.

Step 2. $H_2(a_i) := a_i$.

Step 3. Using $a_i - 17 = -4(p - 80)$, we get $p = K(a_i) = 84.25 - 0.25a_i$.

Step 4. So, $F_1(a_i) = G(K(a_i))$ becomes $2a_i - 12 = 88.5 - 0.5a_i$, which is solved by $a_i^0 = 40.2$.

Step 5. $F_1(a_i) = 68.4 > 33 = F_1(\beta_2)$. So, $M := M + 1 = 2$.

Step 2. $H_2(a_i) := a_i$. Using $F_1(a_i) = F_2(a_i)$ results in $a_i = H_2(a_i) = a_i + 9.5$.

Step 3. Using $a_i + 9.5 - 17 + 5(a_i - 13) = -4(p - 80)$ results in $p = K(a_i) = 98.125 - 1.5a_i$.

Step 4. So, $F_2(a_i) = G(K(a_i))$ becomes $2a_i + 7 = 116.25 - 3a_i$, which is solved by $a_i^1 = 21.85$.

Step 5. $F_2(a_i) = 50.7 > 38 = F_2(\beta_3)$. So, $M := M + 1 = 3$.

Step 2. $H_3(a_i) := a_i$. Using $F_2(a_i) = F_3(a_i) = F_4(a_i)$, we get $a_i = H_3(a_i) = a_i + 17$ and $a_i = H_3(a_i) = a_i + 7.5$.

Step 3. Using $a_i + 17 - 17 + 5(a_i + 7.5 - 13) + 20(a_i - 8) = -4(p - 80)$, we get $p = K(a_i) = 126.875 - 6.5a_i$.

Step 4. So, $F_3(a_i) = G(K(a_i))$ becomes $2a_i + 22 = 173.75 - 13a_i$, which is solved by $a_i^2 = 10.12$.

Step 5. $F_3(a_i) = 42.24 > 41 = F_3(\beta_4)$. So, $M := M + 1 = 4$.

Step 2. $H_4(a_i) := a_i$. Using $F_3(a_i) = F_4(a_i) = F_5(a_i)$, we get $a_i = H_4(a_i) = a_i + 20.5$, $a_i = H_4(a_i) = a_i + 11$, and $a_i = H_4(a_i) = a_i + 5.5$.

Step 3. Using $a_i + 20.5 - 17 + 5(a_i + 11 - 13) + 20(a_i + 3.5 - 8) + 30(a_i - 6) = -4(p - 80)$, we get $p = K(a_i) = 149.125 - 14a_i$.

Step 4. So, $F_4(a_i) = G(K(a_i))$ becomes $2a_i + 29 = 218.25 - 28a_i$, which is solved by $a_i^3 = 6.31$.

Step 5. $F_4(a_i) = 41.62 < 44 = F_4(\beta_5)$. So, the optimal prices are $a_1 = H_4(a_i) = 26.81$, $a_2 = H_4(a_i) = 17.31$, $a_3 = H_4(a_i) = 9.81$, $a_4 = H_4(a_i) = 6.31$, $a_5 = \beta_5 = 4$, $a_6 = \beta_5 = 2$, and $p^* = K(a_i) = 60.81$. The associated return quantities are $9.81, 21.54, 36.17, 9.25, 0, \text{ and } 0$, respectively. The sum of these is equal to the selling quantity $76.78$, and the associated profit rate is $1,730.55$.

Appendix C. Heuristic Solution Using Recellular Data

Applying the heuristic procedure to the ReCellular problem leads to a value of $a'_{\text{new}} = 39.33$ and the acquisition of quality classes 1, 2, and 3. The prices and return quantities are $a_1 = 34.33$, $a_2 = 19.33$, $a_3 = 9.33$, and 17, 32, 27, respectively. The profit resulting from this heuristic solution is $(p^0(a_{\text{new}}') - a_{\text{new}}' R^p(a_{\text{new}}')) = (61.12 - 39.33)/75.33 = 1,645.76$. The graphic procedure is illustrated in Figure 4.

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