The term *inductance* was coined by Oliver Heaviside in February 1886.

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PHYS 212
Faraday’s Law of Induction

We have seen electric flux:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

But we can define the magnetic flux in the same way:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

▶ This is the flux through a loop of wire
▶ If $\vec{B}$ is uniform and perpendicular to the loop: $\Phi_B = BA$
▶ Magnetic flux has units of T m², also called the weber (Wb)
Faraday’s Law of Induction

What happens if I increase the flux through some loop?

- Current flows in the wire!
- The faster we change the flux, the bigger the current
- We induce an emf $\mathcal{E}$ in the loop:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$
Faraday’s Law of Induction

What if there are $N$ turns in my loop (solenoid)?

- Each turn has an induced emf, so we get:

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

- We can change $\Phi_B$ by
  - Increasing/decreasing the magnetic field $B$
  - Increasing/decreasing the area $A$
  - Changing the tilt between $\vec{B}$ and $\vec{A}$
Faraday’s Law of Induction

The induced emf in a loop is due to an electric field pushing charges around!

- Work done: \( W = q \mathcal{E} \)
- Also work done: \( W = \int \mathbf{F} \cdot d\mathbf{s} = q \int \mathbf{E} \cdot d\mathbf{s} \)
- Therefore: \( \mathcal{E} = \int \mathbf{E} \cdot d\mathbf{s} \)
- So Faraday’s Law becomes:

\[
\mathcal{E} = \int \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}
\]
Faraday’s Law of Induction

Lecture Question 13.1

The ring is rotated clockwise at a constant rate. Which graph best represents $\Phi_B$?
Lenz’s Law

So what’s with the negative sign?

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- The change in flux induces a current
- The induced current creates a magnetic field
- This induced magnetic field fights the change in flux
- There is an opposition to the change
Lenz’s Law

For an increasing $\vec{B}$ field

For a decreasing $\vec{B}$ field

(a)

(b)
Lenz’s Law

Lecture Question 13.2

A coil of wire approaches a long current at a constant speed.

(a) A current is induced in the coil.

(b) The rectangle will be pulled in the direction of the current in the wire.

(c) A magnetic force acts on the loop that pushes the loop into the page.

(d) There is no effect on the coil.
When we put a solenoid in a circuit, how does the circuit behave?

- If we pass a current through a solenoid, it produces a magnetic field, \( B = \mu_0ni \)
- The flux through the solenoid is \( \Phi_B = BA \)
- If the solenoid has \( N \) loops, the “total flux” through the whole solenoid is \( N\Phi_B \), called **magnetic flux linkage**
- The ratio of this total flux to the current is the inductance \( L \):
  \[
  L = \frac{N\Phi_B}{i}
  \]
- Compare to \( C \):
  \[
  C = \frac{q}{V}
  \]
The Inductor

For an ideal solenoid,

\[ L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{nl(\mu_0ni)A}{i} = \mu_0n^2lA \]

- The inductance has units of T-m²/A, which we call a henry (H).

- Let’s apply Faraday’s Law to an inductor:

\[ \mathcal{E} = -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt} = -L\frac{di}{dt} \]

- Apparently, an inductor produces its own emf

- This is called **self inductance**
The Inductor

In a circuit,

- an inductor acts similarly to a battery, producing an emf \( \mathcal{E}(t) = -L \frac{di}{dt} \);
- we include it in the loop rule.

\[
\mathcal{E} - iR - L \frac{di}{dt} = 0
\]

\[
\frac{di}{dt} + \frac{R}{L} i = \frac{\mathcal{E}}{L}
\]

- What is \( i \)?
The Inductor

Charging up the current:

- The solution to
  \[
  \frac{di}{dt} + \frac{R}{L} i = \frac{\mathcal{E}}{L}
  \]
  
is
  \[
  i(t) = \frac{\mathcal{E}}{R} \left(1 - e^{-t/\tau_L}\right)
  \]
  
- \(\tau_L = \frac{L}{R}\).

- Compare to the RC circuit:
  \[
  V = \mathcal{E} \left(1 - e^{-t/\tau}\right)
  \]
Dis-charging the current:

- The solution to
  \[
  \frac{di}{dt} + \frac{R}{L}i = 0
  \]
  is
  \[
  i(t) = \frac{\mathcal{E}}{R} e^{-t/\tau_L}
  \]

- \(\tau_L = L/R\).

- Compare to the RC circuit:
  \[
  V = \mathcal{E} e^{-t/\tau}
  \]
The Inductor

Key points:

▷ The inductor is slow to react
▷ Initially, the current is zero
▷ Eventually, it provides no resistance
▷ This process is exponential with $\tau = L/R$
Consider a loop of wire being pushed at a speed $v$:

- An emf is induced in the loop
  \[ E = d\Phi_B/dt = d(BLx)/dt = BLdx/dt = BLv \]
- The resulting current is $i = E/R = BLv/R$
- The resulting force is $F = iLB \sin(90^\circ) = B^2L^2v/R$
- Power is just $P = Fv$, so
  \[ P = \frac{B^2L^2v^2}{R} \]
In fact, even if it’s just a slab of metal, this emf still generates currents:

- These are called eddy-currents
- The eddy currents cause a force to oppose the motion
Power and Mutual Induction

What if we put two coils next to each other?

- The magnetic field from coil 1 changes the flux through coil 2 $\Phi_{21}$
- This induces a current in coil 2
- This is called **mutual inductance**:

$$M = \frac{N_2 \Phi_{21}}{i_1} = \frac{N_1 \Phi_{12}}{i_2}$$
Power and Mutual Induction

The mutual inductance

\[ M = \frac{N_2 \Phi_{21}}{i_1} = \frac{N_1 \Phi_{12}}{i_2} \]

gives rise to an induced emf:

\[ \mathcal{E}_2 = -M \frac{di_1}{dt} \]

\[ \mathcal{E}_1 = -M \frac{di_2}{dt} \]
Lecture Question 13.4

In each of the three cases shown, a time-varying current is flowing through the larger coil that produces a magnetic field. Which orientation has the largest mutual inductance?

(a) Case A  
(b) Case B  
(c) Case C  
(d) All the same  
(e) Not enough information