Surface wind speed (kts) and streamlines

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PHYS 212
What happens when we put two charged objects near each other?

They exert forces on each other!

What causes this “action at a distance?”

We call this the **The Electric Field**.
Let us insert a “test charge” with charge \( q_0 \) to measure this mysterious electric field:

- The force on a charged particle has a direction and so does the electric field.
- The stronger the force, the stronger the electric field.
- \( \vec{F} \propto \vec{E} \)
- What is the proportionality constant?

\[
\vec{F} = q_0 \vec{E} \quad (1) \\
\vec{E} = \frac{\vec{F}}{q_0} \quad (2)
\]
Electric Field

Here is a table of electric field strengths:

<table>
<thead>
<tr>
<th>Field Location or Situation</th>
<th>Value (N/C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the surface of a uranium nucleus</td>
<td>(3 \times 10^{21})</td>
</tr>
<tr>
<td>Within a hydrogen atom, at a radius of (5.29 \times 10^{-11}) m</td>
<td>(5 \times 10^{11})</td>
</tr>
<tr>
<td>Electric breakdown occurs in air</td>
<td>(3 \times 10^{6})</td>
</tr>
<tr>
<td>Near the charged drum of a photocopier</td>
<td>(10^5)</td>
</tr>
<tr>
<td>Near a charged comb</td>
<td>(10^3)</td>
</tr>
<tr>
<td>In the lower atmosphere</td>
<td>(10^2)</td>
</tr>
<tr>
<td>Inside the copper wire of household circuits</td>
<td>(10^{-2})</td>
</tr>
</tbody>
</table>

\[ E = \frac{F}{q}, \] measured in N/C.
Lecture Question 5.1

A charged object sits at the origin, generating an electric field $\vec{E}_0$ a distance $d$ away. If the distance is doubled to $2d$, the electric field:

(a) stays the same;
(b) has the same magnitude but a different direction;
(c) drops to $E_0/2$;
(d) drops to $E_0/4$;
(e) increases to $2E_0$. 
Field lines try to describe a vector quantity (e.g., $\vec{v}$) that has a different magnitude and direction at every point in space:

(source: www.autospeed.com)
If we drop a charge into a field, it feels a force.

If I move the charge, it may experience a different force in a different direction.

There appears to be an invisible sea of electric field vectors, pushing charges around:
Electric Field Lines

Two more examples:
Electric Field Lines

Field Lines:
- Point away from positive charges (by definition)
- Point toward negative charges
- Closely packed: large E-field
- Loosely packed: small E-field
- Shows the direction of the force on a positive test charge


**Lecture Question 5.2**

(a) The electric field is due to a positively charged particle.
(b) The electric field is due to a negatively charged particle.
(c) The electric field is due to particles with opposite charges.
(d) The electric field is due to particles with the same charge.
E-Field from Point Charge

The force on a test charge $q_0$ from another charge $q$ is

$$
\vec{F} = k \frac{qq_0}{r^2} \hat{r}
$$

The E-field is just

$$
\vec{E} = \vec{F} / q_0 = k \frac{q}{r^2} \hat{r}
$$

[Think gravity: $F = G \frac{Mm}{r^2}$, but $F = ma$, so $a = G \frac{M}{r^2}$.

Remember, forces obey superposition—therefore, so do E-fields!

$$
\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n
$$
E-Field from Continuous Charges

So far, we have considered only 0-dimensional charges (points, no extent).

What about distributed charges?

<table>
<thead>
<tr>
<th>Dim</th>
<th>Name</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Charge</td>
<td>$q$</td>
<td>C</td>
</tr>
<tr>
<td>1</td>
<td>Linear charge density</td>
<td>$\lambda$</td>
<td>C/m</td>
</tr>
<tr>
<td>2</td>
<td>Surface charge density</td>
<td>$\sigma$</td>
<td>C/m$^2$</td>
</tr>
<tr>
<td>3</td>
<td>Volume charge density</td>
<td>$\rho$</td>
<td>C/m$^3$</td>
</tr>
</tbody>
</table>
Lecture Question 5.3

At the point P,

(a) the electric field points up.
(b) the electric field points down.
(c) the electric field points right.
(d) the electric field points left.
(e) none of the above.
We know that a test charge \( q_0 \) in an electric field experiences a force:
\[
\vec{F} = q_0 \vec{E} \tag{3}
\]
If we know the force, we can find the charge’s acceleration:
\[
\vec{F} = m\vec{a}.
\]

But, if we know \( \vec{a} \), we can determine the motion of the charged particle!
\[
x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \quad \text{(constant acceleration)} \tag{4}
\]
Charges in Electric Fields

A common example is the electric dipole: two equal but opposite charges $q$ spaced by a distance $d$. The dipole moment is defined to be

$$\vec{p} = q\vec{d} \text{ (points from - to + charge)}$$  \hspace{1cm} (5)

What happens when we put this dipole in a uniform electric field?
Charges in Electric Fields

What is the torque on the dipole?

\[ \tau_{net} = \tau_1 + \tau_2 \]  
\[ = \frac{d}{2} F \sin(\theta) + \frac{d}{2} F \sin(\theta) \]  
\[ = dF \sin(\theta) \]  
\[ = (dq)E \sin(\theta) \]  
\[ = pE \sin(\theta) \]

Or, more generally, \( \vec{\tau} = \vec{p} \times \vec{E} \).
Charges in Electric Fields

This torque tends to bring $\vec{p}$ into alignment with $\vec{E}$ (think of a pendulum).

The work-energy theorem says that $U = -W$ for a conservative force. Taking $\theta = 90^\circ$ as $U = 0$, we have

$$U_f - U_i = -W \quad \quad (11)$$

$$= - \int_{90^\circ}^{\theta} \tau d\theta' \quad \quad (12)$$

$$= - \int_{90^\circ}^{\theta} -pE \sin(\theta') d\theta' \quad \quad (13)$$

$$= -pE[\cos(\theta) - \cos(90^\circ)] \quad \quad (14)$$

$$= -pE \cos(\theta) \quad \quad (15)$$

$$U = -\vec{p} \cdot \vec{E} \quad \quad (16)$$

This is the potential energy in a dipole.