“All the fifty years of conscious brooding have brought me no closer to answer the question, ‘What are light quanta?’ Of course today every rascal thinks he knows the answer, but he is deluding himself.”

-Albert Einstein
Light is not a classical wave of electric and magnetic fields. Light is composed of quanta, with energy

\[ E = hf \]

where \( h = 6.63 \times 10^{-34} \text{ J-s} \) is the Planck constant.
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How can light be both a particle and a wave?
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\[
= \frac{P}{hf}
\]

\[
= \frac{P}{hc/\lambda}
\]

\[
= \frac{100 \times 590 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^8}
\]

\[
= 2.97 \times 10^{20} \text{ photons/s.}
\]
Photon Energy

How do we know that photons have discrete energy? Let’s set up an experiment.
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Incident light is monochromatic and knocks electrons out of the metal.
We increase the voltage until no current is measured. This is the stopping potential $V_{\text{stop}}$.

$$K_{\text{max}} = U = eV_{\text{stop}}$$
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Making the light more intense does not change the stopping potential.
Alternatively, we can change the frequency of light and measure the stopping potential for each frequency.
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There is a minimum photon frequency (energy) under which no electrons are ejected.
Photo Energy

The resulting equation is the conservation of energy: one photon energy turns into potential and kinetic energy of an electron:

\[ hf = K_{\text{max}} + \Phi \]
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\[ hf = K_{\text{max}} + \Phi \]

\( \Phi \) is known as the “work function” and represents the energy required to pull the electron away from the material.
If we replace $K_{\text{max}}$ with $eV_{\text{stop}}$, we get the another form of the photoelectric equation.

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$$V_{stop} = \frac{h}{e} f - \frac{\Phi}{e}$$

The work function depends on the material but is a constant. Therefore, we get a linear relationship.
Lecture Question 6.1
Upon which one of the following parameters does the energy of a photon depend?

(a) the mass of the photon
(b) the amplitude of the electric field
(c) the direction of the electric field
(d) the relative phase of the electromagnetic wave relative to the source that produced it
(e) the frequency of the photon
Although the photon is massless, it carries momentum $p = \frac{h}{\lambda}$. We know from experiments.
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X-rays scatter off of a carbon target. The resulting angle, intensity and wavelength are measured.
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There is a shift in the energy of the x-ray photons as the scattering angle is changed.
The photon is colliding with an electron. Three things can happen.

1. An x-ray heads toward a target electron.
2. Or it can scatter at some intermediate angle $\phi$.
3. The x-ray can bypass the electron at scattering angle $\phi = 0$.
4. No energy is transferred to the electron.
5. Or it can backscatter at the maximum angle $\phi = 180^\circ$.
6. Maximum energy is transferred.
The photon is colliding with an electron. Three things can happen.

During a collision, energy must be conserved: $hf = hf' + K$. 

$hf$ is the initial energy of the photon, $hf'$ is the final energy of the photon, and $K$ is the kinetic energy of the electron.
After the collision, the electron has relativistic kinetic energy: \( K = mc^2(\gamma - 1) \),

with \( \gamma = 1 / \sqrt{1 - (v/c)^2} \).
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with \( \gamma = 1/\sqrt{1 - (v/c)^2} \).

This gives:

\[
\begin{align*}
hf &= hf' + mc^2(\gamma - 1) \\
h/\lambda &= h/\lambda' + mc(\gamma - 1) [I]
\end{align*}
\]

using \( c = f\lambda \).
We can also consider momentum in the x- and y-directions.
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\[ \frac{h}{\lambda} = \frac{h}{\lambda'} \cos(\phi) + \gamma mv \cos(\theta) \quad [II] \]

\[ 0 = \frac{h}{\lambda'} \sin(\phi) - \gamma mv \sin(\theta) \quad [III] \]
When we combine equations I, II and III to solve for $\Delta \lambda = \lambda' - \lambda$, we get:

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Here, we’ve eliminated the unknown electron properties $\nu$ and $\theta$ and $h/mc$ is the Compton wavelength.
How do we bring together the wave nature and particle nature of light?
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During an interference experiment, we say that the photon is spread out in a probability distribution.
We associate different locations with a probability density of detecting the photon there.
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The photon is spread out like a wave while it travels, but behaves particle-like when detected.
An interferometer can be thought of as a photon interfering with itself.
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A photon is emitted, its probability wave splits into two paths, interferes with itself and is detected as a photon (or not) at D.
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$$\lambda = \frac{h}{p}.$$
Electrons or x-rays strike the target and then diffract like waves off of the crystalline structure.
Lecture Question 6.2
Which one of the following experiments demonstrates the wave nature of electrons?

(a) Small flashes of light can be observed when electrons strike a special screen.

(b) Electrons directed through a double slit can produce an interference pattern.

(c) The Michelson-Morley experiment confirmed the existence of electrons and their nature.

(d) In the photoelectric effect, electrons are observed to interfere with electrons in metals.

(e) Electrons are observed to interact with photons (light particles).
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Schrödinger’s Equation

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That is, we solve the differential equation $\vec{F}_{\text{net}} = m\frac{d^2\vec{r}}{dt^2}$ with initial conditions $\vec{r}_0$ and $\vec{v}_0$. The solution is

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$
However, in quantum mechanics, particles do not obey $\vec{F} = m\vec{a}$, and exact positions $\vec{r}(t)$ do not exist. Instead, we have a probability density: $\Psi(x, y, z, t)$
Schrödinger’s Equation

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$\Psi(x, y, z, t)$ is known as the wave function. The probability density is given by its absolute square $p(x, y, z, t) = |\Psi|^2$. 
Schrödinger’s Equation

Often, the wavefunction can be simplified:

\[ \Psi(x, y, z, t) = \psi(x, y, z)e^{-i\omega t} \]

where \( \omega = 2\pi f \) is the angular frequency of the matter wave.
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The probability that a detector will measure a particle between a position $x_1$ and $x_2$ is given by:

$$p = \int_{x_1}^{x_2} |\psi(x)|^2 dx \rightarrow \int_{V} |\psi(x, y, z)|^2 dV.$$
This wavefunction $\Psi$, like $\vec{r}$, is the solution to an important differential equation.
Schrödinger’s Equation

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Schrödinger’s Equation!

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E = K + U \\
= \frac{1}{2}mv^2 + U \\
= \frac{1}{2m}(mv)^2 + U \\
= \frac{p^2}{2m} + U \\
E\psi = \frac{p^2}{2m}\psi + U\psi.
\]
Let’s assume that the wavefunction, which describes our wave-like electrons, is oscillatory.

\[ \psi(x, t) \propto e^{i(kx - \omega t)} \]

with \( k = \frac{2\pi}{\lambda} = \frac{p}{\hbar} \) and \( \hbar = \frac{h}{2\pi} \).
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\[ \frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = \frac{p^2}{2m} \psi \]
Putting this all together, we get the time-independent Schrödinger’s equation.

\[ -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + U \psi = E \psi \]

or,

\[ \frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2} (E - U) \psi = 0 \]
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Caveats:

(a) This is only 1-dimensional

(b) This ignores the time-oscillation

(c) The solution depends on the function $U(x)$
Schrödinger’s Equation

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The general solution is:

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\psi(x) = Ae^{ikx} + Be^{-ikx} \rightarrow Ae^{ikx} \quad \text{(right traveling)}
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\[ = \text{constant} \]
One consequence of this probabilistic behavior is the Heisenberg Uncertainty Principle.

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The less uncertainty in position, the more uncertainty in momentum (and *vice versa*).
Lecture Question 6.3
Which one of the following statements provides the best description of the Heisenberg Uncertainty Principle?

(a) If a particle is confined to a region $\Delta x$, then its momentum is within some range $\Delta p$.

(b) If the error in measuring the position is $\Delta x$, then we can determine the error in measuring the momentum $\Delta p$.

(c) If one measures the position of a particle, then the value of the momentum will change.

(d) It is not possible to be certain of any measurement.

(e) Depending on the degree of certainty in measuring the position of a particle, the degree of certainty in measuring the momentum is affected.
Consider a puck sliding along an icy hill where $U_b = mgh$ is the potential energy at the top.
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In this case, the puck needs $K > U_b$ to pass over this barrier.
However, in quantum mechanics, a particle can tunnel through a barrier even if it does not have enough energy to do so.
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We must solve Schrödinger’s equation in order to find the tunneling probability!
Tunneling

\[ \frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2} (E - U) \psi = 0 \]

For \( x < 0 \) and \( x > L \), we have a free particle \( U = 0 \).

For \( 0 < x < L \), \( E < U_b = eV_b \).

At each boundary, \( \psi(x) \) must be continuous and smooth.
Tunneling

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Tunneling

The term $E - U$ is the kinetic energy.

$$E - U = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{1}{2m} (k\hbar)^2$$

(note: $p = h/\lambda = k\hbar$)
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$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

- This has the same solutions as before ($e^{ikx}$)
- Here, $k = \sqrt{2m(E - U)}\hbar$
- The wavenumber can be imaginary if $U > E$. 
There are three regions when an electron hits a barrier:

- **Left Side** ($k \in \mathbb{R}$): $\psi(x) = Ae^{ikx} + Be^{-ikx}$
- **Middle Section** ($k \in I$): $\psi(x) = Ce^{ikx} + De^{-ikx}$
- **Right Side** ($k \in \mathbb{R}$): $\psi(x) = Ee^{ikx}$

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