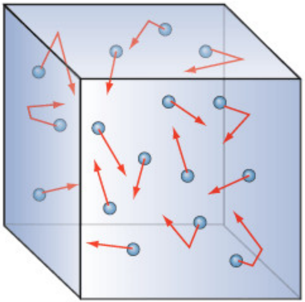
Chapter 2 - The Kinetic Theory of Gases



The ideal gas law is a combination of three intuitive relationships between pressure, volume, temp and moles.

David J. Starling
Penn State Hazleton
Fall 2013

Ideal Gases

Just like the meter, a mole is defined in terms of some physical quantity: 1 mole is the number of atoms in a 12 g sample of carbon-12.

$$N_A = 6.02 \times 10^{23} \text{ atoms/mol}$$

$$n = \frac{N}{N_A} = \frac{M_{sample}}{M} = \frac{M_{sample}}{mN_A}$$

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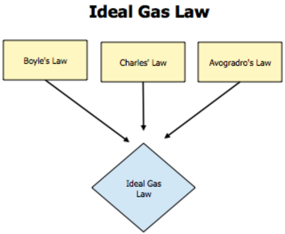
- n: moles of a substance
- N: atoms or molecules of a substance
- M_{sample}: mass of a sample
- m: mass of a single atom or molecule
- M: mass of one mole of an atom or molecule ($= mN_A$)

Ideal Gases

There are three basic laws that describe the behavior of an "ideal gas."



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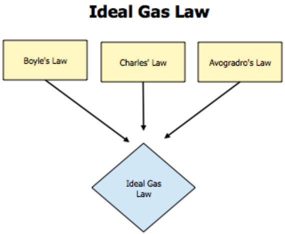

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- Avogadro's Law: $V \propto n$

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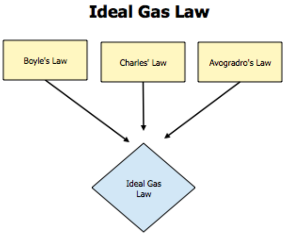

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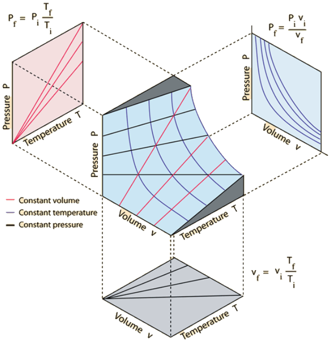
Ideal Gas Law

$$pV = nRT = NkT$$

Ideal Gases

An ideal gas describes all gases as $N \to 0$.



$$pV = NkT = nRT \tag{1}$$

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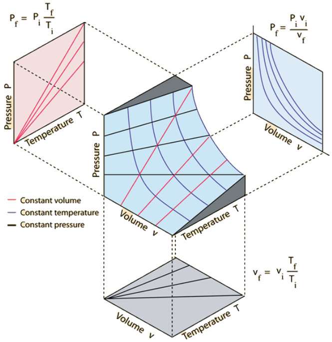
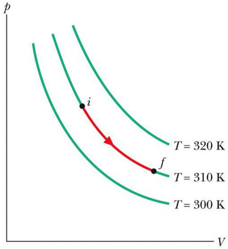


$$pV = NkT = nRT \tag{1}$$

- $Nk = nN_A k = nR \to k = R/N_A$
- $R = 8.314$ J/mol-K
- $k = 1.381$ J/K

Ideal Gases

When a gas expands, it does work on its surroundings. For an isothermal process:



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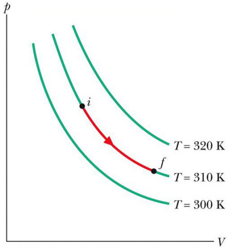


$$\begin{aligned} W &= \int_{V_i}^{V_f} p\, dV \\ &= \int_{V_i}^{V_f} \frac{NkT}{V} dV \\ &= NkT \ln\left(\frac{V_f}{V_i}\right) \end{aligned}$$

Ideal Gases

An ideal gas is enclosed within a container by a moveable piston. If the final temperature is two times the initial temperature and the volume is reduced to one-fourth of its initial value, what will the final pressure of the gas be relative to its initial pressure, P_1?

(a) $8P_1$

(b) $4P_1$

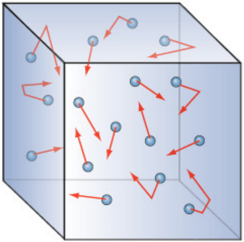
(c) $2P_1$

(d) $P_1/2$

(e) $P_1/4$

Temperature, Pressure and Speed

The kinetic theory of gases relates this macroscopic behavior (p, T, V) *to the microscopic motion of atoms.*



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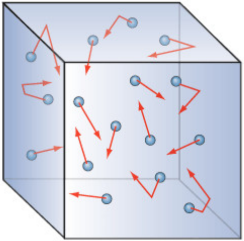

Let's use Newton's Second Law ($F_x = ma_x = dp_x/dt$) to connect pressure to velocity.

Temperature, Pressure and Speed

When a atom/molecule hits the side of a cube container, it rebounds elastically:

$$\Delta p_x = -mv_x - mv_x = -2mv_x$$

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&= \frac{m}{L^3}\left(v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2\right)
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&= \frac{m}{L^3}(v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2) \\
&= \frac{mN\overline{v_x^2}}{L^3} = \frac{Mn\overline{v_x^2}}{V} \\
p &= \frac{nM\overline{v^2}}{3V} \quad (v^2 = v_x^2 + v_y^2 + v_z^2)
\end{aligned}
$$

Temperature, Pressure and Speed

Combine this with $pV = nRT$ and solving for v:

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3RT}{M}}$$

The root mean squared speed is defined as the square root of the average of the square of the speeds of all the molecules/atoms in the gas.

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Table 19-1 Some RMS Speeds at Room Temperature ($T = 300$ K)[a]

Gas	Molar Mass (10^{-3} kg/mol)	v_{rms} (m/s)
Hydrogen (H_2)	2.02	1920
Helium (He)	4.0	1370
Water vapor (H_2O)	18.0	645
Nitrogen (N_2)	28.0	517
Oxygen (O_2)	32.0	483
Carbon dioxide (CO_2)	44.0	412
Sulfur dioxide (SO_2)	64.1	342

[a]For convenience, we often set room temperature equal to 300 K even though (at 27°C or 81°F) that represents a fairly warm room.

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If we consider N atoms/molecules moving at a speed v_{rms}, the kinetic energy is then

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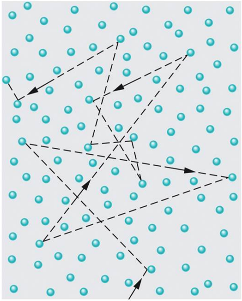
Each particle constitutes $\frac{3}{2}kT$ energy.

Temperature, Pressure and Speed

The average distance a gas particle travels before encountering another gas molecule is called the mean free path λ.

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 (N/V)}$$



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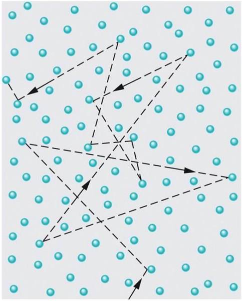

d is the size of the particle

Temperature, Pressure and Speed

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$$P(v) = 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}$$

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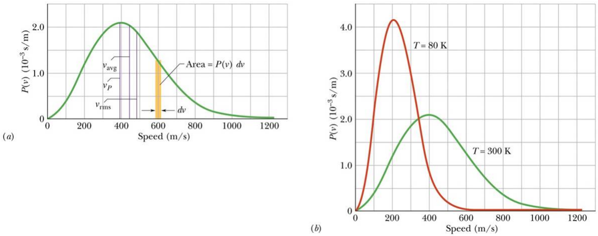
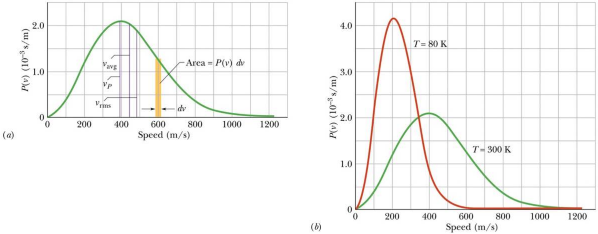


Temperature, Pressure and Speed

We integrate the function to find the probability (or fraction of particles) that have a particular **range of speeds**.

$$\int_{v_1}^{v_2} P(v)\, dv$$



Temperature, Pressure and Speed

$$P(v) = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}$$

- What fraction of particles have a speed of 300 m/s?
- What fraction of particles have a speed of 200 m/s?
- What fraction of particles have a speed of 100 m/s?
- What is the most probable speed?

Temperature, Pressure and Speed

$$P(v) = 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}$$

- What fraction of particles have a speed of 300 m/s? 0
- What fraction of particles have a speed of 200 m/s? 0
- What fraction of particles have a speed of 100 m/s? 0
- What is the most probable speed? $\approx$ 390 m/s

Temperature, Pressure and Speed

remember: $\bar{x} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} = \frac{\sum x_i m_i}{M} \rightarrow \frac{1}{M} \int x \lambda dx$

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There are three important speeds associated with a gas.

- Average: $v_{avg} = \int v P(v) dv = \sqrt{\frac{8RT}{\pi M}}$

Temperature, Pressure and Speed

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- Average: $v_{avg} = \int vP(v)dv = \sqrt{\frac{8RT}{\pi M}}$
- Root-mean-square: $v_{rms} = \sqrt{\int v^2 P(v)dv} = \sqrt{\frac{3RT}{M}}$
- Most probable: $v_{max} = \sqrt{\frac{2RT}{M}}$ (from $dP/dv = 0$)

Temperature, Pressure and Speed

Two identical, sealed containers are filled with the same number of moles of gas at the same temperature and pressure, one with helium gas and the other with neon gas.

(a) The speed of each of the helium atoms is the same value, but this speed is different than that of the neon atoms.

(b) The average kinetic energy of the neon atoms is greater than that of the helium atoms.

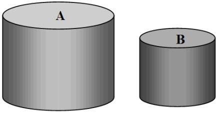
(c) The pressure within the container of helium is less than the pressure in the container of neon.

(d) The internal energy of the neon gas is greater than the internal energy of the helium gas.

(e) The rms speed of the neon atoms is less than that of the helium atoms.

Temperature, Pressure and Speed

Two sealed containers are at the same temperature and each contain the same number of moles of an ideal monatomic gas.



(a) The rms speed of the atoms in the gas is greater in B than in A.

(b) The frequency of collisions of the atoms with the walls of container B is greater than that for container A.

(c) The kinetic energy of the atoms in the gas is greater in B than in A.

(d) The pressure within container B is less than the pressure inside container A.

(e) The force that the atoms exert on the walls of container B is greater than in for those in container A.